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DYNAMIC MODEL FOR A FLEXIBLE SPACE SYSTEM AIMING AT A ROBUST CONTROL DESIGN

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ABSTRACT

In this paper a generic model for a flexible space system consisting of an arbitrary numbers of flexible bodies connected to form a branched geometry is developed using a Lagrangian formulation. This approach is then applied to derive the equations of motion of the US Space Station first assembly flight configuration , referred as MB-1, taking into account the coupled rigid body/flexible structural dynamic interaction. The frequency characterization of the MB-1 model is determined, solving the eigenvalue problem, and the potential problem for control structure interactions (CSI) is evaluated. The dynamic equations derived in physical coordinates are transformed into a set of decoupled equations in modal state space form. It is shown that this procedure leads to computational advantages and facilitates to address issues such as model order reduction, truncation, and robustness with respect parameter variation and unmodelled dynamics in the context of the robust control design methodology, such as LQG/LTR, PRLQG and H_{00}.

INTRODUCTION

The dynamics and control of flexible space structures over the past thirty years or so has lead to an incredibly large volume of research, the reference [1] provides a extensive bibliography to survey the developments of particular importance to dynamics and control of large space structures. Originally dynamics and attitude control of satellites with flexible appendages was the major problem area which becomes more important as the size of solar panels and antenna increased requiring many modes of vibration for accurate representation of the dynamic behaviour. In more recent years the advent of the large flexible structure has compounded the problem of stability and attitude control culminating in the Space Station and the Hubble Telescope. The major problems faced by space engineers is that of manoeuvring flimsy structures and damping out subsequent vibrations by various means of closed loop active damping, ensuring stability, maintaining static shape as in the case of dish antenna and ensuring in the case of Space Station that microgravity experiments are not affected by structural vibrations. A study of the physical characteristics of many space structure components such as mentioned above indicates that dynamic modelling is an approximation to the actual system and can only be verified after the structure is in orbit and its response to disturbance behaviour can be measured. Thus the designer is faced with not knowing exactly how to model the dynamics and control of the system which has various uncertain physical parameters. Besides, due to the fact that the space structures are in general distributed parameters and, in theory, has a infinity number of degrees of freedom, for purposes of efficient computation and easy control implementation, model reduction [2] is an inevitable procedure for dynamic analysis and control design which renders a high level of uncertainty in the mathematical model describing the dynamics of the system. On the other hand, due to the inability of ground testing for model verification and the synthesis error of component modal characteristics of such a large flexible structure, it is expected that structural parameters predicted analytically may contain appreciable errors [3]. Thus, in order to compensate for both kinds of uncertainties, the control system design should be robust to unmodelled dynamics and variations in the structural parameters [4],[5].

SYSTEM OF COORDINATES AND ORIENTATION IN SPACE

The generic system model selected for study consists of an arbitrary numbers of flexible bodies connected to form a branched geometry: the central body B_0 is connected to bodies B_i (i=1,..,N). It should be noted that in deriving the equations of motion the number and locations of the bodies are initially kept arbitrary so that the configuration can be extended and used to study other phases of the Space Station assembly sequence, as well as to study a large and varied class of future spacecraft. Consider the spacecraft model in Fig.1 consisting of a central body B_0 which can be rigid or flexible, with an arbitrary number of beam and/or plate-type flexible appendages fixed to it with a fixed desired orientation. Let X,Y,Z be the inertial coordinate system with its origin located at the Earth's centre. The central body coordinate system X_0,Y_0,Z_0 is attached to it with its origin 0 located at the mass centre of the whole structure in its undeformed configuration. This choice is not mandatory since any other convenient point can be used. Also fixed to the central body, with origin located at the connecting point i , are the coordinates system X_i,Y_i,Z_i, which define relative motion between the bodies B_0 and B_i . Hence, an arbitrary mass element dm_i in body B_i can be reached through a direct path from 0 via i. As a result, the motion of dm_i caused by rigid body and flexible motion of B_0 and B_i can be expressed in terms of the inertial coordinate system. An orthogonal orbiting coordinate system X_r,Y_p,Z_y with its origin also at 0 is so oriented that X_r and Z_y are along the local horizontal and vertical, respectively, while Y_p is aligned with the orbit normal. Any spatial orientation of the frame X_0,Y_0,Z_0 with respect to the frame X_r,Y_p,Z_y can be described by three modified Eulerian rotations [6], where a roll motion is about the X_r axis, a pitch motion is about Y_p axis and a yaw motion is about Z_y axis. It should be noted that one assumes that the origin 0 of the frame X_0,Y_0,Z_0 and X_r,Y_p,Z_y remains at mass centre of the entire structure in its deformed configuration, which implies that the distance of the centre of mass in the undeformed and deformed configuration is negligible, which is quite acceptable for the purpose of the model. As shown in Fig.1, for a beam-type appendage, the Z_i axis is along the nominal beam direction, while X_i axis and Y_i axis complete the orthogonal set. For a plate-type appendage, Y_i axis is taken normal to the nominal X_i-Z_i plane, and Z_i axis normal to X_i axis in the plane of the plate.

