Mecánica Computacional Vol. XXIV
A. Larreteguy (Editor)
Buenos Aires, Argentina, Noviembre 2005

GENETIC ALGORITHMS AND BOUNDARY ELEMENTS: SOME APPLICATIONS IN OPTIMIZATION OF 2D POTENTIAL PROBLEMS

Carla T.M. Anflor[*], Rogério J. Marczak[*], Adrián P. Cisilino[†]

[*] Departamento de Engenharia Mecânica
Universidade do Rio Grande do Sul,
Rua Sarmento Leite 425, 90050-170, Porto Alegre, Brasil
e-mail: anflorgoulart@yahoo.com.br , web page: http://www-mecanica.ufrgs.br

[†] Institution Welding and Fracture Division - INTEMA - CONICET
Universidad Nacional de Mar del Plata
Av. Juan B. Justo 4302 (7600), Mar del Plata, Argentina
e-mail: *cisilino@fi.mdp.edu.ar*

Key words: genetic algorithms, topological optimization, molding die, mold heating line, potential problems, boundary elements.

Abstract. *The present work introduces the application of genetic algorithms (GA) to topology optimizations of 2D potential problems using the boundary element method (BEM). Initially, a brief description of GA basis is presented. Next, some elementary ideas on its application along with BEM procedures are summarized. The proposed approach is able to create cavities where they are less influent or simply move the existing ones, in order to, extremize a given cost function. Therefore it is possible to optimize the location of key features of the domain, like heating lines, as well as to generate the optimal topology of the problem. The performance of the proposed algorithm is assessed by some examples and discussed.*

1 INTRODUCTION

This paper presents the combination of genetic algorithms with the BEM to solve 2D Poisson equation optimization problems. Two main subjects are investigated. Firstly, the proposed algorithm is applied to determine the optimal position of heating lines in a compression molding dies. In this case the design variables are the channel's coordinates inside the domain, in order to keep the die cavity surface as close as possible to a prescribed temperature. Another objective is to verify the feasibility of generating coherent topological solutions by opening and/or closing cavities inside the domain in order to optimize a given cost function. It is important to point out that the number of cavities that will be opened or closed inside the domain is variable.

In general, when the number of cavities to be open becomes large, an optimization process tends to deliver a globally optimized solution, i.e. microstructured designs which, although strictly correct, are not practical from the engineering point of view[7]. Similarly, genetic algorithms converges to solutions with disperse cavities inside the domain. In order to circumvent this problem, the present work restricts number of cavities to be open and their position, avoiding the opening of disperse holes[5].

Some cases subjected to different types of constraints are studied using the proposed GA and discussed.

2 USING BEM FOR TOPOLOGY OPTIMIZATION

The BEM for two-dimensional potential problems is very well established. In what follows only a brief description of the method is given. Further details can be found in the literature[2,11]. Equation (1) relates the potential u and flux q over the boundary, in absence of body sources:

$$\frac{1}{2}u^{i}(x)+\int_{\Gamma}u(x)q^{*}(x,x')d\Gamma=\int_{\Gamma}q(x)u^{*}(x,x')d\Gamma \tag{1}$$

The functions *u** and *q** are the potential and flux fundamental solutions at *x* due to a unit source applied at *x'*.

$$\begin{aligned} u^{*} &= \frac{1}{2\pi}\int_{\Gamma}\ln\left(\frac{1}{r}\right)d\Gamma \\ r &= \| x - x` \| \end{aligned} \tag{2}$$

The next step consists in discretizating the boundary of the domain using *N* discontinuous linear boundary elements (see Fig. 1).

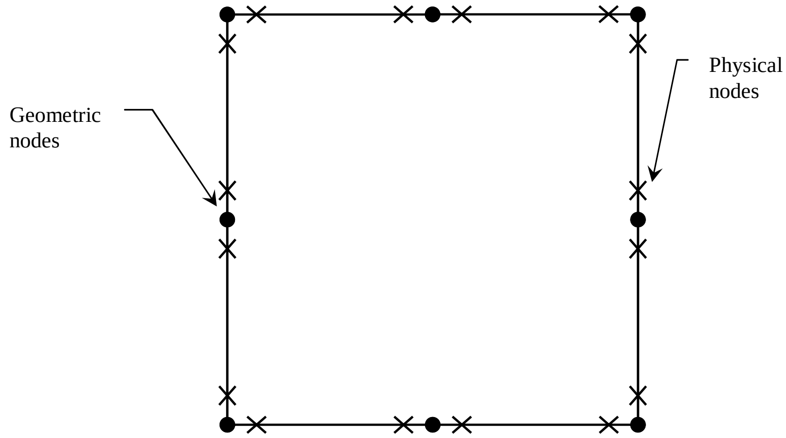


Figure 1: Boundary element discretization.

The values of u and q at any point belonging to an element can be written in terms of the nodal values and the two interpolation functions ϕ_1 and ϕ_2:

$$\begin{aligned} u(\zeta) &= \phi_1 u_1 + \phi_2 u_2 = [\phi_1\ \phi_2]\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \\ q(\zeta) &= \phi_1 q_1 + \phi_2 q_2 = [\phi_1\ \phi_2]\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \end{aligned} \tag{3}$$

where ξ is a local intrinsic coordinate defined in the range [–1,+1] and ϕ_1 and ϕ_2 are the standard discontinuous linear shape functions[2]. Considering the discretized version of Eq. (1), the integral on the left hand side over the element j can be written as

$$\int_{\Gamma j} u q^* d\Gamma = \int_{\Gamma j} [\phi_1\ \phi_2] q^* d\Gamma \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = [h_1^{ij}\ h_2^{ij}]\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \tag{4}$$

where for each element j there are two terms,

$$\begin{aligned} h_1^{ij} &= \int_{\Gamma j} \phi_1 q^* d\Gamma \\ h_2^{ij} &= \int_{\Gamma j} \phi_2 q^* d\Gamma \end{aligned} \tag{5}$$

Similarly, the integral on the right hand side results:

$$\int_{\Gamma j} q u^{*} d\Gamma = \int_{\Gamma j} [\phi_1\, \phi_2] u^{*} d\Gamma \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = [g_1^{ij}\; g_2^{ij}] \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \tag{6}$$

where

$$\begin{aligned} g_1^{ij} &= \int_{\Gamma j} \phi_1 u^{*} d\Gamma \\ g_2^{ij} &= \int_{\Gamma j} \phi_2 u^{*} d\Gamma \end{aligned} \tag{7}$$

