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A NUMERICAL APPROACH TO THE FOREST IMPACT PROBLEM

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Abstract. *We propose a numerical method for the computer simulation of the forest impact on aquifers. By this phenomenon it is mean the raising or the lowering of the groundwater table under the areas felled or recovered by the trees. The mathematical model of the forest impact includes a boundary-value problem with free and contact boundary conditions. Numerical results for an illustrative 2D test problem are discussed. The test shows that even in our model of forest impact, that takes into account only some principal characteristics of this phenomenon, the water table lowering owing to the forest suction is significative enough to be considered as an effective means for control of groundwater.*

1 INTRODUCTION

By the forest impact on aquifers we meant the effect of raising or lowering of the groundwater table under the areas felled or recovered by the trees, see *Fig. 1.* To study this phenomenon the use of experimental methods is common[1,2,3]. The experiments consist in real scaled monitoring of the water table response under a forest area and can take many years. To predict the groundwater level reduction, water balance models are applied[1].

In this paper we propose a numerical method to computer simulation of the forest impact on aquifers. Assuming the hydromechanical point of view we treat this phenomenon as a problem of unconfined flow in porous media with possible fluid discharge (evaporation) through the water table owing to the tree roots suction. The location of the water table under the forest suction effect, the flow characteristics as well as the region of the contact of the aquifer with the tree roots system are the unknowns of this problem.

The two dimensional model of the forest impact phenomenon considered here includes a boundary-value problem with free boundary and contact conditions[4]. This model furnishes all the components necessary to numerical simulation of the forest impact. Having the information about the forest area (location, depth of the tree roots system, trees plantation density and intensity of the species evaporation) and hydrologic data of the region, the behavior of the water table under the controlled area can be simulated for different kind of the natural recharge and the optimal location of the plantation can be defined in order to minimize (maximize) the forest effect at the controlled area.

The phenomenon of unconfined steady flow through porous media belongs to the category of free boundary value problems. The problem is defined over the domain a part of the contour of which, called free boundary, is unknown *a priori* and can be found as a component of the solution.

Amongst the methods to solve the unconfined steady flow problems one can point out analytical, iterative and transformations methods. The analytical solution can be obtained with the theory of analytical functions for linear ordinary differential equations[5]. In the iterative method, one of two conditions defined at the free boundary is chosen to solve at each iteration the direct value problem. Guessing an initial approximation, the location of the free boundary is adjusted at each iteration to make the other boundary condition hold, then the direct problem is re-solved, etc[6]. The method of transformation, known as "Baiocchi's transformation", consists in changing the problem variable to transform the free boundary domain into a fixed domain[7]. The problem on this new domain takes on the appearance of a variational inequality. All this method were proposed to solve the classical seepage problem, i.e. the problem of unconfined steady flow without any infiltration (or vaporization) effect on the water table. Unlikely even for the problem with a prescribed infiltration zone the analytical solution can be obtained only for some particular cases[8]. As for the Baiocchi's like transformation applied to the forest impact problem, its leads us to a quasivariational inequality[4] that is not easy to be implemented numerically.