FLEXIBILITY AND MATRIX OF ADMISSIBLE FUNCTIONS

Constructional phase of the proposed Space Station will involve a constantly evolving structure comprised of lightweight, flexible members in form of beam, plates and rigid bodies. The MB-1 Space Station configuration will consist of a main central truss, a radiator and pair solar panels, these flexible bodies are denoted by B_0, B_1, B_2, and B_3, respectively. The bodies B_0 and B_1 are treated as a free-free beam and as a clamped-free beam. The two solar panels are modelled as two clamped-free plates. The discretization of the continuum MB-1 model is carried out expressing elastic deformations in terms of a set of admissible shape functions which are somewhat arbitrary provided that they satisfy at least the geometric boundary conditions. The modes of a fixed-free and free-free beam are used to describe the plate elastic deformation.For a beam, the transverse oscillations u_x, u_y in orthogonal directions X,Y and the torsion deformation u_z about the Z axis,as well as the lateral deformation u_{XZ} for a plate are assumed to be, respectively, of the form

$$u_{ix}=\sum_{j=1}^{n}\phi_j^s(z)\,q_{xj}(t)\,,\; u_{iy}=\sum_{j=1}^{n}\phi_j^s(z)\,q_{yj}(t)\,,\; u_{iz}=\sum_{j=1}^{1}\phi_j^s(z)\,q_{zj}(t)\,,\; u_{ixz}=\sum_{j=1}^{n}\sum_{k=1}^{m}\phi_j^s(x)\,\phi_k^s q_{xzjk}(t) \quad (1)$$

where : i = the number of the body B_i (i=0,1,2,...,N); s = the type of admissible function, i.e., for a free-free beam, fixed-free beam and free-free beam in torsion, s assumes the values 1,2,3, respectively; n,m = the number of modes considered in the analysis; $\phi_j^1(z)$, $\phi_j^2(z)$, $\phi_j^3(z)$ = the characteristic shape function of a free-free beam, fixed-free beam and free-free beam in torsion, respectively; $\phi_j^1(z)\phi_k^2(x)$ = approximate shape function for a clamped-free plate; $q_x(t)$, $q_y(t)$, $q_z(t)$, $q_{xz}(t)$ = the generalized coordinates associated with the beam vibration in the X and Y direction, beam torsion about Z-axis and plate vibration in the plane XZ, respectively. The matrix of admissible function for the central body B_0 (free-free beam), for body B_1 (beam-type appendage) and bodies $B_{2,3}$ (plate-type appendage) and the elastic rotation matrix γ_i due to deformation of the central body B_0 for a point i are given, respectively, by

$$\Phi_0=\begin{bmatrix}\phi_j^1(Z_0) & 0 & -Y_0\phi_j^3(Z_0)\\ 0 & \Phi_j^1(Z_0) & X_0\Phi_j^3(Z_0)\\ 0 & 0 & 0\end{bmatrix},\;\Phi_1=\begin{bmatrix}\phi_j^2(Z_1) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \phi_j^2(Z_1)\end{bmatrix},\;\Phi_{2,3}=\begin{bmatrix}0 & 0 & 0\\ 0 & \phi_j^1(X_{2,3})\,\phi_k^{2,3}(Z_{2,3}) & 0\\ 0 & 0 & 0\end{bmatrix},\;\gamma_i=\begin{bmatrix}0 & \gamma_i^y & 0\\ \gamma_i^x & 0 & 0\\ 0 & 0 & \gamma_i^z\end{bmatrix} \quad (2)$$

The details of the derivation of Eqs.(2) can be found in [7]. The matrices of admissible functions Φ_0 and Φ_{0i} are associated with the body B_0 and with a point i (i=1,2,3) in body B_0, while $\Phi_{1,2,3}$ and the elastic rotation matrix $\gamma_{1,2,3}$ are associated with bodies B_1, B_2 and B_3, respectively.

