Asociación Argentina de Mecánica Computacional

Mecánica Computacional Vol XXVI, pp.1045-1054
Sergio A. Elaskar, Elvio A. Pilotta, Germán A. Torres (Eds.)
Córdoba, Argentina, Octubre 2007

MODELING THE COLLAPSE OF A LIQUID COLUMN OVER AN OBSTACLE

Marcela A. Cruchaga[a], Ivodinko P. Prado[a], Diego J. Celentano[b] and Tayfun E. Tezduyar[c]

[a]*Departamento de Ingeniería Mecánica, Universidad de Santiago de Chile - USACH, Av. Bdo. O´Higgins 3363, Santiago, Chile, e-mail: mcruchag@lauca.usach.cl*
[b]*Departamento de Ingeniería Mecánica y Metalúrgica, Pontificia Universidad Católica de Chile, Av. Vicuña Mackenna 4860, Santiago, Chile*
[c]*Team for Advanced Flow Simulation and Modeling (T*AFSM), Mechanical Engineering, Rice University - MS 321, Houston, TX 77005, USA*

Keywords: moving interfaces, two-fluid flows, computational fluid mechanics, ETILT, experimental validation.

Abstract. This work presents the numerical and experimental analyses of the collapse of a water column over an obstacle. The physical model consists of a water column initially confined by a closed gate inside a glass box. An obstacle is positioned between the gate and the right wall of the box (inside the initially unfilled zone). Once the gate is opened, the liquid spreads in the container and moves over the obstacle. Measurements of the liquid height along the walls and a middle control section are obtained from videos. The simulations are performed using a moving interface technique, namely the Edge-Tracked Interface Locator Technique (ETILT) to describe the interface motion, and some physical aspects are considered via the model reported in Cruchaga et al., *Comp. Mech.*, 39:453-476 (2007). The analysis involves column aspect ratio of 2 with different obstacle geometries. The numerical predictions agree reasonably well with the experimental trends.

1 INTRODUCTION

Free-surface flow experiments at laboratory scales are commonly used to represent real problems and they are useful to assess the performance of modeling by comparing the numerical predictions with measurements. Several numerical techniques capable of accurately representing the evolution of an interface are proposed in the literature (e.g., see Tezduyar, 1992; Koshizuka and Oka, 1996; Tezduyar, 1998; Osher and Fedkiw, 2001; Sethian, 2001; Idelsohn et al. 2003; Cueto-Felgueroso et al., 2004; Greaves, 2004; Kohno and Tanahashi, 2004; Kulasegaram et al., 2004; and references therein). Focusing on the description of the collapse of a water column, experiments were presented in Martin and Moyce (1952); Koshizuka and Oka (1996). This problem was adopted as a benchmark test to validate the numerical performance of the proposed free-surface flow formulations (e.g., see Koshizuka and Oka, 1996; Idelsohn et al. 2003; Cueto-Felgueroso et al., 2004; Greaves, 2004; Kohno and Tanahashi, 2004; Kulasegaram et al., 2004). In particular, we present in Cruchaga et al. (2007) a set of experiments and their corresponding simulation using a fixed mesh finite element technique. The interface is "captured" with the ETILT, introduced by Tezduyar (2001), using the version described in Cruchaga et al. (2005).

In this paper, we report results from experiments for the collapse of a water column of aspect ratio (height to width ratio of the initial liquid column) of 2 over different obstacle geometries. The experimental data is used for evaluating the performance of the numerical strategy presented in Cruchaga et al. (2007) to model this problem. In particular, the parameters involved in the referred formulation, originally setting by numerical trial in the case of collapse of water column without obstacles, are also applied in the present analysis to evaluate their independency with geometry.

The governing equations are presented in Section 2. The ETILT and the new aspects included in its current version are summarized in Section 3. The details of the experimental procedure are described in Section 4. Experimental and numerical results are presented and discussed in Section 5. Concluding remarks are given in Section 6.