After substitution of Eq.(7) and Eq.(5) for all *j*-elements in the discretized counterpart of Eq.(1) results:

$$c^i u^i + \sum_{j=1}^{N} H^{ij} u^j = \sum_{j=1}^{2N} G^{ij} q^j \tag{8}$$

After the imposition of the boundary conditions the system (8) can be reordered in such a way that all the unknowns are taken to the left hand side, resulting the following system of equations:

$$[A]\{X\}=\{F\} \tag{9}$$

3 IMPLEMENTATION

Genetic algorithms are efficient optimization tools, especially when the objective function has many local minimums[8]. The method aims to imitate a biological process based on the evolution of species. The concepts of genetic evolution can be easily applied in topology optimization. GA begins with an initial, randomly chosen population of chromosomes. In the present context, each chromosome contains the information about the problem topology (discrete coordinates and size of the cavities), and it is numerically represented by a string of bits. The fitness of each member of the population (i.e. their aptitude to satisfy the prescribed boundary conditions) is evaluated in this work by means of BEM models.

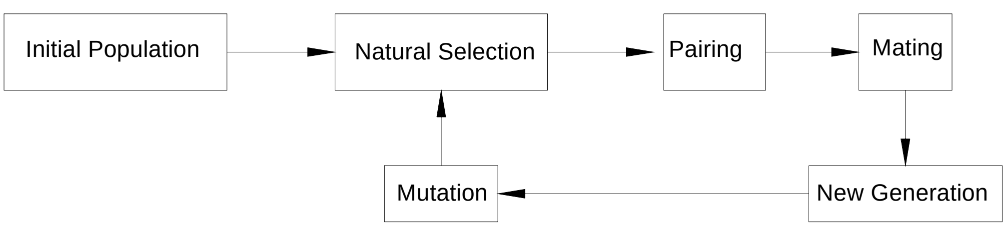


Figure 2: Generic GA Scheme.

Next, a new population is produced by operators which imitate biological processes, and are applied to the chromosomes of the previous generation. The principal genetic operators (Fig. 2) are[6]: a) natural selection; b) pairing; c) mating; and d) mutation. Other operators have been proposed in the literature[10]. In the present work, the selection operator will discard those topologies with the highest cost function values (i.e. those topologies that are worst fitted to fulfill the objective function). The paring operator defines pairs using the best fitted individuals (ie those topologies best fitted to fulfill the objective function). Mating is the next operator, and it consists in creating one or more “offsprings” (new topologies) from the “parents” that were previously selected during the pairing process. This work adopted a single crossover mating operator[13,3]. This means that every pair of parents produces two offsprings, and both will be members of the next generation. The crossover point is determined by a percentage of parent chromosomes that the offprint inherits. Table 1 presents a good demonstration of a pairing. Note that the first offsprings will receive a part of father’s chromosome and another part of mother’s chromosome. The same procedure is applied for the second offspring, but with that portions that was not used to form the first offspring.

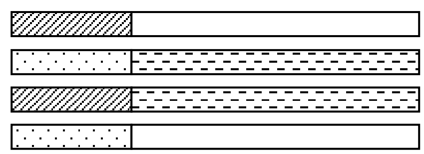
Individuals	Genetic code
Dad – topological structure 1	
Mom – topological structure 2	
Child 1 – new topological structure 3	
Child 2 – new topological structure 4	

Table 1: Crossover Point.

The last operator, the stochastic mutations, randomly alters a small portion of the chromosomes. This operator avoids the algorithm of getting trapped in a local minimum. The above described process is repeated until the maximum number of (prescribed) iterations is attained or the convergence is achieved. Figure 3 presents a scheme of the complete algorithm including the BEM and GA subroutines[4]. In many applications, the optimization process is subjected to constraints. Two types of constrains are applied: the cavities can intersect neither each other nor the external boundary. Those chromosomes (topologies) that do not satisfy the constraints are excluded from the analysis. This is done by penalization, i.e. by assigning to the excluded chromosomes a high cost value. In this way, the chromosome will be automatically eliminated by the selection operator.

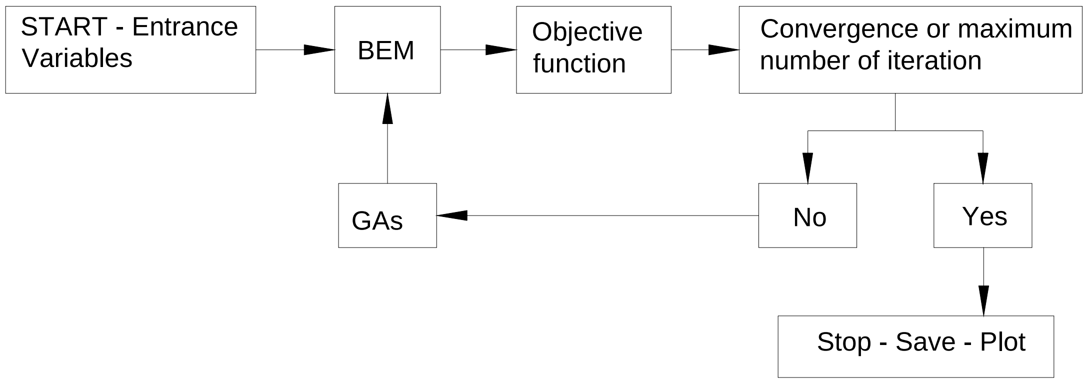


Figure 3: Routine Scheme, AG plus BEM.

4 NUMERICAL EXAMPLES

This section presents simple examples illustrating some applications of the proposed formulation. All cases refer to steady state linear heat transfer. The objective is to verify if and how the algorithm converges to an optimal and coherent topology. Two cases will be studied. The first example consists in a die for flat panel molding[9]. If a thermoset polymer is used as the material, the cavity surface of the die that contacts the polymer should be maintained at a higher temperature, which cures the polymer. This justifies a study of optimization of the heating line's position in order to maintain a linear temperature distribution on the cavity surface. Three heating lines layouts are considered.

