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CONTROL OPTIMO DEL TRAFICO MEDIANTE SEMAFOROS CON OBSERVADORES LOCALES

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Abstract. In order to improve the travel time of surface public transport vehicles (PT) (bus, tramway, etc.), several cities use urban traffic control (UTC) systems enabling to give priority to PT. This article reviews these systems. Further to a debate on their insufficiencies for a global regulation on the urban traffic on a whole network, the paper proposes intermodal regulation strategies, operating on intersection traffic lights to regulate traffic, favoring the PT. These strategies are based on the linear quadratic (LQ) optimal control theory. The article proposes different ways to take the PT into account in the optimisation problem. The simulation tests are carried out on a network of eight intersections and two PT lines.

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1 INTRODUCTION

The mobility of the inhabitants of the agglomerations is in continuous growth. Unfortunately, despite everything its harmful effects on the environment (pollution, noises, occupation of space, etc), it is the private car (PC) which allures more the road users and the growth is done more with its profit. With the end to reverse this tendency and to make competitive public transport (PT), several measurements are used. We can quote: exclusive rights of way reserved for PT, prohibition to station on the roadway system, urban toll aiming at reducing the road traffic, static or dynamic guidance using the panels with variable messages to direct the PC to roads less attended by drunk PT and the priority to the signal-controlled junctions. These measurements make improvements to the PT travel times when the traffic is not very dense. But, at the peak hours when the roads are very loaded, these measurements induce congestions for the PC traffic which deteriorate the time of course of the buses them selves. Let us take for example the exclusive rights of way reserved for PT. PT vehicles have to share the traffic lights green time with the PC in the intersections. If there is not a global strategy for the traffic light control which allows to favor the PT, the PT situation will be debased in these intersections. The cumbersome infrastructure would have improved only of very little the time of course of the PT.

Our objective in this work is to develop a strategy which acts on the intersection traffic lights in order to improve the traffic on a network of urban roads, favouring public transport. To achieve that objective we chose to develop an optimal command on the basis of the Quadratic Linear optimisation theory (LQ), which has the advantage of being able to be used in close loop. The LQ theory has already been applied for the regulation of urban intersections in the TUC strategy (Diakaki and al., 2002). However, TUC considers the PT on the basis of rules only, as it will be explained in paragraph (2.2.1) and not as a variable of the system condition as it is the case in our strategy.

In this paper, the following section will address a state of the art of the PT priority systems. We underline the insufficiency of these systems for traffic regulation on the global level of a whole network. In the third section, we mention the various systems of global management for both modes (PC and PT) and we give some elements explaining the non existence of the global regulation systems for both transport modes. In the fifth section, we describe the used model of command. The sixth section deals with the definition of the optimal command problems, starting with the optimisation criteria. The latter enables to regulate the traffic on the whole network, and has an additional term, enabling to favour the arcs which support the PT vehicles at the instants when they are on these arcs.

Our objective to apply the regulation strategy in real time on a large network, led us to set the optimisation problem on a horizon with infinite time, implying a static equation of Riccati. We show that the use of a combined strategy, using a different optimisation criteria for the arcs supporting the presence of the PT and for the arcs with PC traffic only, can remedy this problem. The results of these various strategies applied in simulation on a network with eight intersections, thirty two arcs and two lines of PT, are given in the eighth section. The conclusion is given in the ninth section.

2 DIFFERENT TYPES OF PT PRIORITY AT TRAFFIC LIGHT INTERSECTIONS

The bus priority at traffic lights can be operated in a passive or in an active way.

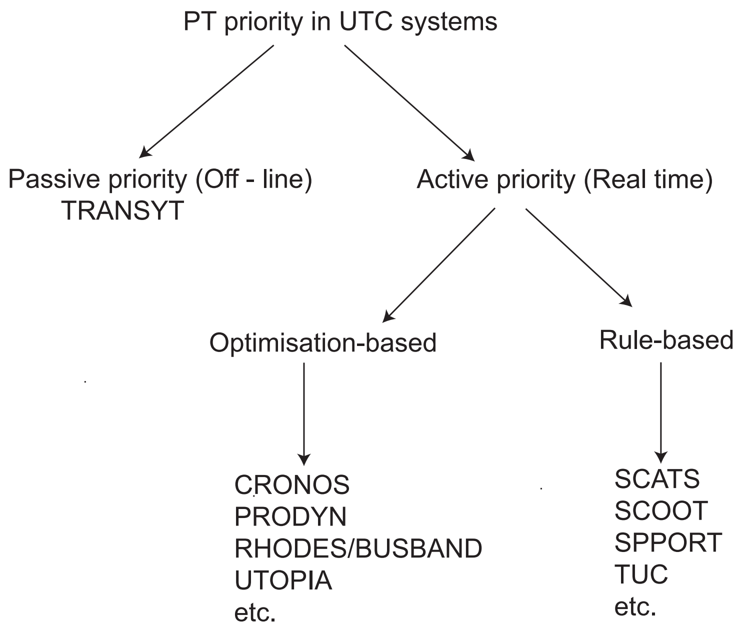

Figure 1: Classification of the PT priority strategies

2.1 Passive priority

The passive priority consists in generating the plans of the traffic lights so as to favour the roads supporting the PT, without detecting these vehicles individually. Some measures can be applied to satisfy this objective, for example :