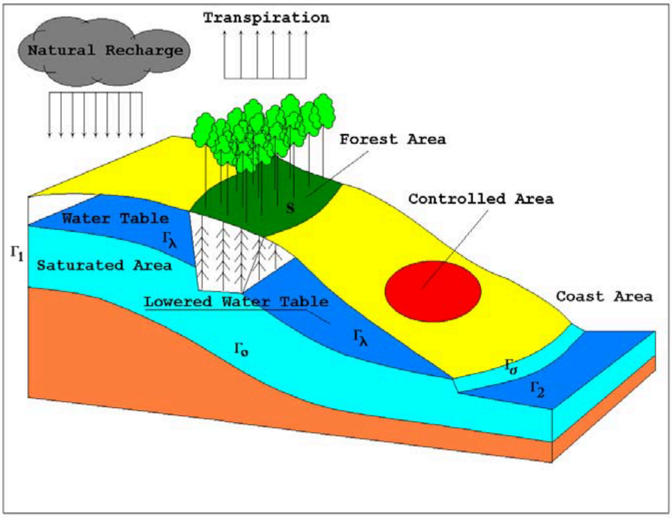

Figure 1: Water table and tree roots system interaction scheme

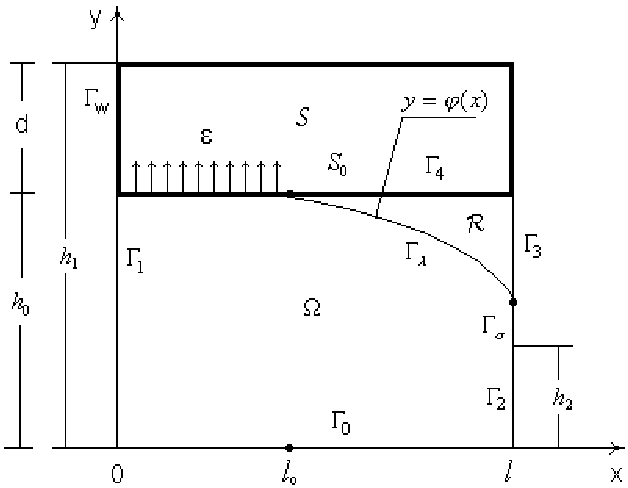

Figure 2: Unconfined fluid flow with suction

Our technique to solve the forest impact problem is based on the shape optimization approach. We transform our free-contact boundary problem into a least squares like shape optimization problem. The objective functional contains one of the free boundary conditions, whereas the state equation together with the rest of the boundary conditions become the problem constraints. It is sought for the minimum of objective with respect to the shape of water table. This approach for classical problem was used with finite elements descretization[9].

In most cases the methods to solve numerically the shape optimization problem use an optimality system. The state variable is considered as a function of the design variable that defines the shape of domain. This involves the following procedure, known as shape sensitivity analysis[10]: for a fixed shape of the domain at each iteration we calculate the derivative of the state variable with respect to design variable.

The method we use in this paper for solving the shape optimization problem is based on the mathematical programming technique[11]. In contrast to methods involving sensitivity analysis, we consider the state of the system and the design variable as independent variables. Performing the boundary elements discretization, we get a mathematical programming problem: find the minimum of a objective function subject to some equality and (or) inequality constraints. The equality constraints arise from the discretization of the state equation and define the relationship between design and state variables. To solve this nonlinear mathematical program we use Herskovits' interior point algorithm[12]. The numerical example of the forest impact problem is shown and compared with different situations including the classical seepage problem.

2 FOREST IMPACT PROBLEM FORMULATION

The difference between the forest impact problem[4] and the classical seepage problem[5] is in the possibility of the flow flux through the water table, which can appear when the aquifer attains the tree roots system. Let $\mathcal{R}$ be an open and, for the sake of convenience, rectangular domain occupied by the porous media and $\mathcal{S}$ the tree roots system of the deepness $d > 0$, see *Fig. 2*. The fluid is assumed to be ideal, the porous media is homogeneous and isotropic with the permeability coefficient equal to 1. We suppose that at the part of the water table that reaches the tree roots system bottom $S_\circ$ there is the suction flux with given rate $\varepsilon(x)$. The left wall Γ_w of $\mathcal{S}$ is assumed impermeable. The contact area between the aquifer and tree roots system is *a priori* unknown and can be defined together with the location of the rest of the water table Γ_λ, seepage Γ_σ and the velocity potential u in Ω. We suppose also that the function $\varphi(x)$ that defines the portion $\Gamma_\lambda \setminus S_\circ$ of the water table, *Fig. 2*, is decreasing and denote $h_\circ \equiv h_1 - d$.

For the forest impact problem we define at the parts $\Gamma_1, \Gamma_2, \Gamma_\circ$ and Γ_σ of the boundary

$\partial\Omega$ the same conditions as for the seepage problem[5], i.e.

$$\begin{cases} u = h_1 & on \quad \Gamma_1, \\ u = h_2 & on \quad \Gamma_2, \\ u = y & on \quad \Gamma_\sigma, \\ q = 0 & on \quad \Gamma_\circ. \end{cases}$$

The part of the water table that does not contact $\mathcal{S}$ remains to be the free boundary and we put here conditions $u = y$ and $q = 0$. When $\Gamma_\lambda \cap S_\circ \neq 0$ we have the flow with given rate $\varepsilon(x)$ through this part of the water table Γ_λ toward the interior of $\mathcal{S}$. Thus, we obtain the following mathematical formulation for the forest impact problem[4]:

$$(\mathcal{P}) \quad \begin{cases} \textit{Find } \varphi(x) \textit{and } u(x,y) \textit{ such that:} \\ \qquad \begin{cases} \Delta u = 0 & in \quad \Omega, \\ u = h_1 & on \quad \Gamma_1, \\ u = h_2 & on \quad \Gamma_2, \\ u = y & on \quad \Gamma_\sigma \cup (\Gamma_\lambda \setminus S_\circ), \\ q = 0 & on \quad \Gamma_\circ \cup (\Gamma_\lambda \setminus S_\circ), \\ q = -\varepsilon(x) & on \quad \Gamma_\lambda \cap S_\circ, \end{cases} \end{cases}$$

where $q \equiv \partial u / \partial n$ and n is the outward normal to $\Gamma_\circ \cup \Gamma_\lambda$.