2 GOVERNING EQUATIONS

The Navier-Stokes equations of unsteady incompressible flows are written as follows:

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho\, \mathbf{u} \nabla \cdot \mathbf{u} + \nabla p - \nabla \cdot (2\mu \boldsymbol{\varepsilon}) = \rho\, \mathbf{f} \qquad in\ \Omega \times \mathrm{Y} \qquad (1)$$

$$\nabla \cdot \mathbf{u} = 0 \qquad in\ \Omega \times \mathrm{Y} \qquad (2)$$

where ρ, $\mathbf{u}$, p, μ, ε and $\mathbf{f}$ are the density, velocity, pressure, dynamic viscosity, strain-rate tensor, and the specific body force. In these equations, Ω denotes an open-bounded domain with a smooth boundary Γ, and Y is the time interval of interest. This system of equations is completed with a set of initial and boundary conditions:

$$\mathbf{u} = \mathbf{u}_0 \qquad in\ \Omega \qquad (3)$$

$$\mathbf{u} = \mathbf{g} \qquad in\ \Gamma_g \times \mathrm{Y} \qquad (4)$$

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \boldsymbol{h} \qquad in\ \Gamma_h \times \mathrm{Y} \qquad (5)$$

where $\mathbf{u_0}$ is the initial value of the velocity field, $\mathbf{g}$ represents the velocity imposed on the part of the boundary Γ_g, and $\boldsymbol{h}$ is the traction vector imposed over Γ_h ($\Gamma_g \cup \Gamma_h = \Gamma$ and $\Gamma_g \cap \Gamma_h = \varnothing$), typically taken as traction-free condition: $\boldsymbol{h} = \mathbf{0}$.

In the present simulation, a simple model to compute the energy dissipated by turbulent effects could be obtained by modifying μ as:

$$\mu = min(\mu + l_{mix}^2 \rho \sqrt{2\,\boldsymbol{\varepsilon}:\boldsymbol{\varepsilon}}; \mu_{max}) \tag{6}$$

where l_{mix} is a characteristic mixing length. In the present work, $l_{mix} = C_t h_{UGN}$ with C_t being a modeling parameter and h_{UGN} a characteristic element length (see Tezduyar, 2003) and μ_{max} is a cut-off value.

3 INTERFACE UPDATE

The interface between the two fluids (Fluid 1 and Fluid 2) represents a strong discontinuity in the fluid properties and the gradients of the velocity and pressure. Nevertheless, these variables are interpolated as continuous functions across the interface. Other types of discontinuities at the interface, e.g. surface tension, are not included in the present model. The interface motion is governed by an advection equation:

$$\frac{\partial \varphi}{\partial t} + \mathbf{u} \cdot \nabla \varphi = 0 \qquad in\ \ \Omega \times \mathrm{Y} \tag{7}$$

where φ is a function marking the location of the interface. In the context of two-fluid flow analysis, all the matrices and vectors derived from the finite element discrete form of equations (1) and (2) are computed including the discontinuity in fluid properties.

In the present work, the interface is updated using the ETILT presented in Cruchaga et al. (2007). From here onwards we summarize such technique following the referred work and references therein. At each time step, the density and viscosity distributions are obtained from:

$$\rho^h = \varphi^{he} \rho_1 + (1 - \varphi^{he}) \rho_2 \tag{8}$$

$$\mu^h = \varphi^{he} \mu_1 + (1 - \varphi^{he}) \mu_2 \tag{9}$$

where φ^{he} is the edge-based representation of φ. To compute φ_{n+1}^{he} at time level $n+1$, given φ_n^{he} at time level n, first a nodal representation φ^h is computed. This is done by using a constrained least-squares projection as given in Tezduyar (2004, 2006):

$$\int_\Omega \psi^h (\varphi_n^h - \varphi_n^{he}) d\Omega + \sum_{k=1}^{n_{ie}} \psi^h(\mathbf{x}_k)\ \lambda_{PEN}\ \ (\varphi_n^h(\mathbf{x}_k) - 0.5) = 0 \tag{10}$$

Here ψ^h is the test function, n_{ie} is the number of the interface edges (i.e., the edges crossed by the interface), $\mathbf{x}_k$ is the coordinate of the interface location along the k^{th} interface edge and λ_{PEN} is a penalty parameter. After this projection, we update the interface by using a discrete formulation of the advection equation (7) governing φ of the form:

$$\int_{\Omega} \psi^h \left(\frac{\partial \varphi^h}{\partial t} + \mathbf{u}^h . \nabla \varphi^h\right) d\Omega + \sum_{e=1}^{n_{el}} \int_{\Omega^e} (\tau_{SUPG} \mathbf{u}^h . \nabla \psi^h) \left(\frac{\partial \varphi^h}{\partial t} + \mathbf{u}^h . \nabla \varphi^h\right) d\Omega + \sum_{e=1}^{n_{el}} \int_{\Omega^e} \nabla \psi^h \, \nu_{DCID} \, \nabla \varphi^h \, d\Omega = 0 \quad (11)$$