- to adjust the traffic lights coordination to the PT speed instead of the PC speed,
- to reduce the duration of the traffic light cycles, in order to reduce the waiting times of the PT when they arrive at traffic lights. This measure cannot be applied in the case of large density traffic since it reduces the intersection capacity.
- to split the green phase attributed to the road supporting the bus, when the traffic light cycle cannot be very short. However, this method also reduces the capacity of the intersection traffic flow.
- to design the traffic light diagrams, taking into account the number of passengers rather than the number of vehicles. However, it implies to know the load in terms of passenger number in each transport mode.

This latter method was used in the regulation system, enabling passive priority, the most well known on the international level, is the TRANSYT system.

PT priority strategy at traffic light intersections

2.2 Active priority

The second method, called dynamic priority, consists in a modification of the intersection signals to authorize the passage of the PT vehicle (PTV) which has been detected. This type of priority is possible on the systems of real time regulations of traffic.

The systems of real time regulation of urban traffic generally belong to one of both families. (See Figure 1). The first system family uses a fixed traffic light cycle on a given period. They gradually adapt the traffic light plan to the variations in real time of traffic conditions (for example : SCOOT Hunt et al. (1982), SCATS **?** and TUC Diakaki et al. (2002)). The second systems consist in adaptive commands, continually optimizing the traffic light plan on a slipping horizon (example : CRONOS Boillot et al. (2000), PRODYN Henry and Farges (1994) et UTOPIA **?**). It influences their way to take into account the priority at traffic lights. In the first category, the priority is given to the public transport vehicle crossing (PTVC) on the basis of the pre-established rule. The strategies of the second category give the priority to the PT vehicle further to the optimisation of some criteria.

2.2.1 Rule-based priority

These methods consist in a short term modification of the traffic light operation to favour the PTV approaching the intersection crossing. The majority of the regulation strategies apply this type of priority. Further to the regulation strategies known on the international level, such as SCOOT, SCATS, SPPORT, TRAFCOD, TUC, several less known systems which were developed by cities or by transport organization authorities, use this kind of priority.

2.2.2 The determined priorities based on the global optimization at the intersection

It is based on the optimisation theory to find the optimal durations of lights enabling the priority crossing of the PTV. It requires PC and PT traffic models and a criteria to be optimised. The advantage of these strategies is that they are not constrained by a traffic light fixed cycle. Among the existing systems, CRONOS, PRODYN, RHODES/BUSBAND can be mentioned.

2.2.3 Limits of TPS regulation systems

The intersection regulation systems which enable to give priority to the PT on the basis of rules cannot manage more than one PTV per traffic light cycle. As we have seen, these systems proceed, attributing an additional duration of green light at the approach of the PTV and restore the order of the phases later on. This procedure cannot be repeated several times during a cycle since the green light durations are limited by maximum values imposed for safety reasons. Thus it limits the use of this type of strategy at the intersections which are not very used by the PTV. The systems which manage the priority through optimisation algorithms can take several criteria into account before attributing the priority ; for example they can attribute the priority to the PT vehicle which deserves it most and not to the one which asks it first, etc. However, these systems often are limited by the computation time. These are systems which were developed to regulate an intersection. They generally use the microscopic models of the intersection and the computation time, very often increases in a sequential way with the number of studied intersections.

>From our viewpoint, a PT priority strategy which is placed on the individual level of buses cannot have a global view of the traffic on a whole region. It can imply twisted effects, since it can enable to feed road network sections or congested intersections, resulting in a deterioration of the traffic general conditions including bus traffic conditions. Thus it is necessary to develop regulation strategies which take into account the intermodal global situations of the traffic on a whole region (a whole route of PT for example).

3 GLOBAL REGULATION STRATEGIES

This awareness of the need for a global management of travels initiated several research on that topic. Concerning intermodality, several research works were carried out on topics such as the problems of flow on a multimodal network (Nes, 2001), of traffic assignment on a multimodal network (Bhouri and Lebacque, 2003), of tariffing (Lotito and al., 2004), etc.

>From the regulation viewpoint, several traffic regulation systems intended to the PT only were enlarged to take the PT mode into account, namely through the "priority to PT" component. If these strategies are effective on the level of the intersection, they are not sufficient to solve the problem of multimodal urban traffic regulation on a whole network.