EQUATION OF MOTION OF THE MULTIBODY SYSTEM

In the Lagrangian formulation, the major dynamic characteristics of the system are contained in its kinetic energy expression. Therefore, expressing the kinetic energy of an arbitrary multibody system in a quadratic form of the system generalized velocities, the associated mass matrix can be written in a clear and concise form suitable for differentiation as required in the Lagrangian procedure.

Kinetic Energy: The kinetic energy expression is derived for a configuration which consist of a flexible body B_i attached to a central flexible body B_0, so that the same procedure can be extended to more complex configurations. To describe the motion of B_0, one denotes the position vector from the of the inertial frame XYZ to the origen of the frame X_0,Y_0,Z_0 by the vector R, the position vector of a generic point with a differential mass element dm_0 in the body B_0 relative to X_0,Y_0,Z_0 by the vector r_0 and the elastic displacement vector of that point relative to X_0,Y_0,Z_0 by the vector u_0 which in turn is given by $u_0=\Phi_0 q_0$. The position vector of a point in the body B_0 relative to the inertial frame is given by $R_0 = R + r_0 + u_0$, and considering that the angular velocity vector of the frame X_0,Y_0,Z_0 is $\dot{\theta}$, its velocity vector is

$$\dot{R}_0 = \dot{R} + \dot{u}_0 + \dot{\theta}\times(r_0+u_0) \quad (3)$$

where the velocity vector $\dot{R}$ and $\dot{\theta}$ are the rigid body translation and rotation, respectively. Similarly, for a generic point with differential mass element dm_i in body B_i, its position vector relative to the frame X_i,Y_i,Z_i is r_i and its elastic displacement vector is u_i. The position vector of that point relative to the frame X_i,Y_i,Z_i is $R_i = R_{0i} + r_i + u_i$ and its velocity vector is given by

$$\dot{R}_i = \dot{R}_{0i} + \dot{u}_i + \dot{\theta}_i\times(r_i+u_i) \quad (4)$$

where $\dot{R}_{0i}$ is $\dot{R}_0$ evaluated at the connecting point i, which is given by

$$\dot{R}_{0i} = \dot{R} + \dot{u}_{0i} + \dot{\theta}\times(r_{0i}+u_{0i}) \tag{5}$$

where r_{0i} and u_{0i} are the position vector and the elastic displacement vector of point i, respectively, relative to the frame X_0,Y_0,Z_0, the later is given by $u_{0i}=\Phi_{0i}q_0$. $\dot{\theta}_i$ is the angular velocity vector of the frame X_i,Y_i,Z_i, given by

$$\dot{\theta}_i = \dot{\theta} + \dot{\beta}_i \tag{6}$$

where $\dot{\beta}_i$ is the elastic angular velocity vector due to the elastic deformation of the body B_0 at the connecting point i, given by $\dot{\beta}_i=\gamma_i\dot{u}_{0i}$. Substituting Eq.(6) and Eq.(5) into Eq.(4), one has

$$\dot{R}_i = \dot{R} + \dot{\theta}\times(r_{0i}+u_{0i}+r_i+u_i) + \dot{\beta}_i\times(r_i+u_i) + \dot{u}_{0i}+\dot{u}_i \tag{7}$$

which is a general expression for the velocity vector of a generic point i in body B_i.

The state of the system, described so far, is defined by relative coordinates emanating from the frame X_i,Y_i,Z_i, located at point i, together with the relative position of frame X_0,Y_0,Z_0 with respect to the inertial frame X,Y,Z. However, for a more complex structure, with several bodies forming a branched tree, the same procedure can be applied, i.e., one evaluates the vectors $\dot{R}_{i+1}$, $\dot{R}_{i,i+1}$ and $\dot{\theta}_{i+1}$ and substitute them back into the $\dot{R}_i$, $\dot{R}_i$ and $\dot{\theta}_i$, respectively. For the case with rotation between the bodies, the angular velocity associated with the rotation needs to be added to the vector $\dot{\theta}_i$ while each body is included in the formulation. All these vectors are not expressed in the same frame, the vectors R and $\dot{R}$ are given in terms of components along the frame X,Y,Z; the vectors $\dot{\theta}$, r_0, u_0, $\dot{u}_0$, r_{0i}, u_{0i}, $\dot{u}_{0i}$ and $\dot{\beta}$ are given in terms of components along the frame X_0,Y_0,Z_0; and the vectors, r_i, u_i, $\dot{u}_i$ and are given in terms of components along the frame X_i,Y_i,Z_i. Therefore, rotation matrices which allows to represent these vectors in a common frame are necessary. Hence, we define the following matrices: 1) a rotation matrix C_0, whose the elements are non-linear functions of the Euler's angles, which permits us to write the velocity vectors $\dot{R}$ in terms of components along X_0,Y_0,Z_0 in the form $V_0 = C_0\dot{R}_0$; 2) a transformation matrix C, whose the elements are trigonometric function of Euler's angles, which permits us to write the angular velocity vector of axes X_0,Y_0,Z_0 in terms of components along X_0,Y_0,Z_0 in the form $\Omega = C\dot{\theta}$; 3) a rotation matrix C_i, whose the elements are functions of the direction cosines between the frames X_0,Y_0,Z_0 and X_i,Y_i,Z_i. Considering the case of interest, where the variation of the angular displacement is small, i.e. the Euler's angles are small, one has $V_0 = \dot{R}_0$ and $\Omega = \dot{\theta}$, which means that C_0 and C are equal to unit matrix. In order to transform the equations from vector to matrix form, one represents the components of the vectors a in terms of vectrix [6], the details of these transformations can be found in [7].