The second example consists in heat conductor. The objective is to remove material where the internal energy density is minimum. In order to obtain a feasible result in terms of engineering manufacturing, a specially devised filter for the resultant geometries is used. The filter allows the creation of new holes in the neighborhood of the existing ones only, therefore avoiding the creation of dispersed holes inside the solution domain.

4.1 Example 1: The objective of this example is to determine the optimal position of heating lines in a compression-molding die (Fig.4) in order to obtain a maximization of the temperature on the cavity surface AB. Three situations are analyzed, namely, considering two, three and four heating lines inside the mold. The boundary conditions are shown in Fig.5. The thermal conductivity of the die material is set as 50 W/m°C. The diameter of the heating lines is 0.02 m and its temperature is prescribed to 150°C. Forty-eight discontinuous linear elements are used to model the die, taking into account its symmetry. The range of possible coordinates to locate the heating lines within the domain in *x* and *y* directions are (0.05..9.95) and (0.05..3.95), respectively, with a 0.05 m step in both directions. The initial position of the heating lines is randomly generated.

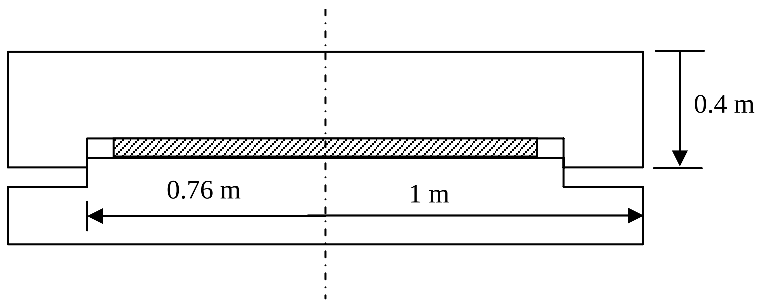


Fig. 4 : Compression mold.

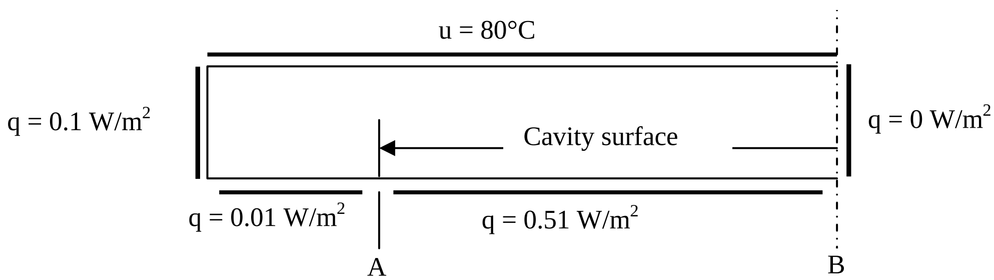


Fig.5 : Boundary conditions.

The objective function is defined by the following boundary integral:

$$\phi = \frac{\int_{AB} u \, dS}{\overline{AB}} - Td \qquad (10)$$

which can be expressed in terms of the boundary unknowns:

$$\phi = \frac{\sum_{i=1}^{n} u_i}{n} - Td \qquad (11)$$

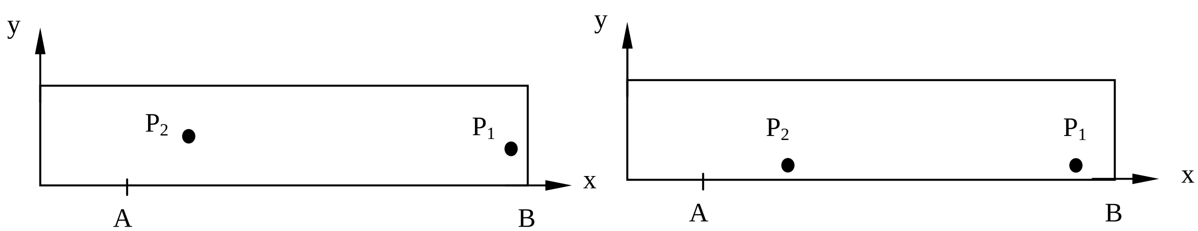
where n is the number of physical nodes along the line AB. Therefore, eq.(11) is used to minimize the difference between the mold cavity temperature and the prescribed target temperature T_d, along the line AB.

<u>Case 1:</u> Two heating lines: In this case the mold has two heating lines, initially placed as depicted in fig.6(a). Table 2 presents the genetic parameters used to set the GA.

Genetic parameters	
Initial population	128
Population size	64
Crossover probability	0.4
Mutation probability	3%

Table 2: Genetic parameters.

Figures 6*a* and 6*b* compares the initial and optimal designs respectively. Figure 7 presents the temperature distribution along cavity surface AB for both designs.



P_1 = (0.90 , 0.15) P_2 = (0.45 , 0.20)
(a)

P_1 = (0.85 , 0.05) P_2 = (0.45 , 0.05)
(b)

Fig.6 : Initial design (a). Optimized design (b) – two heating lines.

