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NEAR-WALL TURBULENCE AND THE EDDY VISCOSITY HYPOTHESIS

Hugo Dario Pasinato
Departamento de Ciencia y Tecnología, Universidad
Nacional de Quilmes, (1876) Bernal, Argentina

Angela O. Nieckele
Departamento de Engenharia Mecânica, PUC/Rio
Rua Marquês de São Vicente 225
(22453-900) Rio de Janeiro RJ, Brasil

SUMMARY

This papers presents a new method to the near-wall boundary condition problem for eddy viscosity turbulence models. By using a novel approach for near-wall turbulence, presented recently in literature, the eddy viscosity is computed as a function of mean flow characteristics and some parameters. A closure for the turbulent kinetic energy equation is proposed through which the turbulent kinetic energy and the turbulent kinetic energy dissipation are obtained for this region. The major properties of the new approach are the physical sound of turbulence and its robustness. Some numerical tests are presented in which the results from standard are examined, and in which, in spite of its simplicity, this method appears as a good predictor of rough effect of turbulence in the near-wall region of shear flow.

INTRODUCTION

Turbulence models used in industrial flow computations today, fall into eddy viscosity and stress transport models. Even though the last models, as the Reynolds stress, (Durbin, 1993) have more precision and physical sound of turbulence phenomena, the eddy viscosity models are still used owing to its simplicity and robustness.

The eddy viscosity main assumption is that the rough effects of turbulence can be mimicked through an additional viscosity. This 'turbulent viscosity' is function of the mean flow characteristics and some universal parameters. The most popular and used eddy viscosity model is the K-ε, (Launder and Spalding, 1974) in which the 'turbulent viscosity' in the high Reynolds region is function of a characteristic velocity fluctuation and a characteristic length. The scaling for these characteristic values gives the known relationship of the eddy viscosity as a function of turbulent kinetic energy and turbulent kinetic energy dissipation. This eddy viscosity formulae works in the high Reynolds flow region away from solid boundary, but fault in the viscous sublayer near-wall boundary. Consequently, the success of turbulent models in the prediction of wall bounded flow has depended, at a large extent, on the form that boundary condition for high Reynolds region have been given at solid wall (Jovanovic, Ye and Durst, 1995).

Two general procedure have been followed to achieve this goal. The most popular is the so-called wall function procedure, which means that high Reynolds turbulent flow is related directly with the wall through a denominated wall function. This function is a form to give the boundary condition at solid wall for the K and ε equation, and to compute an eddy viscosity for the mean flow in this low Reynolds turbulent flow. The second, more complex but not necessarily more precise, is the low Reynolds procedure, in which a new adapted model is used in the near-wall region.

In this work a new-wall function procedure based in the approach of Haritonidis (1989) is proposed. In addition, some numerical tests are shown in which this method appear as a good predictor of rough effect of turbulence in the viscous sublayer.

EDDY VISCOSITY HYPOTHESIS

The eddy viscosity is proposed in analogy with the molecular viscosity, i.e., it is assumed equivalence between the process of momentum transfer among different scales in turbulence and molecular momentum transfer in gas flow. Nonetheless it is known today that these processes are very different, from a phenomenological point of view. In spite of this arguments, the eddy viscosity hypothesis has recently gained momentum because of some results obtained by renormalization procedure applied to turbulence (Yakhot-Orszag, 1986; Kraichnan, 1987,). Basically this hypothesis is

$$\mu_t \approx \rho \langle u \rangle \ell \tag{1}$$

where u and l are the characteristic velocity fluctuation and characteristic length; and ⟨ ⟩ means a mean value. If the scaling for ε is used, and the square root of K is taken as characteristic fluctuation value, the result is

$$\mu_t \approx \rho \frac{K^2}{\varepsilon} \tag{2}$$

which is the known formulae used in the K-ε models.

In the viscous layer, near the wall, this eddy viscosity expression is not valid. Moreover, near the walls the conservation law for ε is not valid as it is used in the K-ε model. An alternative procedure consists in obtaining μ_t with other expression, and to interpolate values for K and ε in the viscous layer.