We call intermodal traffic regulation strategy a strategy which can take into account the regulation objectives of the various transport modes on a whole network. In our context, these are the regulation objectives of the PC and the PT.

3.1 Intermodal Traffic Management Systems

Several systems, the objective of which is the multimodal management of urban traffic have been developed, such as the multi-sites multi-sources platform, SITI developed at INRETS Scemama and Tendjaoui (2003), the project ENTERPRICE in Cologne Riegelhuth et al. (1997), the project MOBINET in Munich Kellermann and Schmid (2000), the project 5T-TITOS in Turin Franco (2000), etc. All of these systems try to integrate the various transport modes in order to operate a combined management of these modes. They generally consist in platforms having the objective to link the systems which act ont the various transport modes and to allow them to have access to the data of the various traffic sources. The objective in turn is to develop intermodal systems which have a combined action on the transport modes. However, the analysis of the condition of these systems shows that the integration still is on the level of the concentration and of the merging of the data sources rather than on the level of the decision Bhouri (2002). None of these platforms has any intermodal regulation system as they are meant in this paper.

3.2 Obstacles for an Intermodal Regulation

The intermodal regulation is confronted with some obstacles on the operational level, for example on the institutional level of the operators' responsibilities, of the operators' habits to work independently from each other, etc. From the theoretical viewpoint, the intermodal regulation is confronted with the problem of the representation of the various networks on the same level of accuracy. Indeed, a traffic global regulation requires a macroscopic representation of traffic. However, it is difficult to represent the PT through a flow as it is the case for the PC.

The macroscopic modeling of intermodal traffic has been studied in many research works (for example, Daganzo and Laval (2005) and Lebacque et al. (1998)). However, the problem in these works is to represent the traffic of the vehicles. In both works, the PT is considered as an obstacle which slows down the PC traffic and not as a component to be modeled.

For our case, the objective is to regulate the global traffic, giving a priority to the PT without changing the rest of the traffic on the network. We are going to develop an intermodal model for the command of two transport modes. Since this command will be applied in closed loop, we don't try to model the intermodal traffic with much accuracy.

A network of urban roads is composed of intersections linked by sections. In order to explain our objective in that work and the used model, we start by giving the variables which characterize the traffic light intersections, before developing the developed intermodal model of command.

4 THE CHARACTERISTICS OF A TRAFFIC LIGHT INTERSECTION

A traffic light plan is associated with every traffic light intersection defining the progress of the condition of all of the command lines of the intersection signals. A traffic light plan is specified thanks to the following variables :

- a phase diagram : a phase is the period during which one or several compatible movements are admitted on an intersection. The phase diagram specifies all of the phases with the potentialities of passing from one to the other. An intersection generally operates according to a single phase diagram, chosen on the one hand to prohibit the simultaneous passages of conflicting movements, on the other hand, respecting the significance of the traffic load on some movements. Figure (2) gives two different phase diagrams for a same intersection.

- A clearance time : it is a minimum period of safety which should be complied with during the transition of two incompatible traffic lights. There is a compulsory safety time between two phases : complete orange and red (all of the intersection branches are at red lights in order to clear the inside of the intersection). One generally programs the intersection light plans with a minimum number of phases in order to minimize this loss of time.

- A cycle : it is the total duration of the green, orange and red lights.

- A gap : it is a duration relative to a reference instant which is used to synchronize the various network controllers.

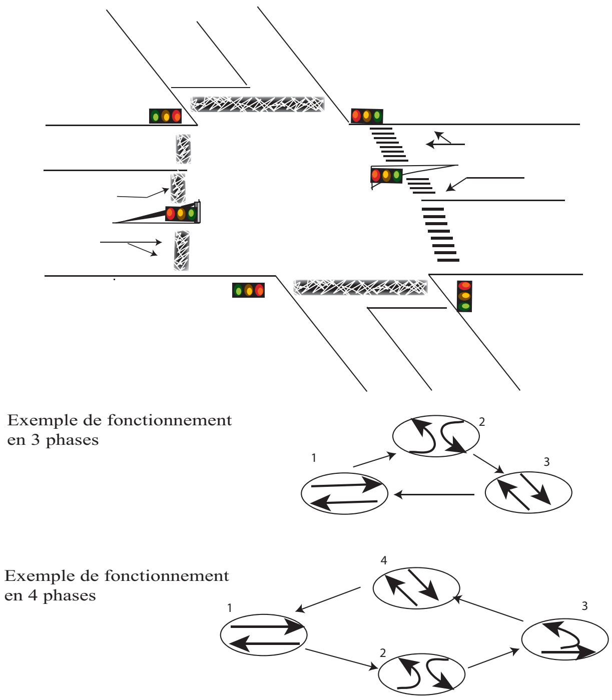

Figure 2: Examples of phase diagrams of an intersection

In urban area, the cycle duration generally is known in advance because of the coordination of the intersections from a same area Scemama (1994). In this work we assume that the duration of the traffic light cycle, the phasing diagram and the gaps are fixed on the considered time horizon. The strategy acts on the duration of the green lights within the cycle in order to improve the traffic conditions.

