Mecánica Computacional, Volumen XVIII - pp. 819-828
Compilado por: E. Dari, C. Padra y R. Saliba
San Carlos de Bariloche, noviembre de 1997

NUMERICAL COMPUTATION OF THE VIBRATION MODES OF A PLATE COUPLED WITH A FLUID

Luis M. Hervella-Nieto
Departamento de Matemática Aplicada, Universidade de Santiago de Compostela
15706 - Santiago de Compostela, Spain.

Rodolfo Rodríguez [1]
Departamento de Ingeniería Matemática, Universidad de Concepción
Casilla 4009, Concepción, Chile.

Jorge E. Solomin
Departamento de Matemática, Universidad Nacional de La Plata, C.C. 172.
1900 - La Plata, Argentina.

RESUMEN

Se introduce y analiza un método de elementos finitos para calcular los modos de vibración de un fluido compresible acoplado con una placa Reissner-Mindlin. Se incluyen experimentos numéricos que muestran el buen comportamiento del método.

ABSTRACT

A finite element method to compute the vibration modes of a compressible fluid coupled with a Reissner-Mindlin plate is introduced and analyzed. Numerical experiments showing the good performance of the method are included.

INTRODUCTION

The approximation of the vibration modes of an elastic solid interacting with a fluid is an important problem which occurs in many engineering applications. During the last years, a large amount of work has been devoted to this subject. A general overview can be found in the monographs by Morand and Ohayon [1] and Conca *et al.* [2], where numerical methods and further references are also given.

This paper deals with a particular fluid-solid interaction problem: the approximation of the small amplitude vibration modes of an elastic plate in contact with an ideal compressible fluid.

The vibration of a fluid alone is usually treated by choosing the pressure as the primary variable. However, for coupled systems, such a choice leads to non-symmetric eigenvalue problems (see, for instance, [3]). To avoid this drawback the fluid has been alternatively described by different variables: velocity potential yielding a quadratic eigenvalue problem ([4]); both, presure and displacement potential, whose discretization leads to symmetric but non-banded problem ([5]), etc.

[1] Partially supported by FONDECYT (Chile) through grant No. 1.960.615 and FONDAP-CONICYT (Chile) through Program A on Numerical Analysis.

On the other hand, the use of displacement variables to describe the fluid, gives rise to symmetric banded eigenvalue problems. However, standard discretizations of this formulation suffer from the presence of non zero frequency spurious circulation modes with no physical entity ([6]).

Recently, an alternative approach of this formulation has been analyzed. It consists of using Raviart-Thomas elements for the fluid and piecewise linear elements for the structure, adequately coupled. Non-existence of spurious modes for this discretization and optimal error estimates have been proved in [7] and [8] for two-dimensional problems. These results have been extended to 3D in [9].

The aim of this paper is to carry out a similar analysis for the interaction between a fluid and a thin structure: a plate which we model by means of Reissner-Mindlin equations in order to allow for small as well as moderately large thickness.

Because of the so called *locking* phenomenon, the standard finite element discretization of these equations leads to wrong results even for the plate alone. In order to avoid this drawback, mixed methods or reduced integration are usually applied (see for instance [10]). Recently ([11]), a locking free method (the lowest order MITC one) has been analyzed in the context of vibration problems. Optimal order error estimates independent of the thickness have been established therein.

We consider a discretization of the coupled problem involving these elements for the bending of the plate and lowest order Raviart-Thomas elements for the fluid, coupled in a non conforming way. To prove that the method is free of locking, a family of problems (one for each thickness $t > 0$) attaining a finite limit as t goes to zero, is considered and approximation results valid uniformly on t are sought.

The spectrum of the continuous problem is characterized for any $t > 0$. An asymptotic analysis is performed by considering that the densities of the plate and the fluid depend adequately on that thickness. It is shown that the spectrum of the coupled problem converge to that of a Kirchhoff plate in contact with a fluid.

Optimal order error estimates, independent of the thickness of the plate, are valid for the eigenfunctions, under mild assumptions. Typical double order estimates for the eigenvalues are also established. Finally, numerical experiments are presented, confirming the theoretical results and showing the good performance of the method.