At the water table we have conditions that take the form of free or contact boundary conditions. We call its "free-contact" boundary conditions. While for the classical seepage problem there exists an equivalent formulation as a variational inequality, that can be used to solve its numerically, we have a quasivariational inequality in the case of the forest impact problem[4].

In fact, let us consider the transformation:

$$w(x,y) = \int_y^{\psi(x)} (u(x,t) - t)dt + w_\circ(x) \text{ in } \Omega, \tag{1}$$

where $\psi(x)$ a function that describe the whole water table Γ_λ and the function $w_\circ(x)$ is defined in the following form:

$$\begin{gathered} w_\circ \in C^1[0,l], \; w_\circ(0) = d^2/2, \;\; w_\circ(l) = 0, \\ w''_\circ(x) = -\varepsilon(x) \;\text{ on } [0,l_\circ], \; w''_\circ(x) = 0 \text{ on } (l_\circ, l]. \end{gathered} \tag{2}$$

Here the interval $[0, l_\circ)$ corresponds to the contact part of the water table and $(l_\circ, l]$ to the free one. Let $g(x,y)$ be a function of class $C^1(\overline{\mathcal{R}})$ such that $g = w$ on $\partial\mathcal{R}$ and $\mathcal{K}$ a nonempty, convex and closed subset of $H^1(\mathcal{R})$:

$$\mathcal{K} = \{v \in H^1(R) \mid v \geq w^\circ \text{ in } \mathcal{R} \;\text{ and }\; v = g \text{ on } \partial\mathcal{R}\}. \tag{3}$$

Then, we have the following result:

Theorem 1. Let $\{\varphi, u\}$ be a solution of problem $(\mathcal{P})$, $\varphi(x)$ is smooth, $u \in H^1(\Omega) \cap C^\circ(\overline{\Omega})$, w is given by formula (1), $w_\circ(x)$ is defined by conditions (2), $w^\circ(x,y) \equiv w_\circ(x)$ for $(x,y) \in \mathcal{R}$ and

$$w(x,y) = \begin{cases} w(x,y), & (x,y) \in \Omega, \\ w^\circ(x,y), & (x,y) \in \mathcal{R} \setminus \Omega. \end{cases}$$

Then w satisfies:

$$w \in \mathcal{K}, \quad \int_R (w_x(v-w)_x + w_y(v-w)_y)dxdy \geq -\int_R (v-w)dxdy, \quad \forall v \in \mathcal{K}, \tag{4}$$

where $\mathcal{K}$ is defined by (3). □

By the definition of function w°, the subset $\mathcal{K}$ depends implicitly on the flow through the contact part of Γ_λ. This part is unknown *a priori* and is defined by the function w. Hence, inequality (4) is a quasivariational one. The next theorem shows that if the solution w of quasivariational inequality (4) exists then the function $u = y - w_y$ together with the curve $\varphi(x)$ that separates two regions of $\mathcal{R}$ where $w = w^\circ$ and $w > w^\circ$, satisfy problem $(\mathcal{P})$. Let $\varphi(x)$ be defined as

$$\begin{aligned} &\varphi(x) = \inf\{y \mid (x,y) \in \mathcal{R} \setminus \Omega\} \text{ for } l_\circ < x < l, \\ &\varphi(l_\circ) = \lim_{x \to l_\circ^+} \varphi(x), \quad \varphi(l) = \lim_{x \to l^-} \varphi(x). \end{aligned} \tag{5}$$

Theorem 2. Let $w \in W^{2,p}(\mathcal{R}) \cap C^1(\overline{\mathcal{R}})$ with $1 \leq p < \infty$ be a solution of quasivariational inequality (4). Let be $\Omega = \{(x,y) \in R \mid w(x,y) > w^\circ(x,y)\}$, $u := y - w_y$ in Ω and vertical discharge corresponding to the velocity potential u in Ω is non negative. Assume $\varepsilon'(x) \geq 0$ and define $\varphi(x)$ by formula (5). Then the pair $\{u, \varphi\}$ is the solution of problem $(\mathcal{P})$. □