5 DYNAMIC MODEL

In order to obtain the dynamic equations for the mathematical model, we will consider the now well established, Store and Forward model due to Gazis and Potts (Gazis and Potts (1963)). The choice of this model is based in the simplifications it imposes on the equations that will allow us to write them as linear equations on the number of vehicles and the green time of the junctions.

The network is represented by a directed graph composed of nodes and arcs. The nodes $j \in J$ represent intersections and the arcs $a \in A$ the unidirectional travel links. On every arc, the model consists in two equations, one of them modeling the progress of the total number of vehicles on the arc, expressed as private vehicle unit (PVU) (for example a bus equals 2,3 PVU). The second equation models the number of PT vehicles on the arc.

5.1 The general traffic dynamic equations

As was said in the introduction, this strategy adopts the same bases as the TUC intersection regulation strategy and both are based on the Store and Forward model. The traffic on each arc a is modeled using the vehicle-conservation equation (Diakaki et al., 2002).

$$x_a(k+1) = x_a(k) + T[q_a(k) - u_a(k)], \tag{1}$$

where x_a is the number of cars on the link expressed in PVU, q_a and u_a are the inflow and the outflow of link a during $[kT, (k+1)T]$ where k is the discrete time step and T is the sampling time. See figure 3 to clarify the relations between the variables. Here we have neglected the traffic generated and consumed in each link, it would be easy to include them without substantially changing the current development.

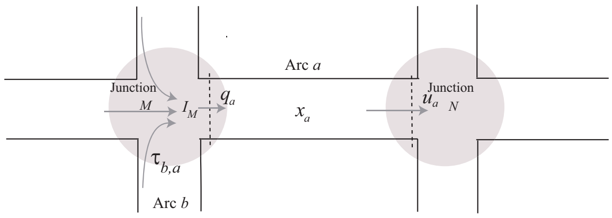

Figure 3: Variable definition.

In order to explicit the equations for q and u we will consider the saturation flow of each link S_a, that represents the maximum traffic flow that can exit the link, expressed in PVU/s. The *Store and Forward* model assumes that the vehicles reaching the arc's end are stored there and exit with rate S_a during the green light. Hence, we can write

$$u_a(k) = \frac{S_a.G_a(k)}{C}, \tag{2}$$

where C is the cycle time and $G_a(k)$ is the effective green time of link a, i.e., the green light duration attributed to arc a during the traffic light cycle C of the intersection situated at the arc exit, and will be the control variable in our approach. If the green light periods are attributed to arc a during different phases (see figure 2), $G_a(k)$ is equal to the sum of all of these green light durations,

$$G_a(k) = \sum_{i \in P_j^a} G_{j,i}(k), \tag{3}$$

where P_j^a is the set of the intersection phases j during which arc a has the green light. It also assumes that the outflow is distributed among the different following links according to the coefficients τ_{ab}, called turning rates, that represent the proportion of outflow from a entering in arc b.

If the link a originates at the junction M, the inflow traffic rate entering arc a can be written as the sum of the outflow traffic rates coming from the arcs entering junction M (other than a). If the arc b precedes arc a, the corresponding flow is $\tau_{ba}u_b$, so the total flow entering arc a is

$$q_a(k) = \sum_{b \in I_M} \tau_{b,a} u_b(k), \tag{4}$$

where I_M is the set of arcs entering junction M, and we have defined $\tau_{aa} = 0$.

Replacing all the previous definitions in the equation (1), we obtain the following model :

$$x_a(k+1) = x_a(k) + \frac{T}{C}[\sum_{\omega \in I_M} \tau_{\omega,a} S_\omega G_{M,\omega}(k) - S_a \sum_{i \in P_j^a} G_{j,i}(k)] \tag{5}$$

or in matrix form :

$$X(k+1) = X(k) + B.G(k), \tag{6}$$

where B is a matrix of dimension $N \times M$, N is the number of links and M is the total number of phases on the network.

This modeling is possible under the following assumptions :

- the sampling time interval T is at least equal to the duration of the light cycle C, we will use $T = C$.
- the gaps between the intersections are not taken into account,
- variations in the queue are neglected, which means that the model considers that all of the input flows on the arc have the green phase at the same time.