Here n_{el} is the number of elements, τ_{SUPG} is the SUPG stabilization parameter (Tezduyar, 2003), and ν_{DCID} is the Discontinuity-Capturing Interface Dissipation (DCID) parameter:

$$\nu_{DCID} = \frac{C_s}{2} h_{UGN}^2 \sqrt{2\boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}} \frac{\left|\nabla \varphi^h\right| h_{UGN}}{\varphi_{ref}} \quad (12)$$

where C_s is a discontinuity-capturing constant and φ_{ref} is a reference value set to 1. We note that the justification behind the expression given by equation (12) is combining some of the features we see in equation (7) and the Discontinuity-Capturing Directional Dissipation (DCDD) given by Tezduyar (2003). φ_{n+1}^h is computed from equation (11) by using a Crank-Nicholson time integration scheme.

From φ_{n+1}^h we obtain φ_{n+1}^{he} by a combination of a least-squares projection and corrections to enforce volume conservation for chunks of Fluid 1 and Fluid 2.

4 EXPERIMENTAL PROCEDURE

In the present work, a simple experimental setup was built to compare the experimental data with the numerical results to be presented in Section 5. The apparatus used is sketched in Figure 1.a and consists of a glass box with two parts delimited by a gate with a mechanical release system. The left part of the box is initially filled with coloured water. Only experiments with column aspect ratio A_r=2 ($A_r = H/L$, with H and L being the initial height and width of the column respectively, with L=0.114 m) are presented. An obstacle is positioned at 3/2L from the gate. The different geometries of the obstacles are presented in Figure 1.b.

Different experiments were carried out and recorded to a film. To measure the interface position, a ruler with tics 0.02 m apart is drawn surrounding the box and over the frame of the gate. A red marker that jointly moves with the gate helps to observe its position during the gate opening. The measurements reported in the present work are average values from the measurements registered for a minimum of five experiments. The maximum experimental errors in space and time are estimated as ±0.005 m and ±0.025 s, respectively.

The observed evolution of the interface at the left and right walls of the box, and in a vertical section in the middle of the obstacle are presented in the next section together with the corresponding numerical predictions.