3 AN EQUIVALENT SHAPE OPTIMIZATION PROBLEM

An equivalent formulation of $(\mathcal{P})$ can be given in terms of shape optimization for the system governed by the Laplace equation. Let Φ be a set of all feasible shapes of the water table, formed by smooth curves. The optimization problem consists in finding

$\psi \in \Phi$ and u such that:

$$
(\mathcal{P}_1) \quad \left\{ \begin{array}{l}
\min\limits_{\psi\in\Phi} \displaystyle\int (q)^2_{\Gamma_\lambda \setminus S_\circ} \\
\textit{where } \; q = \partial u/\partial n \; \textit{and} \; u(x,y) \; \textit{is a solution of problem:} \\
\qquad\qquad \left\{ \begin{array}{lll}
\Delta u = 0 & in & \Omega, \\
u = h_1 & on & \Gamma_1, \\
u = h_2 & on & \Gamma_2, \\
u = y & on & \Gamma_\sigma \cup (\Gamma_\lambda \setminus S_\circ), \\
q = 0 & on & \Gamma_\circ, \\
q = -\varepsilon(x) & on & \Gamma_\lambda \cap S_\circ,
\end{array} \right.
\end{array} \right. \tag{6}
$$

The objective functional contains the square of the flux along the free part of the water table. The choice of the optimal water table location forces the objective to be zero and vice versa. Another alternative optimization formulation uses as optimality criterion the condition on the potential along $\Gamma_\lambda \setminus S_\circ$, see Leontiev & Huacasi[11].

This shape optimization formulation of problem $(\mathcal{P})$ interprets the water table Γ_λ as an optimal boundary. The concept of the optimal boundary includes the values at the contour of the domain only. Thus, it is not necessary to solve the problem in the whole domain Ω to find the optimal boundary. On the other hand, finding Γ_λ, we can obtain $u(x,y)$ in Ω solving value-boundary problem (1). For this reason, below we will be looking for the location of the water table only.

In two-dimensional case for the problem governed by the Laplace equation the values of flux and potential verify on the frontier $\Gamma \equiv \partial\Omega$ the integral equation[13]:

$$
0.5u(\xi) + \int_\Gamma q^*(\xi,\chi)u(\chi)d\Gamma = \int_\Gamma u^*(\xi,\chi)q(\chi)d\Gamma \tag{7}
$$

where $\chi \equiv (x,y) \in \Gamma$, $u^*(\xi,\chi)$ is the fundamental solution of the Laplace equation, $q^*(\xi,\chi)$ its normal derivative, and $\xi \in \Gamma$ is the collocation point.

In this way, to define the location of the water table we have the problem:

$$
(\mathcal{P}_2) \quad \left\{ \begin{array}{l}
\min\limits_{\psi\in\Phi} F(u,q), \\
\textit{where } \; q \textit{ and } u \textit{ verify at } \Gamma \textit{ the integral equation:} \\
0.5u(\xi) + \displaystyle\int_\Gamma q^*(\xi,\chi)u(\chi)d\Gamma = \int_\Gamma u^*(\xi,\chi)q(\chi)d\Gamma
\end{array} \right.
$$

where $F(u,q) = \displaystyle\int (q)^2_{\Gamma_\lambda \setminus S_\circ}$ and the boundary values are defined as in the problems (6).

4 DISCRETIZATION AND MATHEMATICAL PROGRAM

The formulation $(\mathcal{P}_2)$ furnishes an opportunity to use the boundary elements discretization. We introduce E (geometrical) nodes and divide Γ into E elements $\Gamma = \sum_{j=1}^{E} \Gamma_j$. Considering constant functional approximation of the flux and the potential for each $\Gamma_j,\ j = 1, ..., E$, we perform the following discretization of the integral equation (7):

$$0.5u_i + \sum_{j=1}^{E} \left(\int_{\Gamma_j} q_i^* d\Gamma_j \right) u_j = \sum_{j=1}^{E} \left(\int_{\Gamma_j} u_i^* d\Gamma_j \right) q_j, \quad i = 1, ..., E,$$

where $u_i = u(\xi_i),\ u_i^* = u^*(\xi_i, \chi),\ q_i^* = q^*(\xi_i, \chi),\ \xi_i \in \Gamma_i$ and $u(\chi) \equiv u_j,\ q(\chi) \equiv q_j$, $\chi \in \Gamma_j,\ j = 1, ..., E$. Using notations $H_{ij} = \int_{\Gamma_j} q_i^* d\Gamma_j$ for $i \neq j$, $H_{ii} = 0.5$ and $G_{ij} = \int_{\Gamma_j} u_i^* d\Gamma_j$, we can rewrite this equation in the matrix form:

$$[H]u = [G]q.$$

Let (x_i, y_i) be the coordinates of the nodes $i = 1, ..., E$ and $x_{E+1} = x_1,\ y_{E+1} = y_1$. Then, we can obtain explicit formulas for the coefficients of G and H:

$\forall i, j = 1, ..., E, \qquad i \neq j :$

$$G_{ij} = -\sum_{k=1}^{4} 0.5\omega_k \left(a_x^2 + a_y^2\right)^{1/2} \ln\left((x_c - a_x\gamma_k - b_x)^2 + (y_c - a_y\gamma_k - b_y)^2\right), \tag{8}$$

$$H_{ij} = -\sum_{k=1}^{4} \frac{\omega_k \Big(a_y(a_x\gamma_k + b_x - x_c) - a_x(a_y\gamma_k + b_y - y_c) \Big)}{\Big(x_c - a_x\gamma_k - b_x\Big)^2 + \Big(y_c - a_y\gamma_k - b_y\Big)^2}, \tag{9}$$

$\forall i, j = 1, ..., E, \qquad i = j :$

$$G_{ii} = 2\Big(a_x^2 + a_y^2\Big)\Big(1 - \ln\Big(a_x^2 + a_y^2\Big)^{1/2}\Big), \tag{10}$$

$$H_{ii} = \pi, \tag{11}$$

where $a_x = 0.5\Big(x_{j+1} - x_j\Big)$, $b_x = 0.5\Big(x_{j+1} + x_j\Big)$, $a_y = 0.5\Big(y_{j+1} - y_j\Big)$, $b_y = 0.5\Big(y_{j+1} + y_j\Big)$,

$x_c = 0.5\Big(x_i + x_{i+1}\Big)$, $y_c = 0.5\Big(y_i + y_{i+1}\Big)$, and γ_k, ω_k are the abscissa and weight of the Gauss quadrature.

Let n, m, l, r and k be the numbers of the boundary elements located at the segments $\Gamma_\sigma, \Gamma_\lambda, \Gamma_1, \Gamma_\circ$, and Γ_2, respectively. We put m_1 elements at the part $\Gamma_\lambda \setminus S_\circ$ and m_2 elements at the contact part $\Gamma \cup S_\circ$ of the water table, that is $m_1 + m_2 = m$. We assume that the x-coordinates of the nodes at $\Gamma_\lambda \setminus S_\circ$ and the y-coordinates of the nodes at $\Gamma \cup S_\circ$ are fixed. Thus only the y-coordinates define the location of the nodes belonging to seepage and free part of the water table and x-coordinates define the nodes of the contact part of the water table. We also use the following notations: $L \equiv l$, $R \equiv L + r$, $K \equiv R + k$, $N \equiv K + n$, hence $N + m \equiv E$.

For the discrete analog of problem $(\mathcal{P}_2)$ we consider as independent variables the flux at the boundary elements of Γ_1: $X_1 ... X_L$, the potential at the boundary elements of $\Gamma_\circ$: $X_{L+1} ... X_R$, the flux at the boundary elements of Γ_2, Γ_σ and $\Gamma_\lambda \setminus S_\circ$: $X_{R+1} ... X_K$, $X_{K+1} ... X_N$ and $X_{N+1} ... X_{E-m_2}$ respectively, the the potential at the boundary elements of $\Gamma_\lambda \cup S_\circ$: $X_{E-m_2+1} ... X_E$, y-coordinates of the seepage surface nodes (if $n \geq 2$): $X_{E+1} ... X_{E+n-1}$, y- and x-coordinates of the water table nodes: $X_{E+n} ... X_{E+n+m_1-1}$ and $X_{E+n+m_1} ... X_{E+n+m-1}$ respectively.