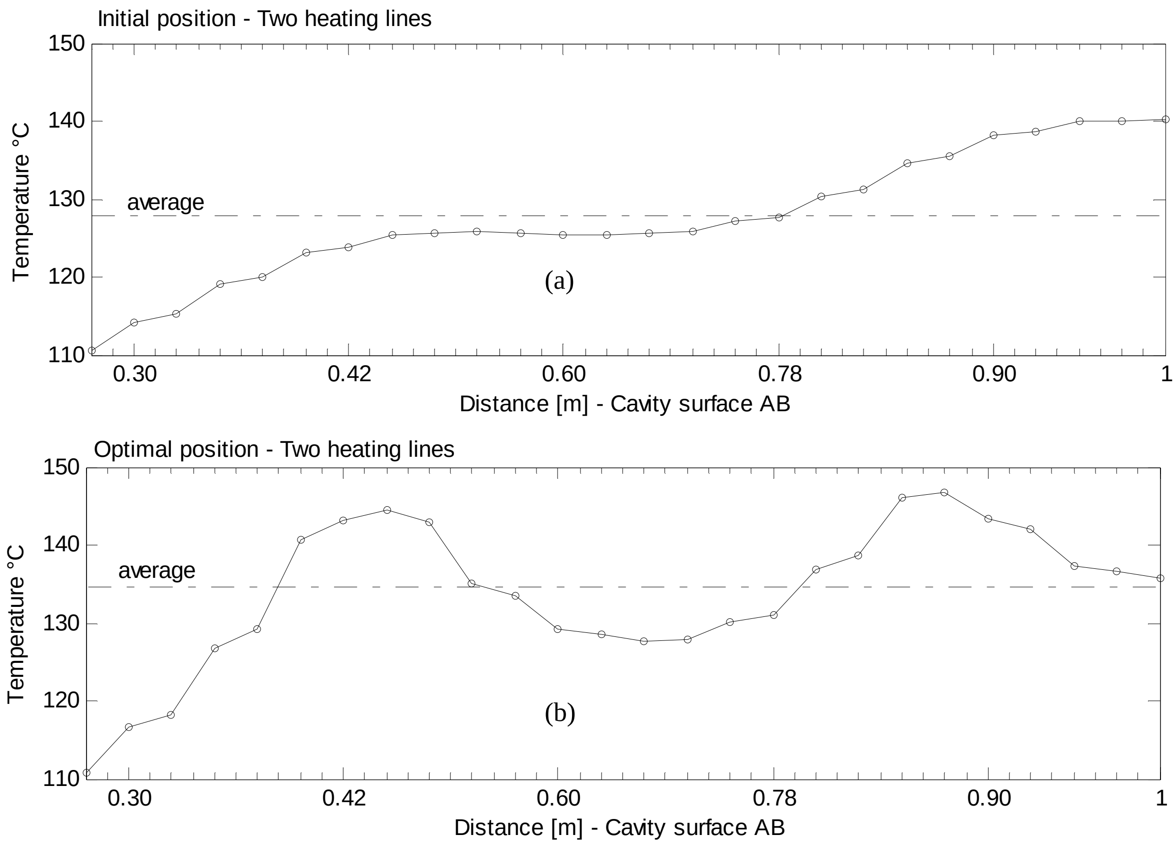


Fig.7 : Temperature distribution along the cavity surface (AB) for two heatlines.

Figure 8 presents the evolution of the normalized cost function

$$Cost = \frac{\phi}{Td} \tag{12}$$

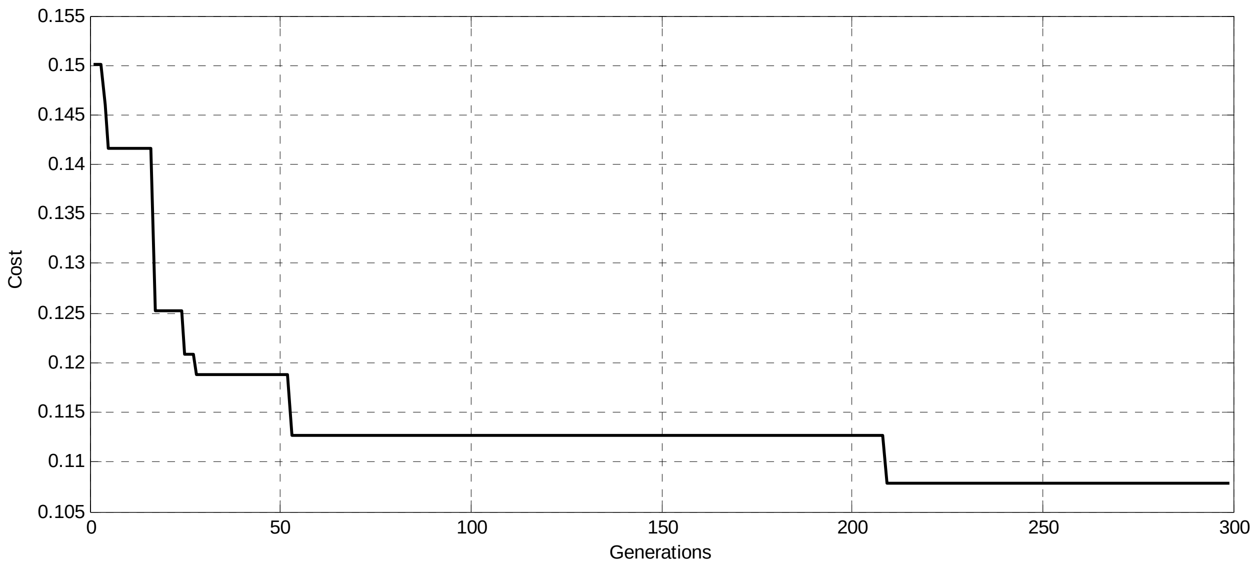


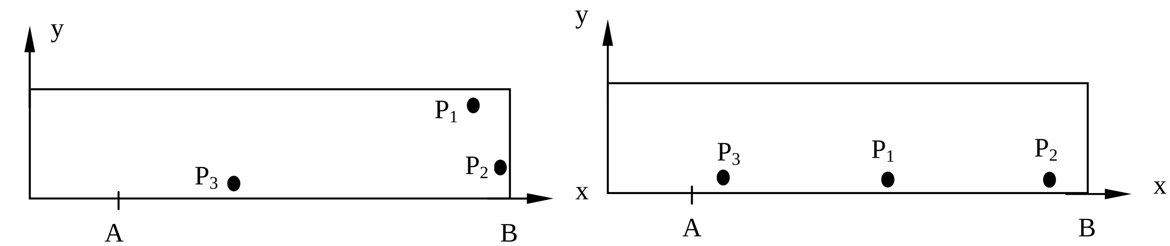
Fig. 8 : Cost Function convergence for two heating lines.

Case 2: Three heating lines: In order to achieve a more uniform temperature distribution along the cavity surface, three heating lines are considered in this examples. Table 3 presents the genetic parameters used to set the GA.

Genetic parameters	
Initial population	128
Population size	64
Crossover probability	0.4
Mutation probability	3%

Table 3: Genetic parameters.

Figures 9*a* and 9*b* compares the initial and optimal designs. As is expected, the algorithm successfully brought the heating lines closer to the mold surface, as shown in fig.9*b*.