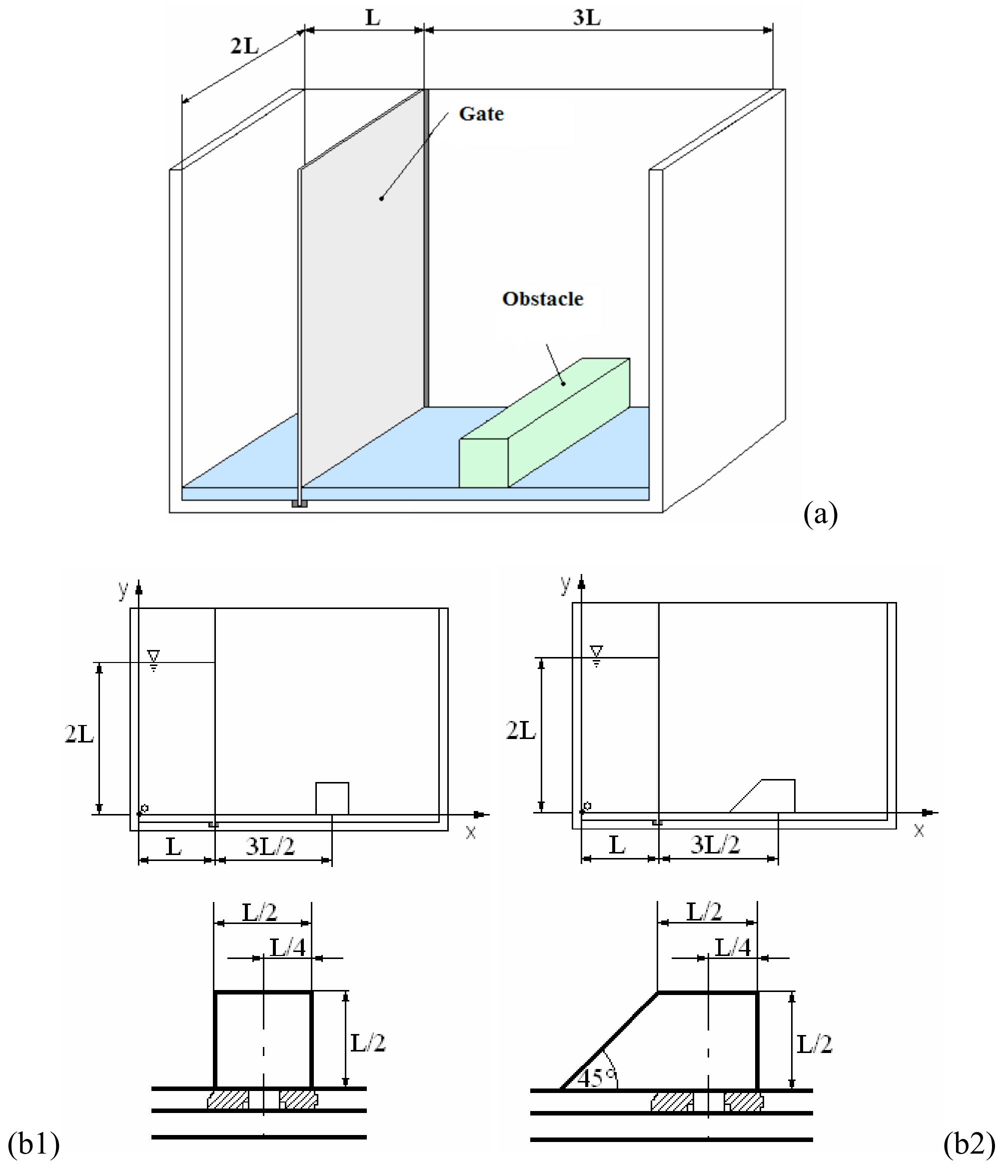


Figure 1: Schematic representation of the physical models (a) and geometry of obstacles (b1: square section, b2: trapezoidal section).

5 COLLAPSE OF A WATER COLUMN OF A_r=2, COMPARISON OF NUMERICAL AND EXPERIMENTAL RESULTS

In the present paper, the numerical simulations are focused on studying the long-term transient behavior for the collapse of a water column described in Section 4. In the simulations, the liquid column is initially at rest and confined between the left wall and the gate. The pressure is set to zero at the top of the rectangular computational domain. Slip conditions are assumed at the solid surfaces. The mesh is composed of 100×75 four-noded isoparametric elements. The time-step size is 0.001 s. The fluid properties are: ρ_1=1000 kg/m^3 and μ_1=0.001 kg/m/s for the water, and ρ_2=1 kg/m^3 and μ_2=0.001 kg/m/s for the air.

Two simulations are carried out: Simulation 1 and Simulation 2. In Simulation 1 the gate is assumed to be suddenly removed at time t=0 s, while in Simulation 2 the gate is not opened instantaneously but with a finite speed. The gate opening speed was extracted from the experiments. In Simulation 2, an average gate opening speed of 0.35 m/s is used. Both simulations include a simple turbulence model defined by equation (6) with C_t=3.57 and μ_{max}=3.0 kg/m/s. Moreover, they also have the DCID defined by equation (12) with C_s=10.

It should be noted that all the adopted parameters and properties used in the present analysis are taken from the analysis of the collapse of a water column without obstacles reported in Cruchaga et al. (2007).

5.1 Collapse of a water column over a obstacle of square section

Figure 2 shows, for the experiments and Simulations 1 and 2, the evolutions of the dimensionless interface vertical position (*y/L*, *y* being the instantaneous interface vertical position) at the left wall, middle section of the obstacle and right wall. The numerical results follow similar trends to those obtained from the experiments. Simulation 2 including the gate opening speed better represents the interface evolution at all the evaluated sections. In particular, the train of waves is captured. Nevertheless, the computed vertical position of the interface is lower than those obtained in the experiments. The interface evolution along the left wall and in the section at the middle of the obstacle is advanced in time, meanwhile at the right wall is delayed with respect to the measurements. However, the magnitude of first maximum along the right wall is properly described.