Let be

$$
\begin{aligned}
&X = (X_1 ... X_{E+n+m-1}),\\
&U = (u_1 ... u_L, X_{L+1} ... X_R, u_{R+1} ... u_K, U_{K+1} ... U_N, U_{N+1} ... U_{E-m_2}, X_{E-m_2+1} ... X_E),\\
&Q = (X_1 ... X_L, q_{L+1} ... q_R, X_{R+1} ... X_K, X_{K+1} ... X_N, X_{N+1} ... X_{E-m_2}, q_{E-m_2+1} ... q_E),
\end{aligned}
$$

where the values of potential at the segments Γ_σ and $\Gamma_\lambda \setminus S_\circ$ are defined corresponding to the boundary conditions of problem (6):

$$
U_{K+i} = 0.5(X_{E+i} + X_{E+i-1}),\ i = 2, ..., n + m_1 - 2,
$$

$$
U_{K+1} = 0.5(X_{E+1} + h_2),\ U_{E-m_2} = 0.5(h_\circ + X_{E+n+m_1-1}),
$$

as well as the remaining values of u and q.

If there are nodes at the seepage surface (what means $n \geq 2$), the following linear constraints for the y-coordinates of these nodes appear: $X_i - X_{i+1} \leq 0,\ i = E+1, ..., E+n-1$. The restrictions of the same kind we have for the x-coordinates of the nodes at the contact part of the water table if $m_2 \geq 2$: $X_i - X_{i-1} \leq 0,\ i = E+n+m_1+1, ..., E+n+m-1$.

The objective function is $F(X) = \sum\limits_{i=N+1}^{E-m_2} X_i^2$. Following formulas (8)-(11), the coefficients of H and G are functions of X, more precisely, of the y-coordinates of seepage and water table free part nodes and of the x-coordinates of the water table contact part nodes:

$$
H(X) \equiv H(X_{E+1} ... X_{E+n+m-1}),\ G(X) \equiv G(X_{E+1} ... X_{E+n+m-1}).
$$

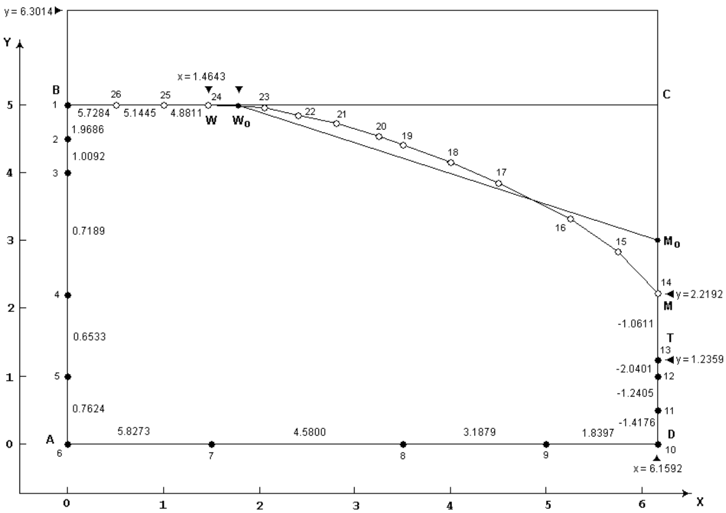

Figure 3: B.E.M. discretization

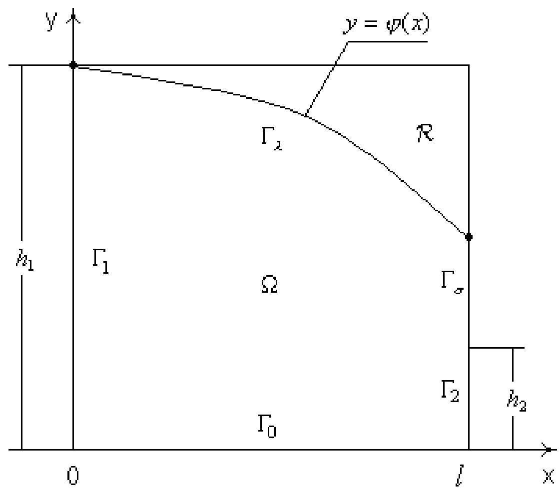

Figure 4: Unconfined fluid flow. Classical case

Performing this kind of discretization for problem $(\mathcal{P}_1)$, we obtain a nonlinear mathematical programming problem:

$$(\mathcal{P}_3) \quad \begin{cases} \min\limits_X F(X) \\ H(X)U - G(X)Q = 0, \\ X_i - X_{i+1} \leq 0, \; i = E+1, ..., E+n-1, \quad (\text{if } n \geq 2), \\ X_i - X_{i-1} \leq 0, \; i = E+n+m_1+1, ..., E+n+m-1, \quad (\text{if } m_2 \geq 2), \\ \text{LOW } \leq X_i \leq \text{ UP}, \; i = E+1, E+n, ..., E+n+m_1-1, \\ \text{LEFT } \leq X_{E+n+m-1}, \; X_{E+n+m_1} \leq \text{ RIGHT}. \end{cases}$$