5.2 The PT Traffic Dynamic Equations

As we will be considering two kinds of traffic, the general one and the PT one, we will distinguish the state variables as x^v for the number of vehicles and x^b for the number of PT vehicles (buses). Knowing the sequence of arcs which are used by each PT line, the progress of the PT vehicles is modeled by a delay equation :

$$x_a^{bi}(k) = x_{a'}^{bi}(k - \zeta_a^i) \tag{7}$$

where x_a^{bi} is the number of vehicles of the PT line number i on arc a, a' is the arc preceding a for the line i and ζ_a^i is a parameter which expresses the mean travel time of the vehicles on line b_i to travel from arc a' to arc a. These values should be real ones, however, in order to be able to write the precedent equation, we take ζ_a^i as integer, meaning that the travel time is a multiple of the sampling interval T. Thus we consider that ζ_a^i is equal to 1 if the bus line has no station on arc a, otherwise ζ_a^i is equal to 2 (for example). Substituting these values in equation (7), the model of the PT becomes the following :

$$x_a^{bi}(k+1) = \begin{cases} x_{a'}^{bi}(k-1), & \text{if line } i \text{ stops on } a \\ x_{a'}^{bi}(k), & \text{otherwise} \end{cases} \tag{8}$$

This simplification complies with the dynamical modeling of the PC, since it consists in assuming that both the PC and PT are "stored" during the red light period and then are "distributed" during the green light period, thus they spend a light cycle on the arc. However, the choice of the cycle duration should be done carefully.

The equation (8) written in vectorial form gives :

$$X^b(k+1) = A_0^b X^b(k) + A_1^b X^b(k-1), \tag{9}$$

where the matrix A_0^b is the adjacency matrix corresponding to the bus line for the arcs without stops, A_1^b is the adjacency matrix corresponding to the bus line for the arcs with a stop, and $X^b(k)$ is the vector of numbers of buses at each traversed arc. It can be further simplified, adding if necessary supplementary state variables, as

$$X^B(k+1) = A^b.X^B(k) \tag{10}$$

where X^B is the vector obtained stacking $X^b(k)$ and $X^b(k-1)$, and matrix A^b is given by :

$$A^b = \left(\begin{array}{c|c} A_0^b & A_1^b \\ \hline 1 & 0 \end{array} \right). \tag{11}$$

5.3 The PT-PC Model

Here we do not talk of coupled model, because, as it is easy to see, both dynamics are not coupled, in fact they will be coupled but through for the objective function in the optimal control problem to be presented in the next section.

The state variable of the whole system consists is a vector of dimension $(N+2N_b)$, where N is the number of arcs in the system, N_b is the number of arcs crossed by the PT lines. The dynamics of the system thus is represented by the following equation

$$X(k+1) = AX(k) + BG(k) \tag{12}$$

where A is a matrix of dimension $(N+2Nb) \times (N+2Nb)$. The matrix B is composed of two stacked blocks the upper one is defined by the topology of the road network, i.e., the coefficient B_{aj} when different from 0 means that phase j is found entering or leaving arc a and its value is defined according to (5). The lower block corresponds to the influence of the green lights on the bus, which, as it is neglected, has to be 0. We have then

$$A = \left(\begin{array}{c|c} \mathrm{I} & 0 \\ \hline 0 & A^b \end{array} \right), \qquad B = \left(\begin{array}{c} B \\ \hline 0 \end{array} \right). \tag{13}$$

With these matrices, it is clear that it will not be possible to command the PT because of the null block of matrix B. However, it doesn't set any problem because in the definition of the model, we suppose that the travel times of the PT are fixed. What we want is to act in such way that bus can comply with their schedules.

6 OPTIMAL CONTROL PROBLEM

Here we pose the optimal control problem. The control means that we will be able to choose the green light times in order to modify the flows. The optimality will be measured in terms of the number of private cars that share the roads with the buses. We will explicit here these definitions. When doing so, we keep in mind that we want to obtain a simply computable global green time. As we have linear dynamics, choosing a quadratic objective function and imposing no restrictions will make the optimal control problem over an infinite horizon belong to the LQ class. The importance of that relies in the fact that the optimal solution can be written as a feedback law and the matrix that defines this law is the solution of a matrix equation (Ricatti equation) stated in terms of the given data.

is given by an equation Our objective is to solve a problem of command LQ to regulate the traffic, respecting the dynamics of the system given by equations (5) and (8).

6.1 Optimisation criteria

From the viewpoint of the traffic regulation, our objective is to improve the traffic conditions of the PT on the network, relative to the PC flow, without deteriorating the global traffic conditions. The objective function need to be quadratic in terms of the state and control variables to rest in the LQ case, the general form of these functions is

$$J(x,u) = \int_0^\infty \alpha_x \|x\|^2_{Q_x} + \alpha_u \|u\|^2_{Q_u}, \tag{14}$$

where Q_x and Q_u are positive definite matrices that allow to weigh differently the components of x and u, and α_x and α_u are non negative coefficientes. These conditions guarantee that the function J will be convex (strongly if $\alpha_{x,u} > 0$) which in turn guarantees the existence (and uniqueness) of the solution over the closed convex set defined by the linear dynamic equations.