STATEMENT OF THE PROBLEM

We consider as a model problem that of determining the natural vibration modes of a coupled system consisting of a compressible fluid contained in a three-dimensional cavity whose walls are all rigid except for one of them which is an elastic plate.

Let Ω be a polyhedral convex three-dimensional domain which we assume completely filled with an inviscid compressible fluid. Its boundary $\partial\Omega$ is the union of the convex surfaces Γ_0, Γ_1, ... , Γ_J. We assume that Γ_0 is in contact with an elastic plate of thickness t. The remaining surfaces are assumed to be perfectly rigid walls. We denote by n the outer unit normal vector to $\partial\Omega$.

The bending of the plate in contact with the fluid is modelled by means of Reissner-Mindlin equations. We denote by Γ its middle surface and consider coordinates such that the 3D reference domain for the plate is $\Gamma \times (-\frac{t}{2}, \frac{t}{2})$.

Throughout this paper we make use of the standard notation for Sobolev spaces $H^k(\Omega)$, $H_0^1(\Gamma)$, $H(\mathrm{div}, \Omega)$, etc. (see for instance [12]) and their respective norms. We also denote $\mathcal{H} := L^2(\Gamma) \times L^2(\Gamma)^2 \times L^2(\Omega)^3$, $\mathcal{X} := H_0^1(\Gamma) \times H_0^1(\Gamma)^2 \times H(\mathrm{div}, \Omega)$ and $\|\cdot\|$ the product norm of the latter.

Let (u_1^P, u_2^P, u_3^P) denote the displacement of a point (x, y, z) of the plate. In the Reissner-

Mindlin model the transversal displacement u_3^P is assumed to be independent of the z-coordinate:

$$u_3^P(x,y,z) = w(x,y), \tag{1}$$

and the "in plane" displacements u_1^P and u_2^P are given by

$$u_1^P(x,y,z) = -z\beta_1(x,y), \qquad u_2^P(x,y,z) = -z\beta_2(x,y), \tag{2}$$

with $\beta := (\beta_1, \beta_2)$ being the rotations of the fibers normal to Γ. For the sake of simplicity we assume that the plate is clamped by its whole boundary; that is, $\beta_1 = \beta_2 = w = 0$ on $\partial\Gamma$.

Under the usual assumptions of this model the dynamic response of the plate to a pressure load q exerted on one of its faces is given by displacements of the form (1)-(2) with $(w,\beta) \in H_0^1(\Gamma) \times H_0^1(\Gamma)^2$ being such that

$$\begin{aligned} t^3 a(\beta,\eta) &+ \kappa t \int_\Gamma (\nabla w - \beta)\cdot(\nabla v - \eta) \\ &+ t\int_\Gamma \rho_{\mathrm{P}} \ddot{w} v + \frac{t^3}{12}\int_\Gamma \rho_{\mathrm{P}} \ddot{\beta}\cdot\eta = \int_\Gamma qv \qquad \forall (v,\eta) \in H_0^1(\Gamma)\times H_0^1(\Gamma)^2 \end{aligned} \tag{3}$$

(see for instance [13]). In the previous equation, the double dot means second derivatives with respect to time, ρ_{P} is the density of the plate, $\kappa := \frac{Ek}{2(1+\nu)}$, where E is the Young modulus, ν the Poisson ratio of the plate and k a correction factor which is usually taken as $5/6$; finally, a is the bilinear form defined on $H_0^1(\Gamma)^2$ by

$$a(\beta,\eta) := \frac{E}{12(1-\nu^2)}\int_\Omega \left[\sum_{i,j=1}^{2} (1-\nu)\varepsilon_{ij}(\beta)\varepsilon_{ij}(\eta) + \nu \operatorname{div}\beta \operatorname{div}\eta\right].$$