P_1 = (0.85 , 0.35) P_2 = (0.95 , 0.1)
P_3 = (0.50 , 0.05)

P_1 = (0.60 , 0.05) P_2 = (0.85 , 0.05)
P_3 = (0.35 , 0.05)

Fig.9 : Initial design (a). Optimized design (b) – three heating lines.

Figure 10 shows the distribution of the temperature along the cavity surface AB for both designs. Clearly, the resulting temperature distribution is more uniform than the one obtained with two heating lines. At the same time the average temperature results higher than the one with two heating lines.

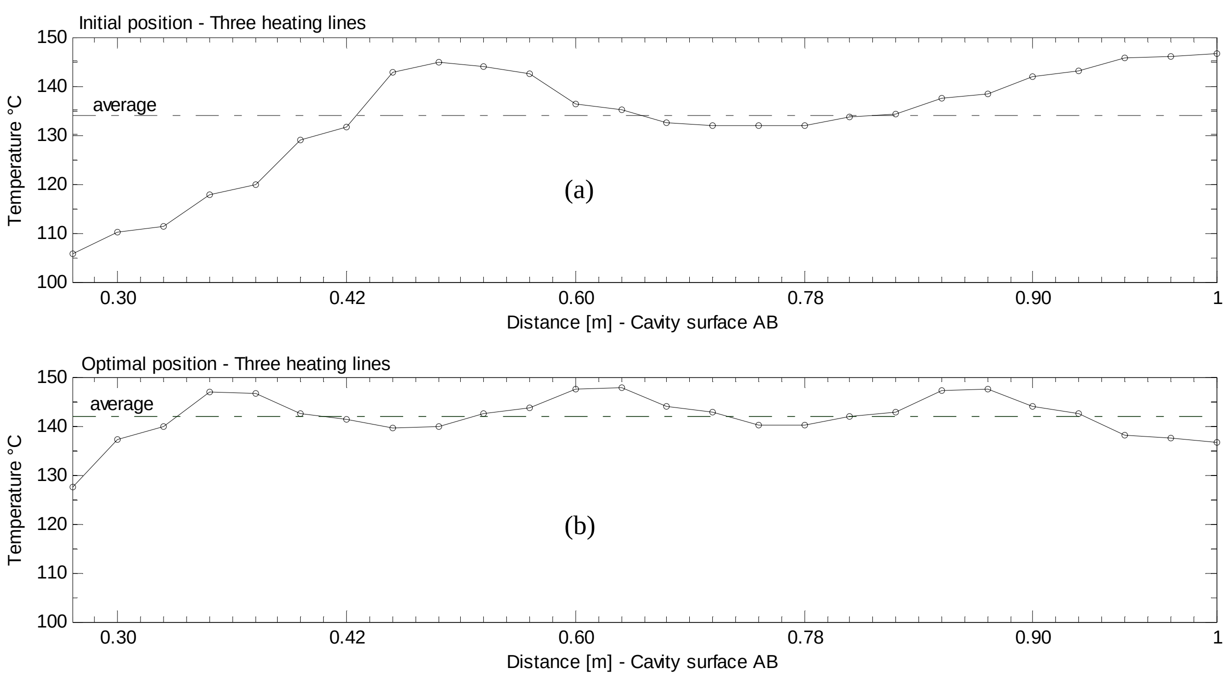


Fig.10 : Temperature distribution along the cavity surface (AB) for three heatlines.

The evolution of the normalized cost function after eq.(12) is presented in Fig. 11. It is interesting to note the increment in the number of generations, due to the increment in population size.

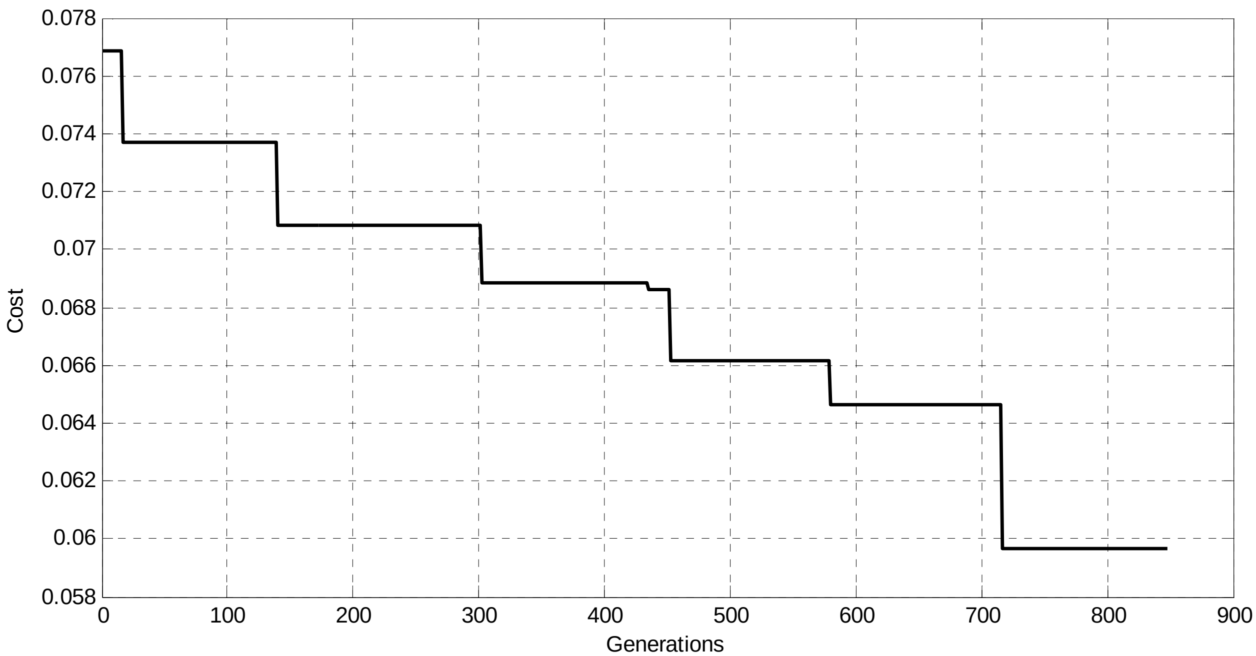


Fig. 11 : Cost Function convergence for three heating lines.