In our (discrete time) case we propose the following objective function

$$\min_G J(G) = \sum_{k=0}^{\infty} (\alpha(X(k)'X^b(k)) + \beta\|X(k)\|^2 + \gamma\|G(k)\|^2), \tag{15}$$

where α, β and γ are non-negative weighting parameters and the X are given by the dynamic equations (5) and (8).

Even if the introduction of the objective function was made for computational simplicity, we can give an interpretation to each term. The first term of the criteria, $(X(k)'X^b(k))$ puts forward the traffic conditions on the arcs crossed by the PT at the time these PT vehicles are present on it. The second member aims at reducing the number of vehicles on every arc on the network and thus to equalize the congestion on every arc. The role of this second term is mainly to not degrade too much the traffic in the other arcs. The last term is used in order to avoid large variations of the control (green light times).

The optimisation criteria (equation 15) has three different terms weighted by parameters α, β and γ. The choice of the values of these parameters enables to modify the objective of the regulation. For example, for $\alpha = 0$, $\beta = \gamma = 1$ the strategy is equivalent to TUC, which doesn't take into account the presence of the PT. On the other hand a significant parameter α ($\alpha >> \beta$) will strongly penalize the arcs which don't support the PT.

6.2 Control Law

The problem of optimal control consists in minimizing the criteria given by equation (15) respecting the dynamics of the system given by the equations (12). In order to avoid working with the input and exit flows we define a nominal green time G^N that solves $BG^N = 0$, in such case the corresponding nominal state is constant and we can work with the following dynamic equation

$$X(k+1) = AX(k) + B\Delta G(k), \tag{16}$$

where $\Delta G(k) = G(k) - G^N$.

Using the LQ optimisation method (Culioli, 1994), the applied command law is given by the following equation

$$G(k) = G^N - F.X(k) \tag{17}$$

where F is the Feedback matrix defined as $F = (R + B^T PB)^{-1} B^T PA$ and the matrix P solves the Riccati matrix equation $P = Q + A^T PA - A^T PBF$ which depends on the coefficients α, β, and γ of the objective function through matrices Q ad R.

Applying the equation (17) to $G(k)$ and to $G(k-1)$, by a simple substraction, one obtains

$$G(k) = G(k-1) - F.(X(k) - X(k-1)), \tag{18}$$

the use of this equation rather than of equation (18), avoids the estimation of the nominal values of the control.

It should be noted that the choice of an infinite time horizon in equation (15) implies that the Feedback matrix F is time independent. This choice is justified by the will for a real time command of the intersection lights and thus by the simplification of the calculations for each command. However it has the drawback to consider a time average of the criteria, reducing the significance of our main objective which is to reduce the number of vehicles on the arcs at the instants when the PT vehicles are on these arcs. This led to the idea to test various strategies, whether a single Riccati matrix or at most a finite matrix combination is used, each of them being calculated for a different system state. We explain this idea in the following sections.

6.2.1 Strategy with PT priority (PPT)

As it was said earlier, the choice of parameters α, β and γ enables to fix various objectives of command. The first tested strategy consists in slightly favoring the PT with the choices $\alpha = \beta = 1$ in equation (15).

6.2.2 Strategy with strong PT priority (PFPT)

In this second strategy, a big significance is given to the first term of the criteria consisting in favoring the arcs that support the buses at the instants when they are on it. In this case we choose $\alpha >> \beta$ (Bhouri and Lotito, 2005).

6.2.3 Combined strategy

This strategy is based on the ability of detecting the presence of buses on the arcs which can be accomplished using appropriated sensors. We use then two different criteria (different Riccati equations) according to the presence or the proved absence of the PT vehicle on the arc.

This is a good compromise between a single Riccati matrix (the same for all of the k) and an infinite or very large sequence of Riccati matrixes (one for every k). The idea is to give the guidelines in a practical and implementable way. The intersection controllers have two Riccati matrixes calculated in the following way : the first one doesn't take the PT into account (TUC for example), on the contrary, the second one strongly takes them into account (α very large). The matrix which corresponds to the situation of the PT is used on each of the intersections on the network.

More precisely, let's consider F_1 the Feedback matrix obtained with TUC (or another independent criteria of the PT position). Given F_2 the Feedback matrix obtained with a criteria which takes into account the position of the PT ($\alpha >> \beta$). The optimal command is given by :

$$G_k = G_{nom} + (P_k F_1 + (I - P_k) F_2)(X_k - X_{k-1}) \tag{19}$$

where P_k is a diagonal matrix, every element of which is equal to 1, if at the moment k there is a PT vehicle waiting on the corresponding arc.