On the other hand, the governing equations for the free small amplitude motions of an inviscid compressible fluid contained in Ω are given by

$$p = -\rho_{\mathrm{F}} c^2 \operatorname{div} u \quad \text{in } \Omega, \tag{4}$$

$$\rho_{\mathrm{F}} \ddot{u} = -\nabla p \quad \text{in } \Omega, \tag{5}$$

where p is the pressure, u the displacement field, ρ_{F} the density and c the acoustic speed of the fluid. Since the fluid is considered inviscid, only the normal component of the displacement vanishes on the rigid part of the cavity boundary $\Gamma_{\mathrm{R}} := \Gamma_1 \cup \cdots \cup \Gamma_J$:

$$u\cdot n = 0 \quad \text{on } \Gamma_{\mathrm{R}}. \tag{6}$$

On the other hand, the normal displacement coincides with the transverse displacement of the plate on Γ_0. Since the latter do not depend on the z-coordinate, it can be considered that the midsurface Γ (instead of Γ_0) is one of the components of $\partial\Omega$ and hence

$$u\cdot n = w \quad \text{on } \Gamma. \tag{7}$$

Now, we multiply equation (5) by a test displacement field ϕ satisfying (6), we integrate by parts and use (4) to obtain

$$\int_\Omega \rho_{\mathrm{F}} \ddot{\mathrm{u}}\cdot\phi + \int_\Omega \rho_{\mathrm{F}} c^2 \operatorname{div} u \operatorname{div}\phi = -\int_\Gamma p\phi\cdot n. \tag{8}$$

In our coupled problem, the unique load q exerted on the plate is the pressure p of the fluid. Therefore, by adding (8) to (3) and choosing test functions (v,η,ϕ) in the space

$$\mathcal{V} := \{(v,\eta,\phi) \in \mathcal{X} :\ \phi\cdot n = 0 \text{ on } \Gamma_{\mathrm{R}} \text{ and } \phi\cdot n = v \text{ on } \Gamma\},$$

we have that

$$t^3 a(\beta,\eta) + \kappa t \int_\Gamma (\nabla w - \beta)\cdot(\nabla v - \eta) + \int_\Omega \rho_F c^2 \operatorname{div} u \operatorname{div} \phi \tag{9}$$
$$= -t\int_\Gamma \rho_P \ddot{w} v - \frac{t^3}{12}\int_\Gamma \rho_P \ddot{\beta}\cdot\eta - \int_\Omega \rho_F \ddot{u}\cdot\phi.$$

To obtain the free vibration modes of this coupled problem we seek harmonic in time solutions of (9). By so doing we obtain the following spectral problem (see for instance [1]):

Find $\lambda \in \mathbb{R}$ *and* $0 \neq (w,\beta,u) \in \mathcal{V}$ *such that*

$$t^3 a(\beta,\eta) + \kappa t \int_\Gamma (\nabla w - \beta)\cdot(\nabla v - \eta) + \int_\Omega \rho_F c^2 \operatorname{div} u \operatorname{div} \phi$$
$$= \lambda\left(t\int_\Gamma \rho_P w v + \frac{t^3}{12}\int_\Gamma \rho_P \beta\cdot\eta + \int_\Omega \rho_F u\cdot\phi\right) \qquad \forall (v,\eta,\phi)\in\mathcal{V}, \tag{10}$$

where λ is the square of the angular vibration frequency.

As usual, when a displacement formulation is used for the fluid, $\lambda = 0$ turns out to be an eigenvalue of this problem; its associated eigenspace is in this case

$$\mathcal{K} := \{(0,0,\phi)\in\mathcal{V} : \ \operatorname{div}\phi = 0 \text{ in } \Omega \ \text{ and } \ \phi\cdot n = 0 \text{ on } \partial\Omega\}.$$

Because of the symmetry of (10), the eigenfunctions corresponding to non zero eigenvalues belong to the orthogonal complement of $\mathcal{K}$ in $\mathcal{V}$ with respect to the bilinear form in the right hand side of that equation. This orthogonal complement can be readily seen to coincide with

$$\mathcal{G} := \{(v,\eta,\phi)\in\mathcal{V} : \ \phi = \nabla q \text{ for some } q\in H^1(\Omega)\}.$$

Since we are interested in considering both, thin as well as moderately thick plates, the method to be used should remain stable as the thickness becomes small. To this goal, in static problems, the loads are typically assumed to depend adequately on the thickness in order to obtain a family of problems with uniformly bounded solutions: volumetric forces are supposed to be proportional to t^3 and surface loads to t^2 (see for instance [10]).