The kinetic energy of a multibody system is the sum of the kinetic energies of the constituent bodies, hence, summing over the entire system consisting of N bodies (i=0,1,2,3,...,N), the total kinetic energy T expression is written as the scalar product

$$T = \frac{1}{2}\int \dot{R}_0\cdot\dot{R}_0\,dm_0 + \sum_{i=1}^{N}\frac{1}{2}\int \dot{R}_i\cdot\dot{R}_i\,dm_i \tag{8}$$

where the integration is performed for the entire spacecraft. The kinetic energy of the central body B_0 is obtained introducing Eq.(3) into the first term in the r.h.s of Eq.(8) which neglecting the quadratic terms is given by

$$T_0 = \frac{1}{2}\int(\dot{R}+\dot{\theta}\times r_0+\dot{u}_0)\cdot(\dot{R}+\dot{\theta}\times r_0+\dot{u}_0)\,dm_0 \tag{9}$$

Similarly, the kinetic energy of the body B_i is obtained introducing Eq.(7) into the second term in the r.h.s of Eq.(8) which given by

$$T_i = \sum_{i=1}^{N} \frac{1}{2}\int \{ \dot{R}+\dot{\theta}\times(r_{0i}+r_i)+\dot{\beta}_i\times r_i+\dot{u}_{0i}+\dot{u}_i\}\cdot\{\dot{R}+\dot{\theta}\times(r_{0i}+r_i)+\dot{\beta}_i\times r_i+\dot{u}_{0i}+\dot{u}_i\}\, dm_i \tag{10}$$

Considering that $u_0=\Phi_0 q_0$ and where M_0 is the mass of body B_0, one obtains the following quadratic terms, in matrix form, for the kinetic energy of body B_0.

$$\begin{gathered}\int \dot{R}\cdot\dot{R}_0\, dm_0 = \dot{R}^T\dot{R}M_0,\ \int \dot{R}\cdot(\dot{\theta}\times r_0)\, dm_0 = \dot{R}^T\int \tilde{r}_0^T dm_0\dot{\theta},\ \int \dot{R}\cdot\dot{u}_0\, dm_0 = \dot{R}^T\int \Phi_0\, dm_0\dot{q}_0 \\ \int (\dot{\theta}\times r_0)\cdot\dot{u}_0\, dm_0 = \dot{\theta}^T\int \tilde{r}\Phi_0\, dm_0\dot{q}_0,\ \int (\dot{\theta}\times r_0)\cdot(\dot{\theta}\times r_0)\, dm_0 = \dot{\theta}^T\int \tilde{r}_0^T\tilde{r}_0\, dm_0\dot{\theta},\ \int \dot{u}_0\cdot\dot{u}_0\, dm_0 = \dot{q}_0^T\int \Phi_0^T\Phi_0\, dm_0\dot{q}_0\end{gathered} \tag{11}$$

Let the time-invariant terms in Eqs.(11) that involve integration of the position vector r_0 and the matrix of admissible function Φ_0 be denoted by

$$\begin{gathered}M^0_{\dot{R}\dot{\theta}} = \int \tilde{r}_0^T dm_0 \quad ,\ M^0_{\dot{R}\dot{q}_0} = \int \Phi_0\, dm_0 \quad ,\ M^0_{\dot{\theta}\dot{q}_0} = \int \tilde{r}\Phi_0\, dm_0 \ , \\ M^0_{\dot{\theta}\dot{\theta}} = \int \tilde{r}_0^T\tilde{r}_0\, dm_0 \quad ,\ M^0_{\dot{q}_0\dot{q}_0} = \int \Phi_0^T\Phi_0\, dm_0 \quad ,\ M^0_{\dot{R}\dot{R}} = M_0\end{gathered} \tag{12}$$