Case 3: **Four heating lines:** Yet another design was test for comparison purposes, this time using four heating lines inside the mold. Table 4 presents the genetic parameters used to set the GA.

Genetic parameters	
Initial population	600
Population size	500
Crossover probability	0.4
Mutation probability	5%

Table 4: Genetic parameters.

Figures 12*a* and fig.12*b* shows the initial and the final designs obtained. Figure12*b* shows one of the heating lines placed near to point A, in order to eliminate the temperature gradient around that point, as depicted in fig.7*b* and fig.10*b*.

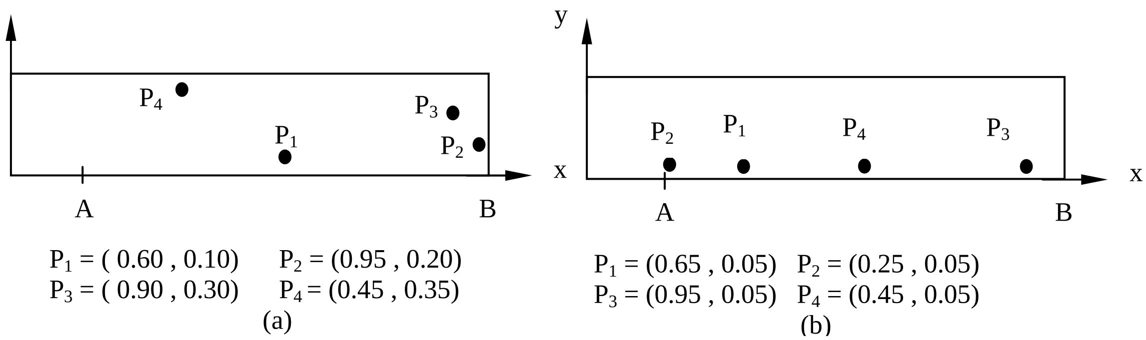


Fig. 12: Initial design (a). Optimized design (b) – four heating lines.

The distribution of the temperature on the cavity surface is presented in Figure 13 for the initial and optimized temperature distributions. Figure 14 presents the evolution of the normalized cost function.

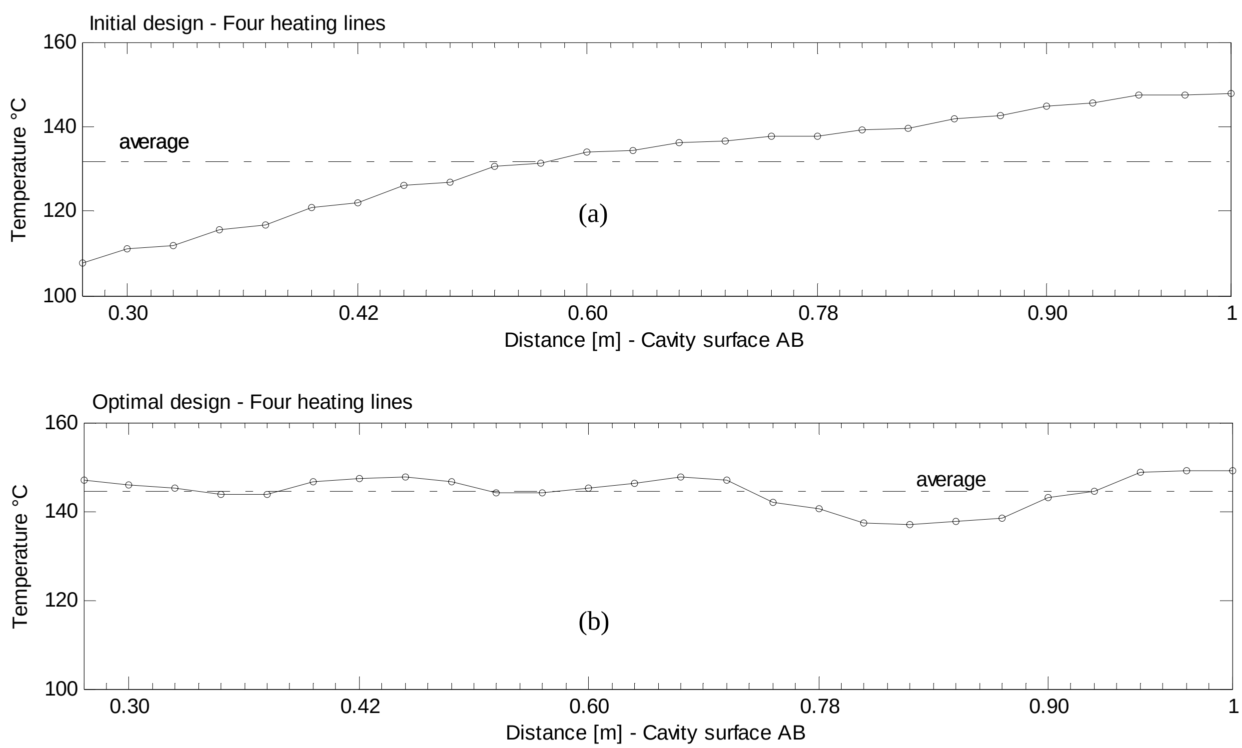


Fig. 13 : Temperature distribution along the cavity surface (AB) for four heatlines.

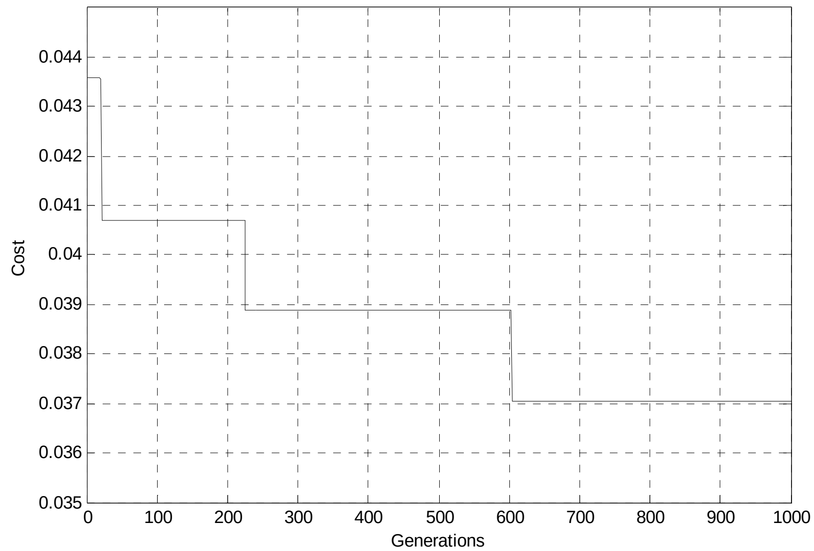


Fig. 14: Cost Function convergence for three heating lines.