We make similar assumptions in our case. A simple way to do it is to consider densities for both, fluid and solid, depending on the thickness of the plate in the following way:

$$\rho_F = \hat{\rho}_F t^3, \qquad \rho_P = \hat{\rho}_P t^2.$$

Under these assumptions, the eigenvalues in (10) and their associated eigenfunctions are the solutions of the following rescaled problem:

Find $\lambda \in \mathbb{R}$ *and* $0 \neq (w,\beta,u) \in \mathcal{V}$ *such that*

$$a(\beta,\eta) + \frac{\kappa}{t^2}\int_\Gamma (\nabla w - \beta)\cdot(\nabla v - \eta) + \int_\Omega \hat{\rho}_F c^2 \operatorname{div} u \operatorname{div} \phi$$
$$= \lambda\left(t\int_\Gamma \hat{\rho}_P w v + \frac{t^2}{12}\int_\Gamma \hat{\rho}_P \beta\cdot\eta + \int_\Omega \hat{\rho}_F u\cdot\phi\right) \qquad \forall (v,\eta,\phi)\in\mathcal{V}, \tag{11}$$

We end this section by performing an asymptotic analysis as the thickness becomes small. For any $t > 0$, by introducing the the shear strain $\gamma := \frac{\kappa}{t^2}(\nabla w - \beta)$, the source problem associated to (11) can be stated in the following way: to find $(w,\beta,u)\in\mathcal{G}$ such that

$$\begin{cases} a(\beta,\eta) + \displaystyle\int_\Gamma \gamma\cdot(\nabla v - \eta) + \int_\Omega \hat{\rho}_F c^2 \operatorname{div} u \operatorname{div}\phi \\ \qquad\qquad = \displaystyle\int_\Gamma \hat{\rho}_P f v + \frac{t^2}{12}\int_\Gamma \hat{\rho}_P \theta\cdot\eta + \int_\Omega \hat{\rho}_F g\cdot\phi \qquad \forall (v,\eta,\phi)\in\mathcal{G}, \\ \gamma = \dfrac{\kappa}{t^2}(\nabla w - \beta). \end{cases} \tag{12}$$

In absence of the fluid, these are the standard Reissner-Mindlin equations whose solutions are known to converge to that of the mixed formulation of Kirchhoff model (see [10]). In our case the limit is $(w_0, \beta_0, u_0) \in \mathcal{G}$ such that there exists $\gamma_0 \in L^2(\Gamma)$ satisfying

$$\left\{ \begin{array}{l} a(\beta_0, \eta) + \int_\Gamma \gamma_0 \cdot (\nabla v - \eta) + \int_\Omega \hat{\rho}_{\mathrm{F}} c^2 \operatorname{div} u_0 \operatorname{div} \phi \\ \qquad\qquad = \int_\Gamma \hat{\rho}_{\mathrm{P}} f v + \int_\Omega \hat{\rho}_{\mathrm{F}} g \cdot \phi \qquad \forall (v, \eta, \phi) \in \mathcal{G}, \\ \nabla w_0 - \beta_0 = 0. \end{array} \right. \tag{13}$$

Notice that, since $\beta_0 = \nabla w_0$, by taking $\eta = \nabla v$ for $v \in H_0^2(\Gamma)$, we obtain the classical variational formulation of Kirchhoff equations coupled with those of the fluid, namely:

$$\int_\Gamma \frac{E}{12(1-\nu^2)} \Delta w_0 \, \Delta v + \int_\Omega \hat{\rho}_{\mathrm{F}} c^2 \operatorname{div} u_0 \operatorname{div} \phi = \int_\Gamma \hat{\rho}_{\mathrm{P}} f v + \int_\Omega \hat{\rho}_{\mathrm{F}} g \cdot \phi,$$

for all $(v, \phi) \in H_0^2(\Gamma) \times H(\operatorname{div}, \Omega)$ such that $\phi \cdot n = v$ on Γ and $\phi \cdot n = 0$ on Γ_{R}.

The arguments used for the plate alone (see [10]) can be easily extended to show that problem (13) satisfies both classical Brezzi's conditions. This ensures the existence of a unique solution of this problem and its continuous dependence on the data $(f, g) \in L^2(\Gamma) \times L^2(\Omega)^3$. Moreover we have the following convergence result which have been proved in [14].