Similarly, considering that $u_0=\Phi_0 q_0$, $u_{0i}=\Phi_{0i}q_0$, $\dot{\beta}_i=\gamma_i\dot{u}_{0i}$, the time-invariant terms that involve integration of the position vector r_i, r_{0i}, as well as the matrix of admissible function Φ_i, Φ_{0i} and the matrix of elastic rotation γ_i, for the kinetic energy of body B_i are given by

$$\begin{gathered}M^i_{\dot{R}\dot{R}} = \sum_{i=1}^{N} m_i \ ,\quad M^i_{\dot{R}\dot{\theta}} = \sum_{i=1}^{N} (m_i\tilde{r}_{0i} + C_i^T\int \tilde{r}_i\, dm_i C_i) \ ,\quad M_{\dot{q}_i\dot{q}_i} = \sum_{i=1}^{N}\int \Phi_i^T\Phi_i\, dm_i \\ M^i_{\dot{R}\dot{q}_i} = \sum_{i=1}^{N} (m_i\Phi_{0i} - C_i^T\int \tilde{r}_i\, dm_i C_i\gamma_i) \quad ,\quad M^i_{\dot{R}\dot{q}_i} = \sum_{i=1}^{N} C_i^T\int \Phi_i\, dm_i\end{gathered} \tag{13}$$

$$M^i_{\dot{\theta}\dot{q}_i} = (\tilde{r}_{0i}C_i^T\int \Phi_i\, dm_i + C_i^T\int \tilde{r}_i\Phi_i\, dm_i) \ ,\quad M^i_{\dot{q}_0\dot{q}_i} = (\gamma_i^T C_i^T\int \tilde{r}_i\Phi_i\, dm_i + \Phi_{0i}^T C_i^T\int \Phi_i\, dm_i) \tag{14}$$

$$\begin{aligned} M^i_{\dot{\theta}\dot{\theta}} &= \sum_{i=1}^{N} (C_i^T\int \tilde{r}_i^T\tilde{r}_i\, dm_i C_i - m_i\tilde{r}_{0i}^2 - \tilde{r}_{0i}C_i^T\int \tilde{r}_i\, dm_i C_i - C_i^T\int \tilde{r}_i\, dm_i C_i\tilde{r}_{0i}) \\ M^i_{\dot{\theta}\dot{q}_0} &= \sum_{i=1}^{N}\{(\tilde{r}_{0i}m_i + C_i^T\int \tilde{r}_i\, dm_i C_i)\Phi_{0i} + (C_i^T\int \tilde{r}_i^T\tilde{r}_i\, dm_i C_i - \tilde{r}_{0i}C_i^T\int \tilde{r}_i\, dm_i C_i)\gamma_i\} \\ M^i_{\dot{q}_0\dot{q}_0} &= \sum_{i=1}^{N} (m_i\Phi_{0i}^T\Phi_{0i} - \Phi_{0i}^T C_i^T\int \tilde{r}_i\, dm_i C_i\gamma_i + \gamma_i^T C_i^T\int \tilde{r}_i\, dm_i C_i\Phi_{0i} + \gamma_i^T C_i^T\int \tilde{r}_i^T\tilde{r}_i\, dm_i C_i\gamma_i)\end{aligned} \tag{15}$$

Adding the respective common terms of Eq.(12), (13), (14) and (15), and defining $\dot{q} = [\ \dot{R}^T\ \dot{\theta}^T\ \dot{q}_i^T]^T$ as the total vector of generalized velocities, the kinetic energy T can be written, in terms of the total mass matrix M of the multi-body system, as

$$T = \frac{1}{2}\dot{q}^T\begin{bmatrix} M_{\dot{R}\dot{R}} & M_{\dot{R}\dot{\theta}} & M_{\dot{R}\dot{q}_0} & M_{\dot{R}\dot{q}_i} \\ & M_{\dot{\theta}\dot{\theta}} & M_{\dot{\theta}\dot{q}_0} & M_{\dot{\theta}\dot{q}_i} \\ & & M_{\dot{q}_0\dot{q}_0} & M_{\dot{q}_0\dot{q}_i} \\ SYM. & & & M_{\dot{q}_i\dot{q}_i}\end{bmatrix}\dot{q} = \frac{1}{2}\dot{q}^T M\dot{q} \tag{16}$$