In this work μ_t is computed as a function of mean flow characteristics, and K and ε are interpolated. The algorithm is based on Haritonidis' (1989) approach, who has recently proposed an improved phenomenological profile of mean flow velocity, from the wall to the beginning of the logarithmic region. This approach is based on a new model of the mixing length and on the experimentally known bursting process in the wall region. The behavior of that new function has shown a very good agreement with experimental date. For a boundary layer without pressure gradient, the Haritonidis' model gives

$$U^+ = \frac{1}{\lambda}\tan^{-1}\left(\lambda y^+\right) \tag{3}$$

$$u_1^+ = \frac{\kappa}{(n\gamma)^{1/2}} y^+ \frac{\partial U^+}{\partial y^+} \qquad\qquad u_2^+ = \frac{(n\gamma)^{1/2}}{\kappa}\lambda^2 y^+ \tag{4}$$

where $U^+ = U/u^*$; $u_1^+ = u_1/u^*$; $u_2^+ = u_2/u^*$; $y^+ = \rho\, y\, u^*/\mu$; and $u^* = \sqrt{\tau_s/\rho}$ is the friction velocity; U is the mean velocity and u_1 e u_2 are the velocity fluctuations in the x and y directions, respectively and τ_s is the wall shear stress. The parameters λ, n, γ and κ are defined as $n = 3$, $\gamma = 0.27$, $\lambda = 0.09365$ and κ (von Kàrmán constant) = 0.41.

Using the velocity profiles given by Eq. (4), it is possible to write the Reynolds stress near the wall as

$$-\langle u_1 u_2 \rangle^+ = \lambda^2 y^{+2} \frac{\partial U^+}{\partial y^+} = \mu_t^+ \frac{\partial U^+}{\partial y^+} \tag{5}$$

therefore, the eddy viscosity in the viscous layer is

$$\mu_t^+ = \frac{\mu_t}{\mu} = \lambda^2 y^{+2} \tag{6}$$

Figure 1 illustrates the dimensionless cinematic viscosity $\nu^+ = \mu^+ = \mu_t/\mu$ as a function of the dimensionless distance y^+. This expression for the eddy viscosity should be used in the viscous layer instead of Eq. (2). The advantage of this latter expression, is that it can be computed based only on the mean flow characteristics.

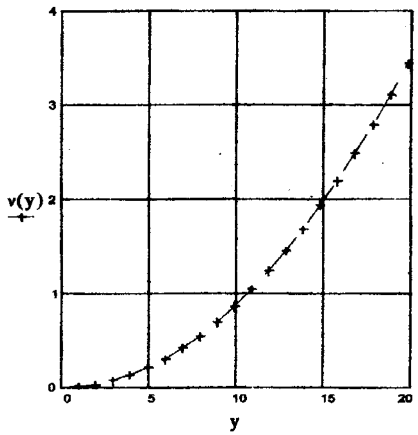

Figure 1. Non dimensional eddy viscosity in the near-wall region as a function of non dimensional wall distance

TURBULENT KINETIC ENERGY IN THE NEAR-WALL REGION

The kinetic turbulent energy K is defined as

$$K = \frac{1}{2}\left(\langle u_1 u_1 \rangle + \langle u_2 u_2 \rangle + \langle u_3 u_3 \rangle\right) \tag{7}$$

where the subscript 1 will be used for mean flow direction; 2 for cross-flow direction and 3 for the third coordinate.

In the near wall region, it is observed experimentally that the velocity fluctuations in the cross-flow directions are of the same order of magnitude, (Schlichting, 1968), therefore K can be estimated as

$$\kappa = \frac{1}{2}\left(\langle u_1 u_1 \rangle + 2 \langle u_2 u_2 \rangle \right) \tag{8}$$

The turbulent fluctuations in dimensionless form can be obtained from Eq. (4) as

$$\langle u_1 u_1 \rangle^+ = \frac{\kappa^2}{\left(n\ \gamma\ \lambda^2\right)} \lambda^2 y^{+2} \left(\frac{\partial U^+}{\partial y^+} \right)^2 \tag{9}$$

$$\langle u_2 u_2 \rangle^+ = \lambda^4 \frac{(n\gamma)}{\kappa^2} y^{+2} \tag{10}$$

Comparing Eq. (9) and (10), it is observed that $<u_2u_2>$ is much smaller than $<u_1u_1>$, thus

$$K \cong \frac{1}{2} \langle u_1 u_1 \rangle \tag{11}$$

Therefore, the expression for K in dimensionless form is

$$K^+ = \frac{1}{2}\left(\frac{k^2}{n\ \gamma\ \lambda^2} \lambda^2 y^{+2} \left(\frac{\partial U^+}{\partial y^+} \right)^2 \right) \tag{12}$$