4.2 Example 2: This example refers to a square heat conductor of dimensions 9m × 9m, submitted to the boundary conditions depicted in Fig.15*a*. The thermal conductivity is set to $k = 1$ W/m^2. Thirty-two linear discontinuous elements are used in the discretization of the external boundary. The positions indicated in Fig. 15*b* are the possible locations for the cavities. The cavities are square shaped, and they are discretized using eight boundary elements. The boundaries of the cavities are considered insulated. The number of cavities is set equal to 8 and kept constant during the optimization process in order to remove a constant volume fraction (32%). The cost function is chosen as the difference between the external work of the original domain and the external work of the current solution. This allows the removal of material where the internal energy density is less significant.

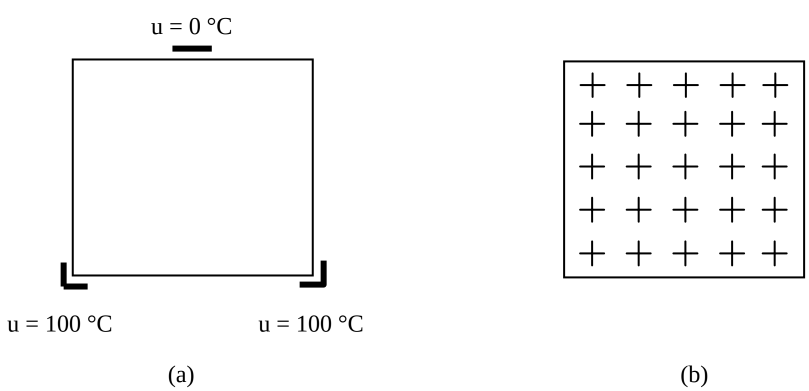


Figure 15: Example 2. (a) Boundary conditions. (b) Admissible hole locations.

A specially devised filter is used to eliminate disperse cavities, which could generate microstructured designs. This filter allows the creation of a new cavity in the neighborhood of an existing one, only. Figure 16 depicts an example of undesirable topology and its new placement after filter action.

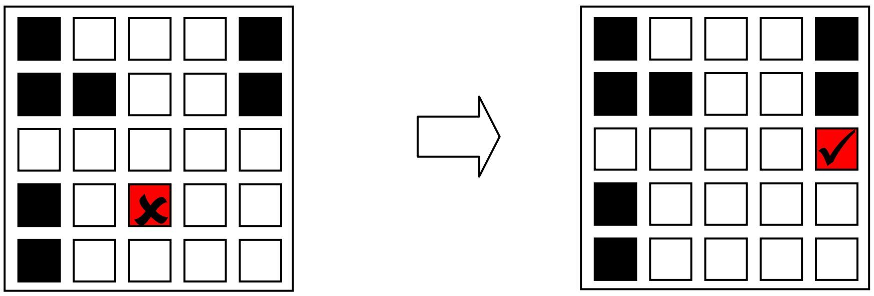


Figure 16: Filtering the creation of isolated cavities.

Genetic parameters	
Initial population	128
Population size	64
Crossover probability	0.4
Mutation probability	3%

Table 5: Genetic parameters.

Figure 17 shows the evolution of the topology along the iterative process and Fig.18 presents the corresponding cost evolution history.

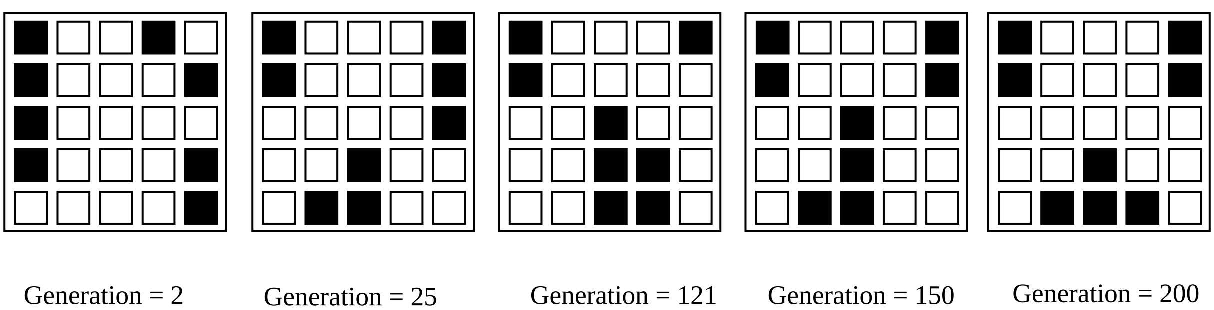


Figure 17: Intermediary solutions of example 2.