It should be noted that the mass matrix M is divided into submatrices, which are associated with system velocities and their interactions. The submatrix $M_{\dot{R}\dot{R}}$ is associated with rigid body translation motion of the frame X_0,Y_0,Z_0, which is diagonal with its elements equal to the total mass of the structure. These elements are invariant with respect the system coordinates. The submatrix $M_{\dot{R}\dot{\theta}}$ is the sum of first mass moment of the body B_0 and B_i about the frames X_0,Y_0,Z_0 and X_i,Y_i,Z_i, respectively. It is associated with the coupling between the translation and rotational motions. Considering the rigid body dynamics and that the frame, X_0,Y_0,Z_0 is located at the mass centre of body B_0, the term in the submatrix $M_{\dot{R}\dot{\theta}}$ associated with body B_0 is zero, which means that the translation and rotation are dynamically decoupled, the coupling that remain is due to the elastic deformation. The submatrix $M_{\dot{\theta}\dot{\theta}}$ is the sum of second mass moment of the body B_0 and B_i about the frames X_0,Y_0,Z_0 and X_i,Y_i,Z_i, respectively. It represents the conventional mass moment of inertia, and it is associated with the rigid body rotational motion. The submatrix $M_{\dot{q}\,\dot{q}}$ is associated with the deformation of body B_0 due to it own deformation and the body B_i deformation. The submatrix $M_{\dot{q}\,\dot{q}}$ is only associated with elastic deformation of the body B_i and also system coordinate independent. The off-diagonal submatrices $M_{\dot{R}\dot{q}}$,$M_{\dot{R}\dot{q}}$, $M_{\dot{\theta}\,\dot{q}}$, $M_{\dot{\theta}\,\dot{q}}$ correspond to coupling between rigid body translation, rotation and elastic deformation of body B_0 and B_i, respectively. The submatrix $M_{\dot{q}\,\dot{q}}$ is the coupling between the elastic deformations of body B_0 and B_i. All of the off-diagonal submatrices are system coordinate and matrix admissible function dependent and they can be designated as mixed-mass matrices.

Potential Energy: The potential energy of the spacecraft has contribution from two sources: gravitational potential energy and strain energy due to elastic deformation. In this study the first one is neglected. The total strain energy expression can be given by

$$V = \frac{1}{2} q^T K q \tag{17}$$

where K is the total symmetric positive definite stiffness matrix.

For the MB-1 model the bodies B_0 and B_1 are treated as beam and the strain energy expressions associated with its torsion and bending are given by

$$V_{torsion} = \frac{GJ}{2}\int_0^L \left(\frac{\partial u_{1x}}{\partial x}\right)^2 dx \quad , \quad V_{bending} = \frac{EI}{2}\int_0^L \left\{\left(\frac{\partial u_{1x}}{\partial x}\right)^2 + \left(\frac{\partial u_{1y}}{\partial x}\right)^2\right\} dx \tag{18}$$

The bodies B_2 and B_3 are modelled by plate and the strain energy expression associated with then is given by

$$V_{plate} = \frac{D}{2}\int_0^L\int_0^L \left\{\left(\frac{\partial^2 u_{1zz}}{\partial x^2}\right)^2 + \left(\frac{\partial^2 u_{1zz}}{\partial z^2}\right)^2 + 2\nu\left(\frac{\partial^2 u_{1zz}}{\partial x^2}\right)\left(\frac{\partial^2 u_{1zz}}{\partial z^2}\right) + 2(1-\nu)\left(\frac{\partial^2 u_{1zz}}{\partial x \partial z}\right)^2\right\} dx\,dz \tag{19}$$

where GJ, EI and D are the flexural rigidities for a beam (torsion and bending) and a plate, respectively, and ν is the Poisson's ratio. The strain energy used here implies small elastic deformation and rotation with shear deformation ignored.

Equations of Motion: Using the general expression of kinetic energy, potential energy and the Lagrange's equation, the linearized equations of motion can be written as

$$M \ddot{q} + K q = F_q \tag{20}$$

where F_q represents the generalized forces associated with the generalized coordinates q. The details of the derivation of the mass matrix M and stiffness K matrices can be found in [7].