Theorem 1 *There exists a constant C, independent of t, such that, for all $(f, \theta, g) \in \mathcal{H}$, if (w, β, u) is the solution of (12) and (w_0, β_0, u_0) is the solution of (13), then*

$$\|\beta - \beta_0\|_{1,\Gamma} + \|w - w_0\|_{1,\Gamma} + \|u - u_0\|_{H(\mathrm{div},\Omega)} \le Ct \left(\|f\|_{0,\Gamma} + t^2 \|\theta\|_{0,\Gamma} + \|g\|_{0,\Omega} \right).$$

In [14] it has also been proved that the eigenpairs of Kirchhoff equations coupled with the fluid are limit of those of problem (11).

DISCRETIZATION

Let $\{\mathcal{T}_h\}$ be a regular family of partitions of Ω in tetrahedra (h stands for the maximum diameter of the elements). Each $\mathcal{T}_h$, induces a triangulation on Γ:

$$\mathcal{T}_h^\Gamma := \{T \subset \Gamma : \ T \text{ is a face of a tetrahedron } K \in \mathcal{T}_h\}.$$

To approximate the fluid displacements we use lowest order Raviart-Thomas elements (see [15]):

$$R_h := \{\phi_h \in H(\operatorname{div}, \Omega) : \ \phi_h|_K \in \mathcal{P}_0^3 \oplus (x, y, z)\mathcal{P}_0 \ \ \forall K \in \mathcal{T}_h\}.$$

For the plate we consider a method analyzed in [16]. It is based on different finite element spaces for the rotations, the transverse displacement and the shear strain. For the former we take piecewise linear functions augmented in such a way that they have quadratic tangential components on the boundary of each element. Namely, for each $T \in \mathcal{T}_h^\Gamma$, let n be a unit normal on ∂T and define

$$\mathcal{Q}(T) := \{\eta \in \mathcal{P}_2(T)^2 : \ \eta \cdot n|_\ell \in \mathcal{P}_1(\ell) \ \text{ for each edge } \ell \text{ of } T\};$$

then, the finite element space for the rotations is defined by

$$H_h := \{\eta_h \in H_0^1(\Gamma)^2 : \ \eta_h|_T \in \mathcal{Q}(T) \ \ \forall T \in \mathcal{T}_h^\Gamma\}.$$

For the transverse displacements we take standard piecewise linear elements, namely,

$$W_h := \{v_h \in H_0^1(\Gamma) : \ v_h|_T \in \mathcal{P}_1(T) \ \ \forall T \in \mathcal{T}_h^\Gamma\}.$$

Finally, to discretize the shear strain we use the lowest order rotated Raviart-Thomas space

$$\Gamma_h := \{\psi_h \in H_0(\mathrm{rot}, \Omega) : \ \psi_h|_T \in \mathcal{P}_0^2 \oplus (-y, x)\mathcal{P}_0 \ \ \forall T \in \mathcal{T}_h^\Gamma\}$$

and the *reduction operator*

$$\Pi : H^1(\Gamma)^2 \cap H_0(\mathrm{rot}, \Omega) \longrightarrow \Gamma_h,$$

locally defined for each $\psi_h \in H^1(\Gamma)^2$ by (see [10, 15])

$$\int_\ell \Pi\psi_h \cdot \tau = \int_\ell \psi_h \cdot \tau,$$

for every edge ℓ of the triangulation (τ being a unit tangent vector along ℓ).

We impose weakly the interface condition (7); in fact, we take as dicrete space for the coupled problem

$$\mathcal{V}_h := \{(v_h, \eta_h, \phi_h) \in W_h \times H_h \times R_h : \ \phi_h \cdot n = 0 \text{ on } \Gamma_{\mathrm{R}} \ \text{ and } \ \int_T \phi_h \cdot n = \int_T v_h \ \ \forall T \in \mathcal{T}_h^\Gamma\}.$$

Note that, for elements in $\mathcal{V}_h$, the equality $\phi_h \cdot n = v_h$ must hold only at the baricenter of the triangles in $\mathcal{T}_h^\Gamma$. So $\mathcal{V}_h \not\subset \mathcal{V}$, giving rise to a variational crime for our method. Let us remark that if the interface condition were imposed strongly (i.e., $\phi_h \cdot n = v_h$ on Γ) then $v_h \equiv 0$.

