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A surface remeshing for floating-like bodies with a moving free surface

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RESUMEN

Se describe un remallado de superficies, de tipo Lagrangiano, para problemas de "tenida a la mar". Para tal fin, se considera una malla inicial de elementos finitos para la superficie mojada hidrostática del cuerpo flotante. El remallado se hace en función de la posición instantánea de la curva de intersección, entre la superficie libre móvil del flujo y la superficie rígida del casco. El desplazamiento de los nodos sobre la superficie del casco, constituye una restricción no-lineal del problema numérico. Para resolverlo, se introduce una estrategia de desplazamiento de tipo pseudo-elástico, en conjunción con una técnica mediante multiplicadores de Lagrange.

ABSTRACT

A surface remeshing, of Lagrangian type, for seakeeping-like problems is outlined. For this, an initial finite element like mesh is considered for the hydrostatic weted surface of a floating body. The remeshing is done as a function of the instantaneous position for the intersection curve between the moving free surface and the rigid body surface. The node displacements on the body surface, is a non-lineal restriction of the numerical problem. For solving, an pseudo-elastic displacement strategy is introduced, in conjunction with a Lagrangian multipliers technique.

INTRODUCTION

In previous works, numerical methods for potential flows with a free surface have been considered, for instance, sloshing and ship-like hydrodynamics. In the later case, wave-resistance [3, 7, 6] and seakeeping [11, 4] ones. The basic numerical scheme chosen was the boundary element method, or panel method, e.g. see Morino [8] in the aerodynamics context, and Ohkusu [10] in the naval one. For seakeeping ship motions see [15, 1, 14, 2, 9].
In [5], a Lagrangian-type panel method, in the time domain, for potential flows with a moving free surface was proposed. After a spatial semi-discretization, with a low-order scheme, the instantaneous velocity-potential and normal displacement on the moving free surface, were obtained by means of a time-marching scheme. The kinematic and dynamic boundary conditions, at the free surface, were non-lineal restrictions over the related Ordinary Differential Equation

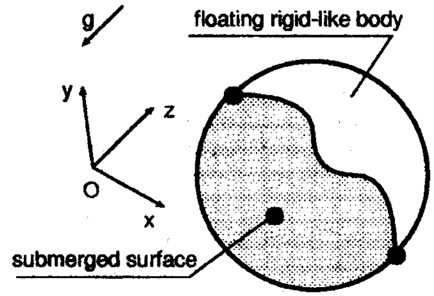

Figure 1: Geometrical description for a moving wetted surface on a floating rigid body.

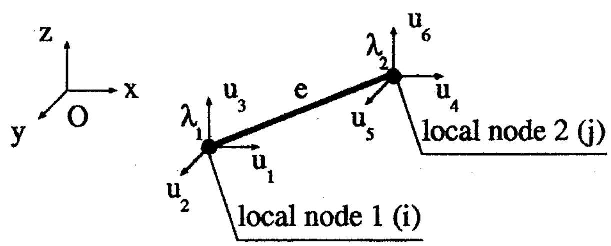

Figure 2: Extended degree of freedom for a truss e-element: u_1, u_2, u_3, λ_1 at the local node 1, and u_4, u_5, u_6, λ_2 at the local node 2.

system and, for handle them, an alternative Steklov-Poincaré operator technique was proposed. But, an unsolved geometrical problem was a remeshing strategy for the instantaneous wetted hull surface so, in this work, we will concentrate in a strategy for this.

Thus, let us consider an initial finite element triangular mesh, for the hydrostatic weted surface of a floating body, for instance, on an hemispherical one, see figure 1. Let us suppose, that we replace the triangular elements by truss ones, which are assumed as uniform, linearly elastic, pin-connected at its ends, and 3D axially loaded [13]. Now, let us introduce a prescribed perturbation on the boundary curve of the hemispherical surface, and we wish to move the nodes over the prescribed rigid surface, in such way, that the new mesh should fit with the perturbed boundary curve, and its mesh quality should not degrade too much. The instantaneous intersection curve, between the moving free surface and the hull surface of the floating body, is known from solving the dynamics of the free surface and its projection on the surface hull.