EIGEN-STRUCTURE OF MB-1 DYNAMIC SYSTEM

In the MB-1 structure the main body B_0 is modeled as a free-free beam, with transverse bending in X_0-Z_0 and Y_0-Z_0 planes, and torsion about Z_0 axis. The body B_1 (radiator) is modeled as a clamped-free beam with transverse bending in X_1-Z_1 and Y_1-Z_1 planes and the bodies B_2 and B_3 are the two solar panels, modeled as clamped- free plates with its mode shape given by the product of modes of the free-free beam in $Z_{2,3}$ direction and clamped-free beam in $X_{2,3}$ direction. All bodies are rigidly attached to the core structure and the radiator and the solar panels are assumed torsionally stiff.

The frequency characterization of the MB-1's model can be determined by setting $F_q = 0$ in Eq.(20) and solving the eigenvalues problem for the undamped open-loop case, to yield the eigenvalues(natural frequency) ω_i and the modes shapes ϕ_i. Considering a model with 15 degree of freedom (DOF), consisting of 6 rigid body motions (translation and rotation); the first bending mode in X_0-Z_0 and Y_0-Z_0 planes plus torsion about Z_0 axis for the central truss; the first two bending mode in Y_1-Z_1 plane for the radiator; and the first two bending mode in $X_{2,3}$-$Y_{2,3}$ plane for the two solar panels. The frequency spectrum and the associated mode shapes of the MB-1 structure are shown in Table 1. The dimensions, masses and material properties of the MB-1 used here can be found in [8].

In order to characterize the frequencies of the MB-1, let us consider a nominal orbital altitude of 400 Km for which the orbital period is 92.61 minutes or 1.8×10^{-4} Hz. A low bandwidth attitude control system, for example, a Control Moment Gyro (CMG), which one assumes that for the Space Station will have a bandwidth in the range of 0.01 Hz to 0.05 Hz. A typical attitude control system using thrusters, for example, a Reaction Control System (RCS), with bandwidth centred at 0.5 Hz, and the range of frequencies found for the MB-1 model with 15 DOF between ω_7 and ω_{12}. These range of frequencies are shown in Fig.2 which indicates that the spectral separations of the orbital rate, the CMG, and the low frequency modes of the MB-1 structure are reasonable. However, the same cannot be said about the structural modes and the RCS, since the RCS bandwidth falls between the frequencies ω_7 and ω_{10} of the solar panels, which suggests that there is a potential control structure interaction (CSI) [9] problem between the structural flexibility and the on-board controller. A approach to control system design for purposes of avoiding control structure interaction has been to keep the loop bandwidth an order of magnitude smaller than the first structural mode.

MODAL COORDINATES AND MODAL STATE SPACE MODEL

The dynamic equations in physical coordinates derived previously can be transformed into modal coordinates considering that the mode shape vector can be combined as column vectors into the so called modal matrix Φ which contains the eigenvectors (modal modes shapes) and the square of the natural frequencies can be given by a diagonal matrix $\Omega^2 = \mathrm{diag}\{\omega_1^2, \omega_2^2, ..., \omega_n^2\}$. As a result, a coordinate transformation between the physical coordinate (q) and the modal coordinate (η) can be performed using $q = \Phi\eta$ and $F = B^*U$, where B^* is the input distribution matrix ($n x n_a$ dimensional) which describes the placement of the n_a actuators and their effect on the structure and U is the control signal. Substituting these expression into Eq.(20) and premultiply both sides by Φ^T, result in the following dynamic equation relating structure's modal coordinates to the actuator's control input

$$\ddot{\eta} + diag\{\omega_1^2, \ldots, \omega_n^2\}\,\eta = \Phi^T B^* U \qquad (21)$$

The damping can be introduced into the model expressed in physical coordinates by the modal damping matrix

$$D = \Phi^{-T}\, diag(2\zeta_1\omega_1, \ldots, 2\zeta_n\omega_n)\, \Phi^{-1} \tag{22}$$

Therefore, the system equations of motion in modal coordinates can be written as

$$\ddot{\eta} + diag\,\{2\zeta_1\omega_1, \ldots, 2\zeta_n\omega_n\}\,\dot{\eta} + diag\,\{\omega_1^2, \ldots, \omega_n^2\}\,\eta = \Phi^T B^* U \tag{23}$$

where ζ represents the assumed structural modal damping ratio.