5.2 Collapse of a water column over a obstacle of trapezoidal section

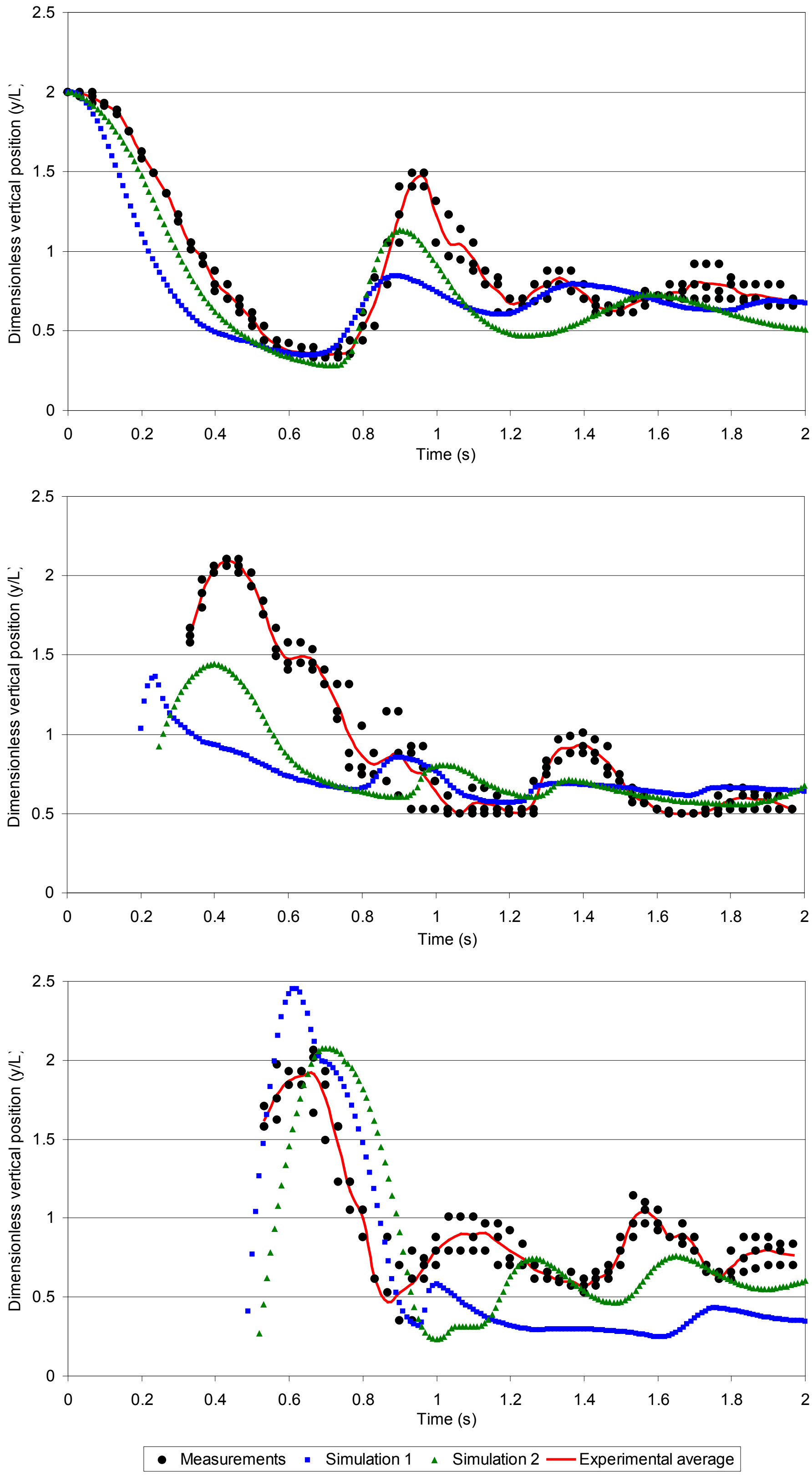
Figure 3 shows, for the experiments and Simulations 1 and 2, the evolution of the dimensionless interface vertical position at the left wall, middle section of the obstacle and right wall. In this case, the results obtained with Simulation 2 are in reasonably good agreement with the experimental data. The computed vertical position of the interface at the left wall practically coincides with the experimental results. The first and second maximums are well described not only in time but also in magnitude. The numerical behavior of the interface vertical position at the middle of the obstacle also shows good agreement between the numerical and experimental results. Although the time at which the interface reaches the right wall is slightly delayed, the computed vertical interface position at such a wall captures reasonably well the interface evolution from such instants onwards.