The discrete eigenvalue problem reads:

Find $\lambda_h \in \mathbb{R}$ and $0 \neq (w_h, \beta_h, u_h) \in \mathcal{V}_h$ such that

$$\left\{\begin{aligned} & a(\beta_h, \eta_h) + \int_\Gamma \gamma_h \cdot (\nabla v_h - \Pi\eta_h) + \int_\Omega \hat{\rho}_{\mathrm{F}} c^2 \operatorname{div} u_h \operatorname{div} \phi_h \\ & \qquad = \lambda_h \left(\int_\Gamma \hat{\rho}_{\mathrm{P}} w_h v_h + \frac{t^2}{12} \int_\Gamma \hat{\rho}_{\mathrm{P}} \beta_h \cdot \eta_h + \int_\Omega \hat{\rho}_{\mathrm{F}} u_h \cdot \phi_h \right) \qquad \forall (v_h, \eta_h, \phi_h) \in \mathcal{V}_h, \\ & \gamma_h = \frac{\kappa}{t^2}(\nabla w_h - \Pi\beta_h). \end{aligned}\right. \tag{14}$$

Note that the use of the reduction operator Π leads to a second variational crime. On the other hand, $\lambda_h = 0$ turns out to be an eigenvalue of this problem with corresponding eigenspace

$$\mathcal{K}_h := \{(0, 0, \phi_h) \in \mathcal{V}_h : \ \operatorname{div} \phi_h = 0 \text{ in } \Omega \text{ and } \phi_h \cdot n = 0 \text{ on } \partial\Omega\}.$$

Under mild assumptions it can be proved that the strictly positive eigenvalues and the corresponding eigenfunctions of the discrete problem above converge to those of the continuous problem with optimal order. Furthermore, the obtained estimates are shown to be independent of the thickness of the plate (for the proof and further discussions see [14]).

Theorem 2 *Let λ_m be the m-th strictly positive eigenvalue of problem (11) and assume that it is uniformly separated of the rest of the spectrum (as t goes to zero). Let λ_{mh} be the m-th strictly positive eigenvalue of problem (14). Let (w, β, u) and (w_h, β_h, u_h) be the corresponding eigenfunctions normalized in the same manner. Then, for t and h small enough, it holds*

$$\|\beta - \beta_h\|_{1,\Gamma} + \|w - w_h\|_{1,\Gamma} + \|u - u_h\|_{H(\mathrm{div},\Omega)} \leq Ch$$

and

$$|\lambda_m - \lambda_{mh}| \leq Ch^2.$$

with a constant C independent of t and h.

NUMERICAL EXPERIMENTS

In this section we present some numerical experiments showing the good performance of the method described above. The FORTRAN code has been previously validated by applying it to test problems and comparing the results with those of a method introduced in [9] for the vibration modes of 3D structures coupled with a fluid.

We have computed the lowest frequency vibration modes of a coupled system consisting of a clamped rectangular steel plate (4 m × 6 m) moderately thick (0.5 m), in contact with a 3D cavity (4 m × 6 m × 1 m) filled with water, with its remaining walls being perfectly rigid.

We have used different meshes obtained by succesive uniform refinements of that in Figure 1 for the fluid (Figure 2 shows the corresponding induced mesh on the plate). The refinement parameter N refers to the number of vertical layers on the fluid.

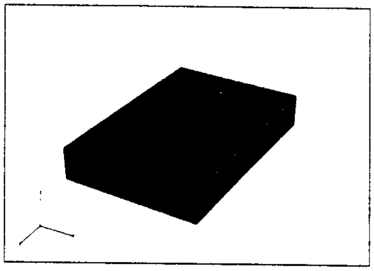

Figure 1: Mesh in the fluid for $N = 1$.

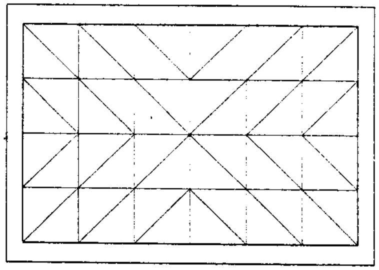

Figure 2: Mesh in the plate for $N = 1$.