The problem has $E+m+n-1$ variables, E nonlinear equality constraints, $n+m_2-2$ linear inequality constraints and m_1+1 "box" constrains, where LOW and UP define the limits for the unknown y-coordinates, and LEFT and RIGHT for the unknown x-coordinates of the water table nodes. We note that the flux and the potential $X_1...X_E$ at the boundary elements and the design variables $X_{E+1}...X_{E+n+m+1}$ are independent variables of the mathematical program $(\mathcal{P}_3)$. Thus, the objective function $F(X)$ is quadratic with respect to the problem variables.

To solve nonlinear mathematical program $(\mathcal{P}_3)$ we use Herskovits' interior point algorithm[12]. We find the y-coordinates of free part of the water table and seepage surface nodes as well as x-coordinates of the contact part of the water table and values of potential and flux at the corresponding segments of the boundary.

5 NUMERICAL TESTS

For the test problem we choose: $h_1 = 6.3014$, $h_2 = 1.2359$, $\ell = 6.1592$ and $d = 1.3014$ $(h_\circ = 5.0)$. This data is taken in order to compare the solution of the forest impact problem with the seepage one, considered in Leontiev & Huacasi[11]. The suction flux is taken as $\varepsilon = 1$.

The discretization includes 26 boundary elements ($l = 5$, $r = 4$, $k = 3$, $n = 1$, $m_1 = 10$, $m_2 = 3$), see *Fig. 3*. We are looking for the y-coordinates of ten nodes at the free part of the water table $W - M$ and the x-coordinates of three nodes at the contact part of the water table $B - W$. The position of the node 24 defines the location of the contact point of the water table (point W). The coordinates of the rest of the nodes are fixed. The water table initial position, used at the first iteration of the algorithm, is given by the line $B - W_\circ - M_\circ$ in *Fig.3*. For the "box" constraints we take: $UP = h_\circ$, $LOW = h_2$, $LEFT = 0.0$, $RIGHT = 2.0$.