6 CONCLUSIONS

The performance of the ETILT was assessed in the simulation of the collapse of a water column over obstacles. A set of experiments has been carried out in order to obtain the evolution of the interface position. Two different geometries of obstacles were studied.

Simple concepts were included in the model to describe turbulence effects in the analyses. The involved parameter used to characterize the water behavior where taken from the analysis of the collapse of liquid column without obstacles. Moreover, the effect of the gate opening was found to change the overall behavior of the interface in the range of gate opening speed analyzed.

Overall, the numerical results compare satisfactorily with the measurements.



(a) (b) (c)

Figure 2: Evolution, for the obstacle of square section, of the dimensionless interface vertical position at the left wall (a), section at middle of the obstacle (b) and right wall (c).

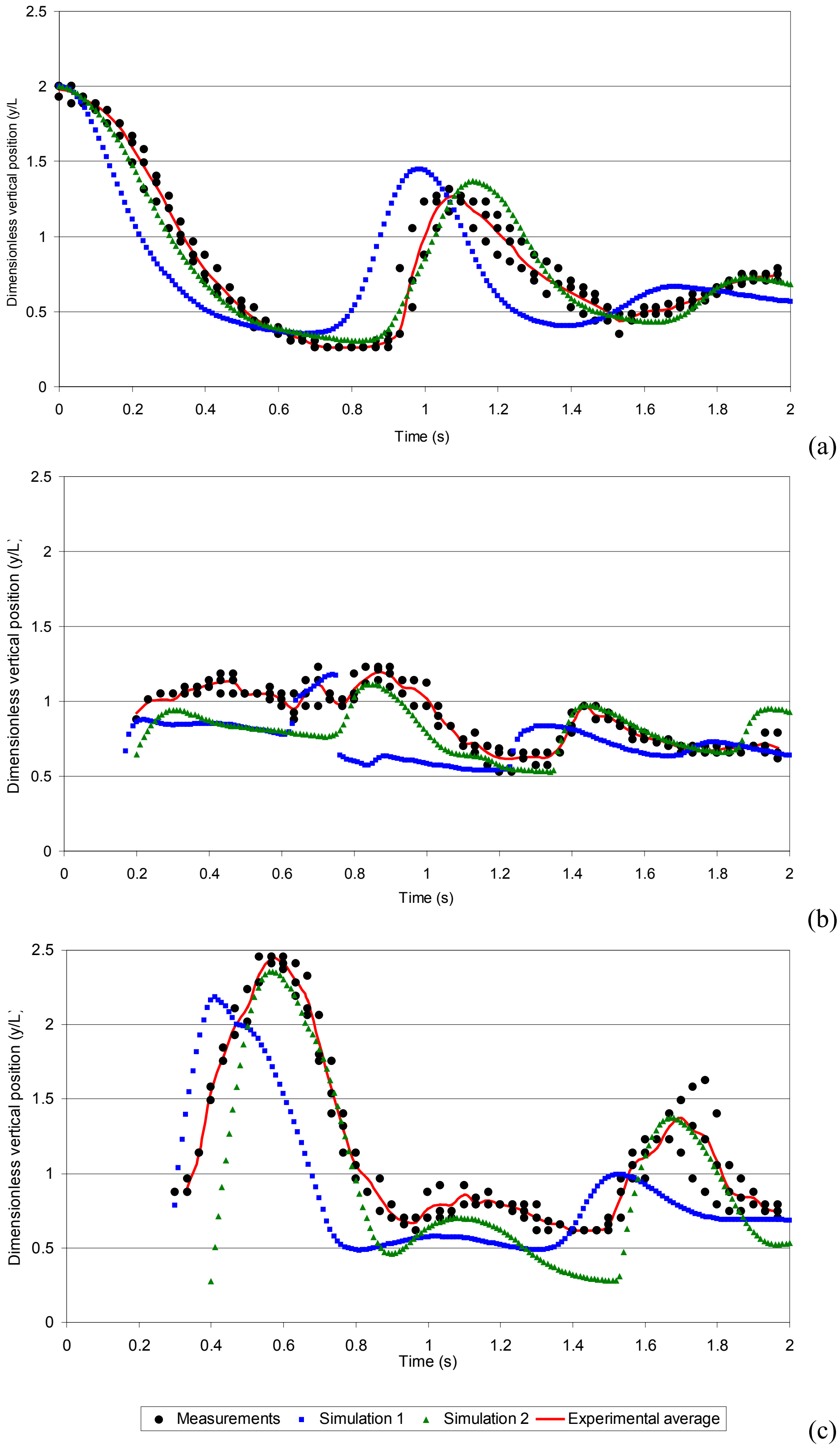


Figure 3: Evolution, for the obstacle of trapezoidal section, of the dimensionless interface vertical position at the left wall (a), section at middle of the obstacle (b) and right wall (c).