The following table shows the lowest vibration frequencies of the coupled problem computed with a coarse mesh ($N = 1$) and a refined one ($N = 5$). We also include a more accurate approximation of each frequency obtained by extrapolating the results obtained for $N = 3, 4, 5$. Finally we include the corresponding extrapolated values for the uncoupled problem (i.e., the plate in vacuo and the fluid in a rigid cavity).

Table I

Mode	$N=1$	$N=5$	Extrapolated	Uncoupled
ω_1	693.931	697.293	697.558	748.748
ω_2	1045.643	1016.307	1014.627	990.784
ω_3	1077.227	1081.678	1081.908	1123.117
ω_4	1576.777	1461.715	1456.015	1473.072
ω_5	1480.657	1506.247	1507.294	1497.489

By comparing the last two columns of Table I it can be observed that the effect of the coupling is rather significative. It can also be seen that the results obtained with our method are highly precise even for the coarser mesh.

A similar experimentation has been performed with thin plates and the results are of the same quality, showing that the method is free of locking.

Figures 3-12 show the pressure in the fluid and the deflections of the plate for each coupled mode.

References

[1] H.J-P. Morand and R. Ohayon, *Interactions Fluides-Structures*, Recherches en Mathématiques Appliquées **23**, Masson, Paris, 1992.

[2] C. Conca, J. Planchard and M. Vanninathan, *Fluids and Periodic Structures*, Masson, Paris, 1992.

[3] O.C. Zienkiewicz and R.L. Taylor, *The Finite Element Method, Vol. 2*, McGraw-Hill, 1989.

[4] G.C. Everstine, *A symmetric potential formulation for fluid-structure interaction*, J. Sound Vib., **79** (1981) 157–160.

[5] H. Morand and R. Ohayon, *Substructure variational analysis of the vibrations of coupled fluid-structure systems. Finite element results*, Internat. J. Numer. Methods in Eng., **14** (1979) 741–755.

[6] M. Hamdi, Y. Ouset and G. Verchery, *A displacement method for the analysis of vibrations of coupled fluid-structure systems*, Internat. J. Numer. Methods Eng., **13** (1978) 139–150.

[7] A. Bermúdez, R. Durán, M.A. Muschietti, R. Rodríguez and J. Solomin, *Finite element vibration analysis of fluid-solid systems without spurious modes*, SIAM J. Numer. Anal., **32** (1995) 1280–1295.

[8] A. Bermúdez and R. Rodríguez, *Finite element computation of the vibration modes of a fluid-solid system*, Comp. Methods in App. Mech. and Eng. **119** (1994), 355-370. .

[9] A. Bermúdez, L. Hervella-Nieto and R. Rodríguez, *Numerical solution of three-dimensional elastoacustic problems*, in *Numerical Methods in Engineering '96*, J. Wiley & Sons, 1996, pp. 874–880.

[10] F. Brezzi and M. Fortin, *Mixed and Hybrid Finite Element Methods*, Springer-Verlag, New York, 1991.

[11] R. Durán, L. Hervella-Nieto, E. Liberman, R. Rodríguez and J. Solomin, *Approximation of the vibration modes of a plate by Reissner-Mindlin equations* (to appear).

[12] V. Girault and P.A. Raviart, *Finite Element Methods for Navier-Stokes Equations*, Springer-Verlag, Berlin, Heidelberg, New York, Tokio, 1986.

[13] T.J.R Hughes, *The Finite Element Method: Linear Static and Dinamic Finite Element Analysis*, Prentice-Hall, Englewood Cliffs, NJ, 1987.

[14] R. Durán, L. Hervella-Nieto, E. Liberman, R. Rodríguez and J. Solomin, *Finite element analysis of the vibration problem of a plate coupled with a fluid* (to appear).

[15] P.A. Raviart and J.M. Thomas, *A mixed finite element method for second order elliptic problems*, in *Mathematical Aspects of Finite Element Methods*, Lecture Notes in Mathematics **606**, Springer Verlag, Berlin, Heidelberg, New York, 1977, pp. 292-315.

[16] R. Durán and E. Liberman, *On mixed finite element methods for the Reissner-Mindlin plate model*, Math. Comp., **58** (1992) 561–573.