TRUSS FINITE ELEMENT MESH ON A PRESCRIBED SURFACE

A Lagrangian function W for the pseudo-elastic energy and restrictions on the displacement of the mesh, e.g. see [12], is chosen as

$$W = \frac{1}{2}\mathbf{u}^T\mathbf{K}\mathbf{u} + \boldsymbol{\Lambda}\mathbf{g} \; ; \qquad (1)$$

where $\mathbf{u}^T = \{\mathbf{u}_1, ..., \mathbf{u}_i, ..., \mathbf{u}_n\}$, $\boldsymbol{\Lambda}^T = \{\lambda_1, ..., \lambda_i, ..., \lambda_n\}$ and $\mathbf{g}^T = \{g_1, ..., g_i, ..., g_n\}$ are the global vectors for the displacements, Lagrange multipliers and the surface equation, respectively, evaluated at the nodes. The (implicit) surface equation $g(x, y, z) = 0$ is assumed differentiable enough. The stiffness matrix $\mathbf{K}$ is obtained by means of a standard finite element discretization with truss elements, while $\mathbf{u}_i = (u, v, w)_i$, λ_i and $g_i = g_i(x, y, z) = 0$ are the displacement,

Lagrangian multiplier and the (implicit) surface equation, respectively, at the i-node, for $i = 1, .., N_n$, where N_n is the node number on the mesh.
The minimum "pseudo-elastic" energy of the global truss, is found performing the first derivative of Eq. 1 with respects to the global nodal displacements $\mathbf{u}$, subject to the surface restrictions $\mathbf{g} = \mathbf{0}$, non-lineal in general. As a "pseudo-elastic" modulus, the reciprocal of the bar length L^e is generally chosen, that is, $E = 1/L^e$, since thus the bar stiffness is bigger when the bar length is smaller.
As usual in finite element analysis, the total elastic energy is the sum $W = \sum_e W^e$ of the contributions W^e of each element, for $e = 1, .., N_e$, where N_e is the element number on the mesh. Choosing the local numeration shown in figure 2, we have the parameters $(u_1, u_2, u_3, \lambda_1)$ at the local node 1, and $(u_4, u_5, u_6, \lambda_2)$ at the local node 2, so the element contribution to the pseudo-elastic energy of the whole structure is reduced to

$$W^e = \frac{1}{2}(u_1 K_{11} u_1 + ... + u_6 K_{66} u_6) + \lambda_1 g_1(u_1, u_2, u_3) + \lambda_2 g_2(u_4, u_5, u_6) \; ; \qquad (2)$$

For an hemisphere of radius R, for example, we have

$$\begin{array}{l} g_1(u_1, u_2, u_3) \equiv g_i(u_1, u_2, u_3) = (x_i - u_1)^2 + (y_i - u_2)^2 + (z_i - u_3)^2 - R^2 \\ g_2(u_4, u_5, u_6) \equiv g_j(u_4, u_5, u_6) = (x_j - u_4)^2 + (y_j - u_5)^2 + (z_j - u_6)^2 - R^2 \end{array} \; ; \qquad (3)$$

The residual element vector for Eq. 2 is

$$\begin{array}{l} r_1 = W_{,u_1} = K_{11} u_1 + ... + K_{16} u_6 + \lambda_1 g_{1,1} \\ r_2 = W_{,u_2} = K_{21} u_1 + ... + K_{26} u_6 + \lambda_1 g_{1,2} \\ r_3 = W_{,u_3} = K_{31} u_1 + ... + K_{36} u_6 + \lambda_1 g_{1,3} \\ r_4 = W_{,u_4} = K_{41} u_1 + ... + K_{46} u_6 + \lambda_2 g_{2,4} \\ r_5 = W_{,u_5} = K_{51} u_1 + ... + K_{56} u_6 + \lambda_2 g_{2,5} \\ r_6 = W_{,u_6} = K_{61} u_1 + ... + K_{66} u_6 + \lambda_2 g_{2,6} \\ r_7 = W_{,u_{\lambda_1}} = g_1 \\ r_8 = W_{,u_{\lambda_2}} = g_2 \end{array} \; ; \qquad (4)$$

where $g_{\alpha,i} = \partial g_\alpha / \partial u_i$, for $\alpha = 1, 2$ (local nodes) and $i = 1, 2, 3$ (global Cartesian coordinates). Since the surface restrictions g_1 and g_2 are non-lineal in general, we employ a standard iterative solution, for instance, a Newton-Raphson scheme