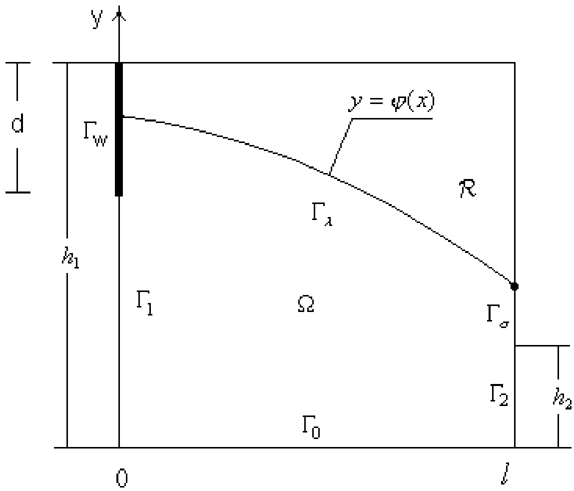

Figure 5: Unconfined fluid flow with vertical wall

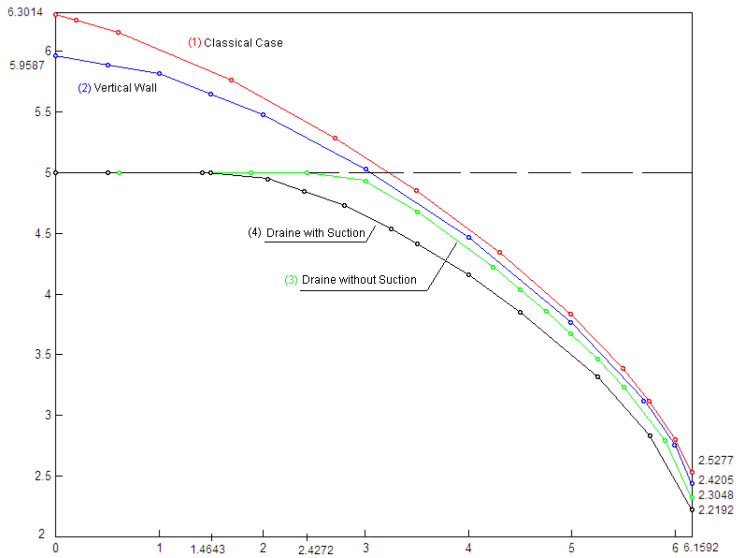

Figure 6: Water table location. Numerical results

Table 1: WATER TABLE COORDINATES AND BOUNDARY VALUES

node	x *fixed value	y *fixed value	q *value of u
14	6.1592*	2.2192	-1.17240E-07
15	5.7500*	2.8368	-2.82183E-08
16	5.2500*	3.3175	2.23091E-07
17	4.5000*	3.8532	5.45910E-07
18	4.0000*	4.1583	-1.23321E-07
19	3.5000*	4.4158	-2.90582E-07
20	3.2500*	4.5345	2.36618E-07
21	2.8000*	4.7249	-2.24024E-07
22	2.4000*	4.8467	-2.45834E-07
23	2.0500*	4.9553	3.35504E-07
24	1.4643	5.0000*	4.88110*
25	0.9270	5.0000*	5.14453*
26	0.4977	5.0000*	5.72835*

Table 2: ITERATIVE HISTORY

ITER.	$(q)^2_{\Gamma_\lambda \setminus S_\circ}$	EQUALITY CNSTR.
1	1.02936E-00	3.12027E-00
2	1.14977E-00	2.35687E-00
3	8.44161E-01	1.71197E-00
4	6.31515E-01	7.37943E-01
5	5.66052E-01	2.76270E-01
6	3.66079E-01	1.96013E-01
7	2.59572E-01	8.91051E-02
8	1.18700E-01	4.82199E-02
9	7.08363E-03	2.72489E-02
10	3.35875E-03	7.91583E-03
11	3.72563E-03	2.48889E-03
12	5.91536E-04	1.15717E-03
13	5.95424E-05	2.52348E-04
14	3.31008E-06	7.45031E-05
15	2.94575E-07	2.25893E-05
16	5.04176E-09	6.79022E-06
17	2.09812E-10	8.67709E-07
18	2.63430E-11	8.59921E-09
19	6.91086E-13	4.60299E-10

The mathematical program have 39 variables, 26 nonlinear equality constraints, 12 "box" constraints and 2 linear inequality constrains. We adopt the algorithm stopping criterion with precision 10E−6 (see Herskovits[12] for details). With the different initial data, the convergence of the algorithm was obtained in no more than 20 iterations.

The coordinates of the water table nodes and the value of flux and potential calculated at the corresponding boundary elements of the water table are given in Table 1. In this Table the first column indicates the node number, second and third present x and y coordinates of the water table nodes obtained numerically, fourth column shows the value of the flux (potential) calculated at the corresponding boundary elements.Table 2 shows the history of iterations: first column gives the number of iteration, second shows the objective function value, third presents the maximal error in the equality constrains that corresponds to the residual error of the discrete boundary integral equation. *Fig. 3* shows the location of the water table (continuous line $B - W - M$) and corresponding nodes (14-26) positions calculated numerically as well as boundary data, i.e. flux at the segments $A - B$, $D - T$ and $T - M$ and potential for the segments $A - D$ and $B - W$.

We compare the location of the water table in the forest impact problem with the solution of another unconfined problems, considered for the same geometrical and piezometric parameters. The first one is the classical seepage problem, *Fig. 4*. We consider also the situation then only the vertical impermeable wall Γ_w is present, *Fig. 5*. Finally, we solve the forest impact problem assuming that the bottom $S_\circ$ is impermeable i.e. the suction rate $\varepsilon = 0$. The results are presented in *Fig. 6*. Here line (1) defines the location of the water table for the classical seepage problem, line (2) gives the location of the water table for the unconfined problem with vertical impermeable wall, line (3) is the water table in the case of impermeable bottom $S_\circ$, line (4) is the solution of the forest impact problem with constant suction rate $\varepsilon = 1$. We can observe in these examples that the forest suction provides sufficiently lowering of the groundwater table.

6 CONCLUSIONS

The approach proposed in this paper for numerical simulation of the unconfined steady flow in porous media with possible discharge through the water table combines three principal aspects: the original problem transformation to a shape optimization problem, boundary elements discretization and mathematical programming technique to solve the discrete problem. The state and free boundary variables are considered as independent variables of the mathematical program. Thus, there is no need to perform any kind of sensitivity analysis and the objective function we have is quadratic with respect to the problem variables. The method is simple to be applied and can be used for 3D problems. The numerical simulation shows that even for our model of forest impact on aquifers, that takes into account only some principal characteristics of this phenomenon, the water table lowering owing to the forest suction is significative enough to be considered as an effective means for the control of groundwater.

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