ACKNOWLEDGEMENT

The authors thank the support provided by the research projects CONICYT-FONDECYT 1060141 and PCCI 7070012, and by DICYT-USACH.

REFERENCES

Cruchaga, M.A., Celentano, D.J., and Tezduyar, T.E., Moving-interface computations with the Edge-Tracked Interface Locator Technique (ETILT). *International Journal for Numerical Methods in Fluids*, 47:451-469, 2005.

Cruchaga, M.A., Celentano, D.J., and Tezduyar, T.E., Collapse of a liquid column: numerical simulation and experimental validation. *Computational Mechanics*, 39:453-476, 2007.

Cueto-Felgueroso, L., Colominas, I., Mosqueiras, G., Navarrina, F., and Casteleiro, M., On the Galerkin formulation of the smoothed particle hydrodynamics method, *International Journal for Numerical Methods in Engineering,* 60:1475–151, 2004.

Greaves, D., Simulation of interface and free surface flows in a viscous fluid using adapting quadtree grids, *International Journal for Numerical Methods in Fluids*, 44:1093-1117, 2004.

Idelsohn, S., Storti, M., and Oñate, E., A Lagrangian meshless finite element method applied to fluid-structure interaction problems, *Computer and Structures*, 81:655-671, 2003.

Kohno, H., and Tanahashi, T., Numerical analysis of moving interfaces using a level set method coupled with adaptive mesh refinement, *International Journal for Numerical Methods in Fluids*, 45:921-944, 2004.

Koshizuka, S., and Oka, Y., Moving-Particle Semi-Implicit Method for Fragmentation of incompressible Fluid, *Nuclear Science and Engineering*, 123:421-434, 1996.

Kulasegaram, S., Bonet, J., Lewis, R.W., and Profit, M., A variational formulation based contact algorithm for rigid boundaries in two-dimensional SPH applications, *Computational Mechanics*, 33:316-325, 2004.

Martin, J., and Moyce, W., An experimental study of the collapse of liquid columns on a rigid horizontal plane, *Phil. Tran. Royal Society London*, 244:312-324,1952.

Osher, S., and Fedkiw, P., Level set methods: and overview and some recent results, *Journal of Computational Physics*, 169:463-502, 2001.

Sethian, J.A., Evolution, Implementation, and Application of Level Set and Fast Marching Methods for Advancing Fronts, *Journal of Computational Physics*, 169:503-555, 2001.

Tezduyar, T.E., Stabilized Finite Element Formulations for Incompressible Flow Computations, *Advances in Applied Mechanics*, 28:1-44, 1992.

Tezduyar, T., Aliabadi S., and Behr M., Enhanced-Discretization Interface-Capturing Technique (EDICT) for Computation of Unsteady Flows with Interfaces, *Computer Methods in Applied Mechanics and Engineering*, 155:235-248, 1998.

Tezduyar, T.E., Finite element methods for flow problems with moving boundaries and interfaces, *Archives of Computational Methods in Engineering*, 8:83-130, 2001.

Tezduyar, T.E., Computation of moving boundaries and interfaces and stabilization parameters, *International Journal for Numerical Methods in Fluids*, 43:555-575, 2003.

Tezduyar, T.E., Finite Element Methods for Fluid Dynamics with Moving Boundaries and Interfaces", Chapter 17 in *Encyclopedia of Computational Mechanics, Volume 3: Fluids* (eds. E. Stein, R. De Borst and T.J.R. Hughes), John Wiley & Sons, 2004.

Tezduyar, T.E., Interface-Tracking and Interface-Capturing Techniques for Finite Element Computation of Moving Boundaries and Interfaces", *Computer Methods in Applied Mechanics and Engineering,* 195:2983-3000, 2006.