$$\mathbf{r}^{k+1} = \mathbf{r}^k + \mathbf{J}^k \Delta \mathbf{x}^k \; ; \qquad (5)$$

where $\mathbf{r} = (\mathbf{r}_u, \mathbf{r}_\lambda)^T$ is the residual vector, $\mathbf{J}$ is the Jacobian matrix, assumed as not singular $\det \mathbf{J} \neq 0$, and $\Delta \mathbf{x}$ is the increment solution for $\mathbf{x} = (\mathbf{u}, \lambda)^T$. Imposing null next residual $\mathbf{r}^{k+1} = \mathbf{0}$, we arrive to

$$\begin{cases} \mathbf{J}^k \Delta \mathbf{x}^k = -\mathbf{r}^k & ; \\ \mathbf{x}^{k+1} = \mathbf{x}^k + \Delta \mathbf{x}^k & ; \end{cases} \qquad (6)$$

where the element Jacobian matrix is

$$\mathbf{J} = \begin{bmatrix} \frac{\partial r_1}{\partial u_1} & \cdots & \frac{\partial r_1}{\partial u_6} & \frac{\partial r_1}{\partial \lambda_1} & \frac{\partial r_1}{\partial \lambda_2} \\ \cdots & & & & \\ \frac{\partial r_8}{\partial u_1} & \cdots & \frac{\partial r_8}{\partial u_6} & \frac{\partial r_8}{\partial \lambda_1} & \frac{\partial r_8}{\partial \lambda_2} \end{bmatrix} \; ; \qquad (7)$$

performing the derivatives in Eq. 7, reordering and replacing in Eq. 6.1, we obtain the element Newton-Raphson system equation

$$\begin{bmatrix} \mathbf{K}_{11} + \lambda_1 \mathbf{H}_1 & \mathbf{K}_{12} & \mathbf{b}_1^T & \mathbf{0} \\ \mathbf{K}_{21} & \mathbf{K}_{22} + \lambda_2 \mathbf{H}_2 & \mathbf{0} & \mathbf{b}_2^T \\ \mathbf{b}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{b}_2 & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{u}_{123} \\ \Delta \mathbf{u}_{345} \\ \Delta \lambda_1 \\ \Delta \lambda_2 \end{bmatrix} = \begin{bmatrix} \mathbf{r}_{123} \\ \mathbf{r}_{456} \\ \mathbf{r}_7 \\ \mathbf{r}_8 \end{bmatrix} \; ; \qquad (8)$$

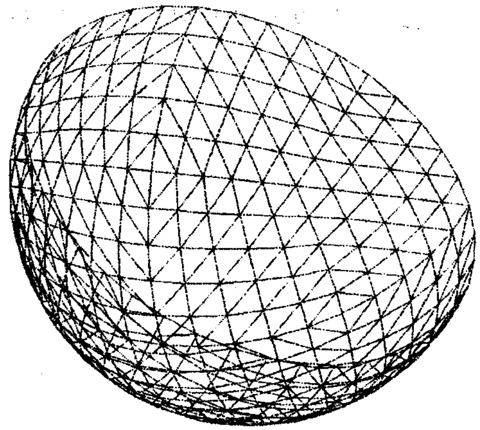

Figure 3: A 3D view of a undeformed panel mesh with 321 nodes and 600 triangular elements.

where

$$\mathbf{H}_\alpha = \begin{bmatrix} g_{\alpha,11} & g_{\alpha,12} & g_{\alpha,13} \\ g_{\alpha,21} & g_{\alpha,22} & g_{\alpha,23} \\ g_{\alpha,31} & g_{\alpha,32} & g_{\alpha,33} \end{bmatrix} ; \tag{9}$$

for $\alpha = 1,2$ are the local nodes (i,j), with $g_{\alpha,jk} = \partial^2 g_\alpha / \partial u_j \partial u_k$, where $j,k = 1,2,3$ corresponding to the x,y,z global directions, respectively, and the gradient vector

$$\mathbf{b}_\alpha = [g_{\alpha,1} \quad g_{\alpha,2} \quad g_{\alpha,3}] ; \tag{10}$$