This strategy appears to be clever since it enables to reduce the congestion on the arcs where there are PT vehicles at the instants when they are on it, without increasing it on the arcs where there is none. The simulation results will be detailed in section (**??**).

6.3 The constraints

The solution of the optimal command problem by the LQ method doesn't enable us to take the constraints into account because the Ricatti equation will no longer be valid. However, for operative needs 4), at every intersection j, the durations of green lights should comply with a certain number of constraints :

- the cycle duration (C),
- the phase diagram : all of phases P_j should have their green light within the cycle,
- the clearance times between phases R_j,

which implies :

$$\sum_{i \in P_j} G_{j,i} + R_j = C. \tag{20}$$

On the other hand, the duration of every green light is limited by a maximum and a minimum. Indeed, a too long red light duration can be interpreted by users as a malfunction of the intersection lights and imply their non-compliance :

$$G_{j,i,min} \leq G_{j,i} \leq G_{j,i,max}. \tag{21}$$

We solved this problem through a projection of the obtained command values onto the set of feasible values defined by the above constraints. It means to obtain the closest (in some distance) values to the optimal but not feasible ones. The projection step means to solve the following quadratic optimization problem that includes the constraints (20) and (21),

$$\begin{array}{ll} \min\limits_{\overline{G}} & \sum_{i \in P_j} (G_{j,i} - \overline{G}_{j,i})^2, \\ \text{s.t.} & (20) \text{ and } (21). \end{array} \tag{22}$$

This problem belongs to the class of Quadratic Knapsacks problems and the numerical solution was done according to the algorithm presented in (Lotito, 2005).

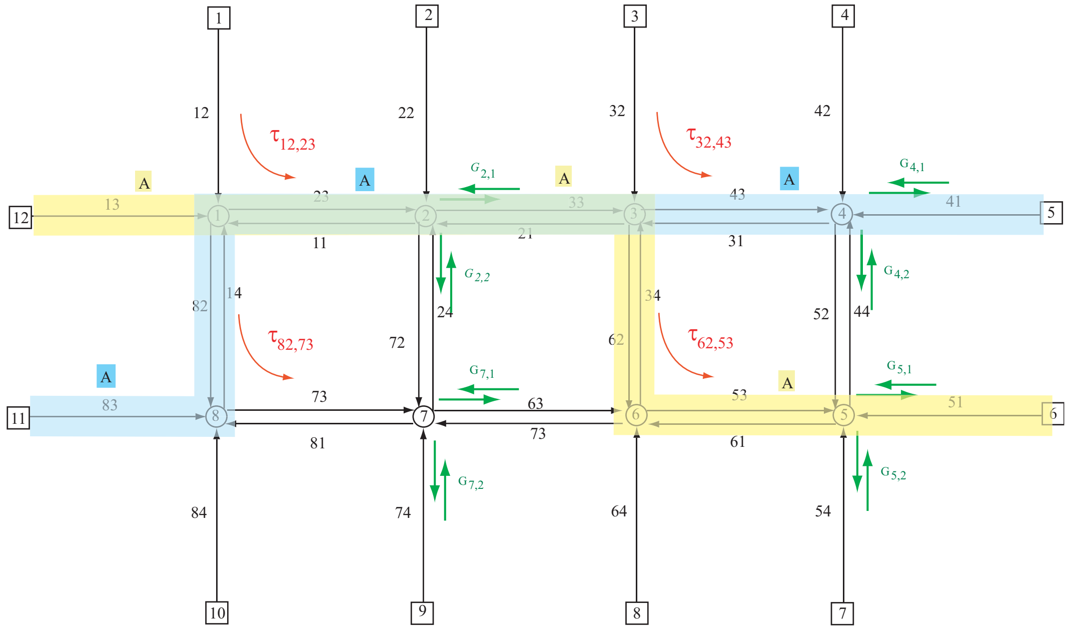

Figure 4: The example network

7 NUMERICAL RESULTS

In this section we expose numerical tests that we have made with a small academic example of a network consisting in 8 intersections, 32 links and two bus lines as it is shown in the figure 4. Each intersection has two phases and each one is given 50The entering flows over links 1, 3, 8, 10, 11, 12 is 0.5 veh./s and 0 over the other entering links. The turning rates are chosen as 25% turn left, 25% turn right and 50% pass through. The saturation flows are chosen equal to 1 everywhere. The bus lines are two, the first one enters on 12 and traverse intersections 1, 2, 3, 6 and 5, making a stop before 1, 3 and 5; the second one enters on 11 and traverses the intersections 8, 1, 2, 3, 4, making a stop before 8, 2 and 4. The frequencies of the buses is 1 bus for 3 time steps for the first line and 1 bus for 5 time steps for the second line.