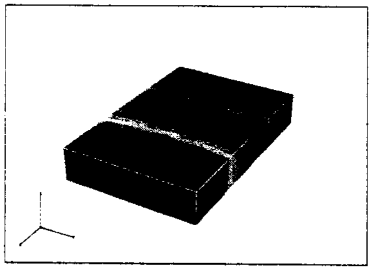

Figure 3: Pressure in the fluid, mode ω_1.

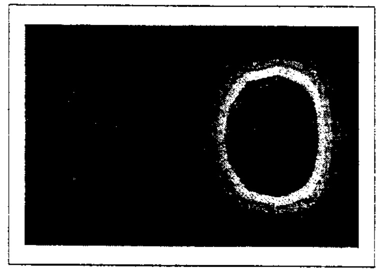

Figure 4: Deflections of the plate, mode ω_1.

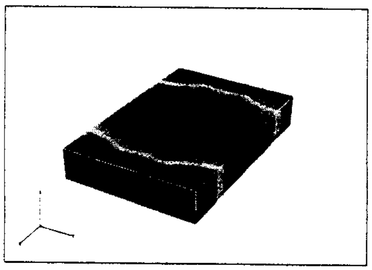

Figure 5: Pressure in the fluid, mode ω_2.

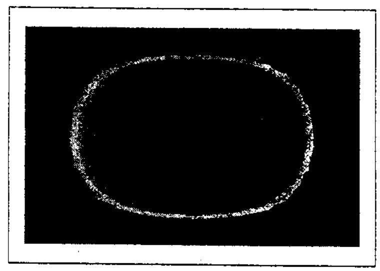

Figure 6: Deflections of the plate, mode ω_2.

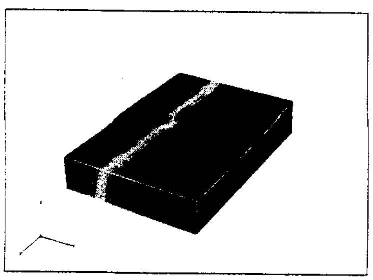

Figure 7: Pressure in the fluid, mode ω_3.

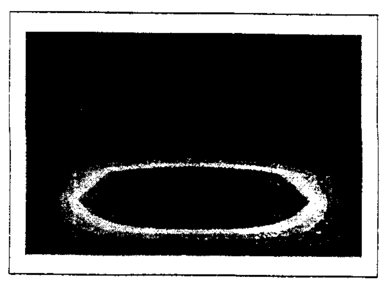

Figure 8: Deflections of the plate, mode ω_3.

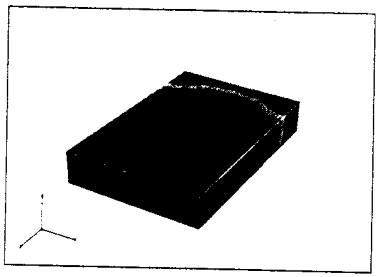

Figure 9: Pressure in the fluid, mode ω_4.

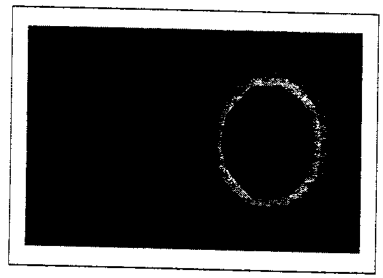

Figure 10: Deflections of the plate, mode ω_4.

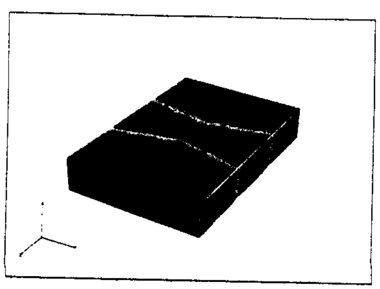

Figure 11: Pressure in the fluid, mode ω_5.

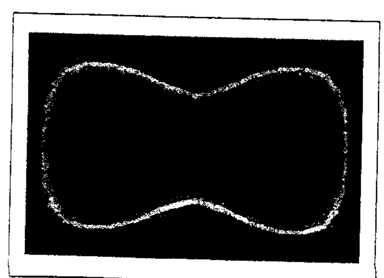

Figure 12: Deflections of the plate, mode ω_5.