The local mapping on the element is chosen as

$$\begin{aligned} \mathbf{u}_{123} &= (\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)^T \\ \mathbf{u}_{456} &= (\mathbf{u}_4, \mathbf{u}_5, \mathbf{u}_6)^T ; \\ \lambda_{12} &= (\lambda_1, \lambda_2)^T \end{aligned} \tag{11}$$

for the element nodal displacements, and

$$\begin{aligned} \mathbf{r}_{123} &= (\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)^T \\ \mathbf{r}_{456} &= (\mathbf{r}_4, \mathbf{r}_5, \mathbf{r}_6)^T ; \\ \mathbf{r}_{78} &= (\mathbf{r}_7, \mathbf{r}_8)^T \end{aligned} \tag{12}$$

for the element nodal residuals. The element stiffness matrix $\mathbf{K}$ is on the 3D space (6×6), and it is obtained from the local one

$$\mathbf{K}_p = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} ; \tag{13}$$

by means of the standard rotation $\mathbf{K} = \mathbf{R}^T \mathbf{K}_p \mathbf{R}$, where

$$\mathbf{R} = \begin{bmatrix} l_1 & m_1 & n_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & l_1 & m_1 & n_1 \end{bmatrix} ; \tag{14}$$

where (l_1, m_1, n_1) are the director cosines of the truss respect to the global axis x, y, z, and A, E are "pseudo" sectional area and elastic modulus, respectively. An obvious assumption for the solution of the Newton-Raphson scheme, is that the Jacobian matrix must be regular. On the other hand, an appropriate relative order for the global equations is necessary to prevent null

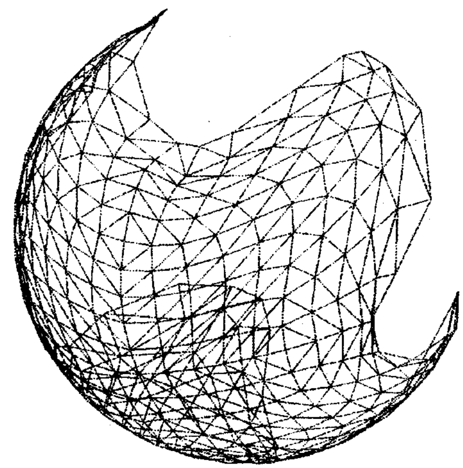

Figure 4: A 3D view of the deformed mesh obtained with the proposed method, when a large perturbation of 70 % of the sphere radius R, is imposed on its great circle.

pivots on the principal diagonal. As Cook remarks [13], in a Gauss elimination solution with pivoting on the principal diagonal, a zero pivot appears if a constraint equation is processed before any of the degree of freedom to which it is coupled. Otherwise, the null sub-matrix fills in, and the solution proceeds normally if the stiffness matrix $\mathbf{K}$ is positive definite. Then, we choose the global numeration $(u_i, v_i, w_i, \lambda_i)$ on the i-node, for $i = 1, .., N_n$. Finally, the Dirichlet boundary conditions, at the nodes with prescribed displacements, make its related Lagrangian multipliers as *passives* [12], so we impose null values for them.

NUMERICAL EXAMPLE

A simple example is included as a first validation of the proposed method, where a relative large perturbation is introduced on the intersection curve between the free surface of a fluid and a floating hemisphere. The hemisphere radius is $R = 1$, and the perturbation is a sine curve of amplitude $A = 0.7R$ and 4 periods along its boundary, which is also the intersection curve between the free surface and the floating body. For the computation of the deformed mesh on the sphere surface, we have considered a 3D pseudo-elastic problem with restrictions. Its components are truss finite elements and, for the restriction given by displacement on a prescribed surface, the sphere equation in this simple case, a Lagrange multiplier is introduced by node. Then, we have 3 displacements u, v, w and a Lagrange multiplier, as unknowns, by node. The original panel mesh is shown in figure 3, with 321 nodes, 600 triangular panels, so, with 4 degree of freedom by node, we have 1284 unknows. When a large perturbation is imposed on its maximum circle, as 70 % of its radius, the resulting deformed mesh obtained with the proposed method, is shown in figure 3.

CONCLUSIONS

Next stages should include an extension to B-splines representations for ship-like configurations, a coupled formulation between the dynamics of the free surface and the floating body, and a version for a parallel computation on the Beowulf cluster "Gerónimo" of the CIMEC Group, for instance, the HPF (High Performance Fortran) or OpenMP paradigms.

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