It is interesting to plot some lines of the Ricatti matrices because in some sense it shows the impact on the green ligths of the flows of the precedent arcs, in figure **??** we show the coefficients in line 3, corresponding to the first phase of the second junction. We can see that the flows impact the most come from arcs

In the figures 6 and 7 we show the the resulting flows and green times for arcs 7 and 27.

8 CONCLUSIONS

In this paper we have presented three traffic regulation strategies for urban networks. They were tested in simulation and compared to the TUC strategy which doesn't include the PT in the command. The paper shows that the best way to favour the PT without deteriorating the general traffic conditions is to use a combined command : the green light duration of the intersections are calculated without taking into account the position of the PT for the arcs used by the PT. On the contrary in the optimisation criteria, we give a strong weight to the arcs which support the PT at the instants when they are on these arcs. This strategy based on the LQ theory is realistic from the viewpoint of its implementation and the numerical results show its effectiveness. More advanced tests and analyses on the networks with traffic real data will be achieved to validate it completely.

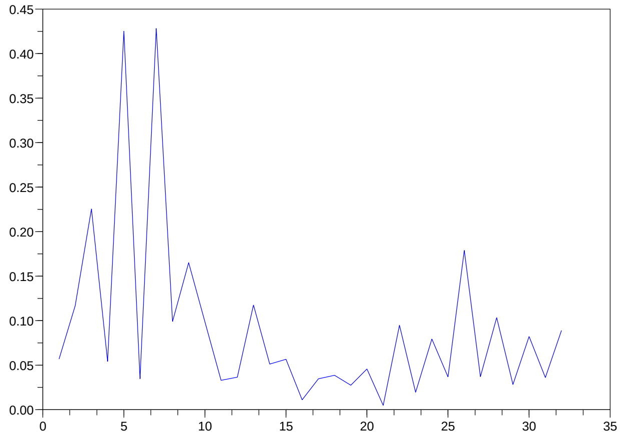

Figure 5: Third line of the control matrix.

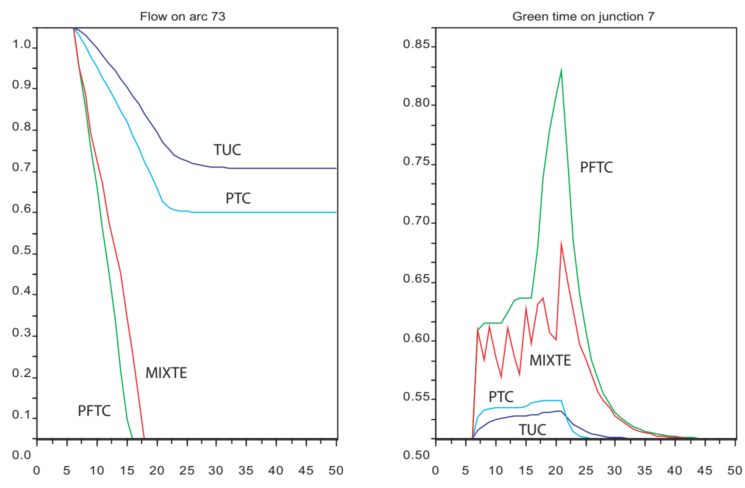

Figure 6: Results on arc 7, flow and green time of the corresponding light.

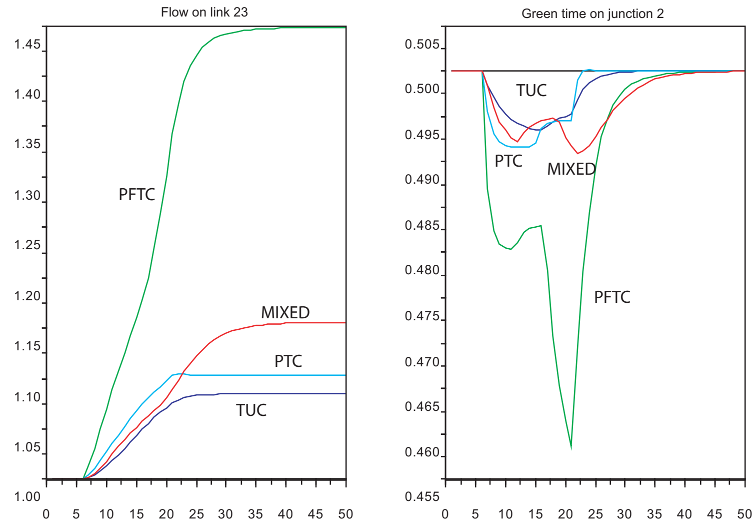

Figure 7: Results on arc 27, flow and green time of the corresponding light.

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