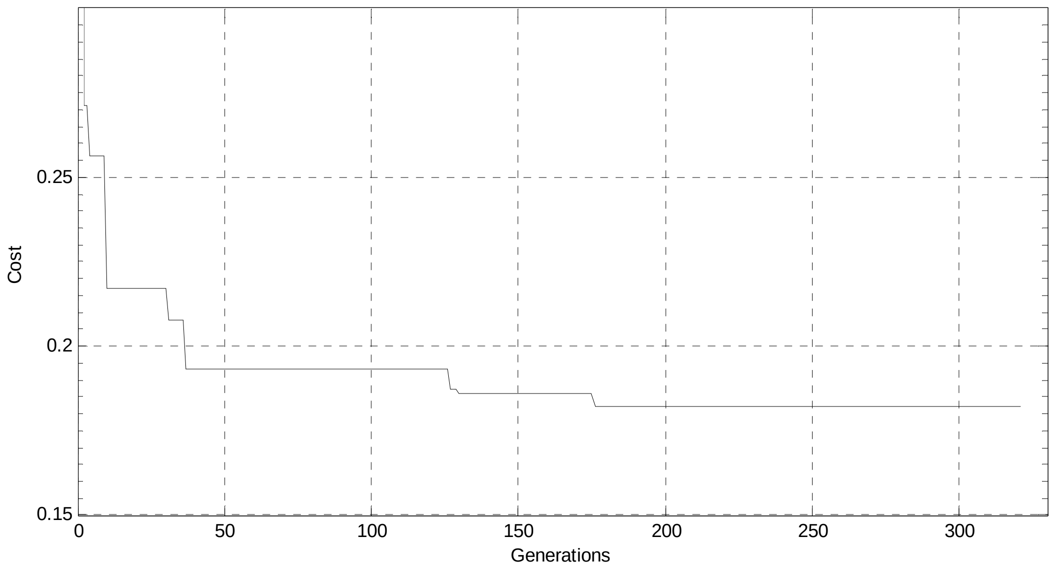


Figure 18: Cost function evolution for example 2.

Figure 19 presents a comparison of the results for example 2 with other available solution obtained by a BEM topological-shape-sensitivity approach[1,12]. It is clear that the proposed algorithm is able to remove material where it is less necessary. In a fully automated structure optimization procedure, the final topology (Fig. 19*a*) should be interpreted to generate engineering designs.

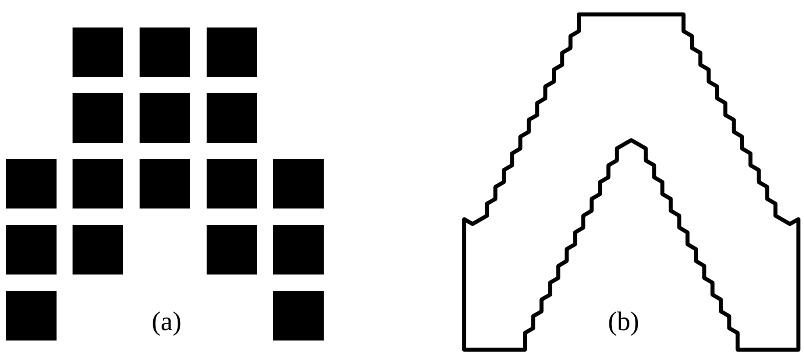


Figure 19: Comparison of the results. (a) Present work. (b) Topological-shape-sensitivity result[12].

5 CONCLUSIONS

This work presented a topological optimization strategy for 2D potential problems using boundary elements and genetic algorithms. The BEM formulation is based on discontinuous linear boundary elements and it was applied to solve two heat transfer examples.

The solutions for the first example show satisfactory results for all cases presented. The final results demonstrate that GA+BEM can be used as an efficient tool for determining the number and positions of the heating lines.

The proposed approach was applied to solve a more complex case in the second example, and proved that the correct topology can be achieved after a few iterations. The good numerical performance was due to the application of the aforementioned filter, which reduced drastically the number of necessary iterations and also avoided the use of penalization, two common drawbacks in GA algorithms.

From the results obtained, it is clear that the proposed methodology has potential to be extended to other classes of problem.

6 ACKNOWLEDGEMENTS

Carla T.M. Anflor thanks the CNPq for a fellowship grant. Rogério J. Marczak wishes to acknowledge the financial support provided by FAPERGS, through grant 02/1288-9. A. Cisilino wishes to acknowledge the financial support provided by ANPYCT of the República Argentina through grant PICT 12-12528.

7 REFERENCES

[1] A. Nowotny, R. Feijóo, E. Taroco and C. Padra, “Topological-shape sensitivity Analysis”, *Comput. Methods Appl. Mech. Engrg.*, **192**, 803–829, (2003).

[2] C.A. Brebbia and J. Dominguez, *Boundary Elements an Introductory Course*, McGraw-Hill, (1992).

[3] C. Anflor, H.Santanna, R.Marczak and A. Cisilino, “Topology optimization of 2D potential problems using boundary elements and genetic algorithms”, *International Conference on Boundary Element Techniques VI*, Edited by A P S Selvadurai, C L Tan and M H Aliabadi, 305-310 (2005).

[4] C. Anflor, A. Cisilino and R.J. Marczak, “Searching topological optimization of 2D potential problems with genetic algorithms and BEM”, *Proc. 6th World Congresses of Structural and Multidisciplinary Optimization,* (2005).

[5] C. Anflor, R.Marczak and A. Cisilino, “Using Genetic Algorithms and Boundary Elements to optimize the topology of 2D potential problems”, *In: Proceedings of the XXVI Iberian Latin-American Congress on Computational Methods in Engineering - CILAMCE 2005, Brazilian Assoc. for Comp. Mechanics & Latin American Assoc. of Comp. Methods in Engineering, Guarapari - ES, Brazil* (submitted for publication).

[6] D.E. Goldberg, *Genetic Algorithms in search optimization and machine learning Reading*, Addison-Wesley, (1989).
[7] M.F.Bendsoe, *Method for the optimization of structural topology shape and material*, The technical university of Denmark, (1994).
[8] K.L. Katsifarakis, D.K. Karpouzos and N. Theodossiou, "Combined use of BEM and genetic algorithms in groundwater flow and mass transport problems", *Engineering Analysis with Boundary Elements*, **23**, 555–565 (1999).
[9] M.R. Barone and D.A.Caulk, "Optimal arrangement of holes in a two-dimensional heat coduction by a special boundary integral method". *International Journal for Numerical Methods in Engineering*, **18**, 675-685 (1982).
[10] N.S. Mera, L. Elliott,and D.B. Ingham, "Numerical solution of a boundary detection problem using genetic algorithms". *Engineering Analysis with Boundary Elements*, **28**, 405–411, (2004).
[11] P.K. Banerjee, *The Boundary Element Methods in Engineering*, McGraw-Hill, (1994).
[12] R.J. Marczak, "Topology Optimization and Boundary Elements – A Preliminary Implementation for Linear Heat Transfer", *Engineering Analysis with Boundary Elements* (accepted for publication).
[13] W. Cerrolaza, W. Annicchiarico and, W. Martinez, "Optimization of 2D boundary element models using b-splines and genetic algorithms", *Engineering Analyses with boundary Elements*, **24**, 427-440 (2000).