A state space representation of the n second order Eq.(23) can be written by selecting the (2nx1) modal state vector as $X = col\{\eta, \dot{\eta}\}$, where the first n elements of the state vector X are the modal displacement η and the last n elements of X are the modal rate $\dot{\eta}$. This results in the standard state space form in modal coordinates given by

$$\dot{x} = \begin{bmatrix} 0 & I \\ -\omega_N^2 & -2\zeta_N\omega_N \end{bmatrix} x + B \begin{bmatrix} 0 \\ \Phi^T B^* \end{bmatrix} \quad , \quad y = [0 \;\; C^*\Phi]\, x \tag{24}$$

It should be noted that the modal state-space form obtained appears quite appropriated to deal with a mixed uncertainty model [4],[5]. First, because of the neglected dynamics can be relatively easily separated from the design model by a model reduction approach [2]. Second, because of the structural characteristics like modal frequencies, damping ratio and mode shape appear explicit in the model as physically meaningful parameters, which is suitable to deal with parameter variation. As a result, the overall model allows the incorporation of both sources of uncertainty into the robust controller design.

CONCLUSIONS

A general coupled rigid body/flexible structural dynamic model is developed using a Lagrangian formulation. This approach can be extended and used to study other phases of the Space Station assemble sequence, as well as to study a large and varied class of future rigid /flexible spacecraft. The frequency characterization of the MB-1 model is evaluated which has shown that there is a potential problem of control structure interactions (CSI). The dynamic set of decoupled equations obtained in modal state space form has the attractive feature of providing a controller design procedure which can deal with state-space representation form appropriate for representing errors due to parameter variation and unmodelled dynamics. As a result, this procedure leads to computational advantages and facilitates to address issues such as model order reduction, truncation, and robustness with respect parameter variation and unmodelled dynamics in the context of the robust control design methodology, such as LQG/LTR, PRLQG and H_{00}.

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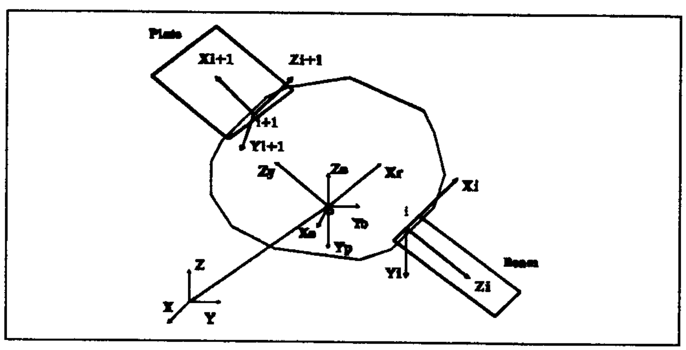

FIGURE - 1 : Coordinate System used in the Formulation.

Table -1 : Frequency Spectrum for the MB-1 Model

Frequency(Hz)	Associated Mode Shape
$\omega_{i=1,\dots,6}=0$	rotational rigid body modes
$\omega_7=0.101$	first sym. bending of solar panels.
$\omega_8=0.184$	first bending of radiator
$\omega_9=0.337$	first asym. bending of solar panels
$\omega_{10}=0.594$	second sym. bending of solar panels
$\omega_{11}=0.767$	second bending of radiator
$\omega_{12}=1.415$	second asym. bending solar panels
$\omega_{13}=5.216$	first bending X_0Z_0 plane main truss
$\omega_{14}=7.589$	first torsion about Z_0 main truss
$\omega_{15}=8.365$	first bending Y_0Z_0 plane main truss

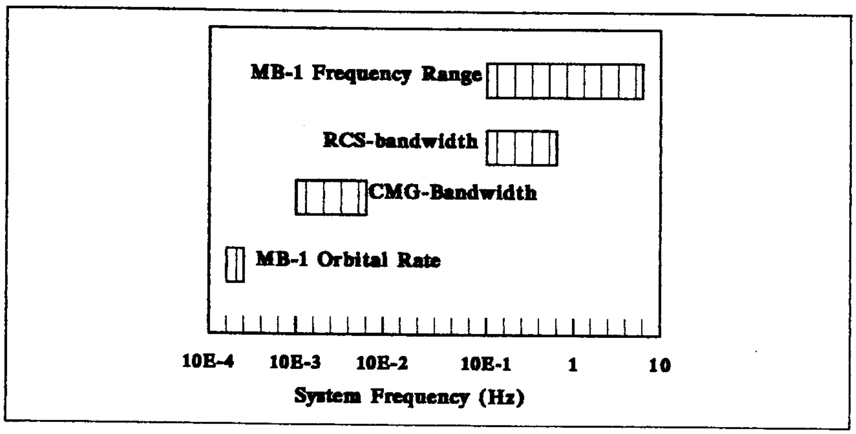

FIGURE - 2: Frequency Characterization of the MB-1 Model