Based on the mean flow velocity given by Eq. (3) and with $\kappa^2 / \left(n\ \gamma\ \lambda^2\right) = 23.47$, the dimensionless kinetic energy can be obtained as

$$K^+ = \frac{23.47}{2} \frac{\lambda^2 y^{+2}}{\left(1 + \lambda^2 y^{+2}\right)^2} \tag{13}$$

DISSIPATION OF TURBULENT KINETIC ENERGY IN THE NEAR-WALL REGION

The dissipation of turbulent kinetic energy for the near wall region can be derived from the conservation equation of the turbulent kinetic energy, which is obtained from the Navier-Stokes equation as

$$\frac{\partial K}{\partial t} + U_j \frac{\partial K}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\langle K u_j \rangle + \langle p u_j \rangle \right) + \langle u_i u_j \rangle \frac{\partial U_i}{\partial x_j} = \nu \left(\frac{\partial^2 K}{\partial x_j^2} \right) + \nu \left(\frac{\partial^2 \langle u_i u_j \rangle}{\partial x_i \partial x_j} \right) - \varepsilon \tag{14}$$

In the near-wall region changes in mean flow direction are slower than in cross-flow direction , therefore the characteristic length in the mean and cross-flow direction are very different. Further, it can be assumed that the flow field is 2-D and in steady state. With this hypothesis, the dissipation can be obtained from Eq. (14), for a boundary layer near the wall equation, as

$$\varepsilon = \nu\frac{\partial^2 K}{\partial x_2^2} + \nu\frac{\partial^2\langle u_2u_2\rangle}{\partial x_2^2} - \left\langle u_2\frac{\partial p}{\partial x_2}\right\rangle - \frac{\partial}{\partial x_2}\langle Ku_2\rangle - \langle u_1u_2\rangle\frac{\partial U_1}{\partial x_2} \quad (15)$$

The first term in Eq. (15) can be obtained directly from Eq. (12). The second term is negligible in relation to the first one. The third and fourth terms in Eq. (15) are usually modeled as a diffusion term in K-ε models, in the high Reynolds region. They are the most complex terms in the K equation. In the present work, the following approximation is done

$$\frac{\partial}{\partial x_2}\left(\langle pu_2\rangle + \langle Ku_2\rangle\right) \cong \frac{\partial}{\partial x_2^+}\left\langle\frac{1}{2}u_1u_1u_2\right\rangle^+ = -2.42\frac{\partial}{\partial y^+}\left[\lambda^3 y^{+3}\left(\frac{\partial U^+}{\partial y^+}\right)^2\right] \quad (16)$$

In the viscous layer by using the turbulent fluctuation and the Reynolds stress, this term can be taken as

$$\frac{\partial}{\partial x_2^+}\left\langle\frac{1}{2}u_1u_1u_2\right\rangle^+ = -2.42\frac{\partial}{\partial y^+}\left[\lambda^3 y^{+3}\left(\frac{\partial U^+}{\partial y^+}\right)^2\right] \quad (17)$$

The so-called production term in the K equation, in the viscous layer is

$$\langle u_1u_2\rangle\frac{\partial U_1}{\partial x_2} \quad (18)$$

and can be expressed in dimensionless form as

$$-\langle u_1u_2\rangle^+\frac{\partial U_1^+}{\partial x_2} = \lambda^2 y^{+2}\left(\frac{\partial U^+}{\partial y^+}\right)^2 = \frac{\lambda^2 y^{+2}}{\left(1+\lambda^2 y^{+2}\right)^2} \quad (19)$$

Thus, the turbulent kinetic energy dissipation equation in the near-wall region can be obtained from as

$$\varepsilon^+ = \frac{23.47}{2}\frac{\partial^2}{\partial y^{+2}}\left(\frac{\lambda^2 y^{+2}}{\left(1+\lambda^2 y^{+2}\right)^2}\right) + \frac{\sqrt{23.47}}{2}\frac{\partial}{\partial y^+}\left(\frac{\lambda^3 y^{+3}}{\left(1+\lambda^2 y^{+2}\right)^2}\right) + \left(\frac{\lambda^2 y^{+2}}{\left(1+\lambda^2 y^{+2}\right)^2}\right) \quad (20)$$

resulting in (21)

$$\varepsilon^+ = 11{,}73\left(\frac{2\lambda^2}{\left(1+\lambda^2y^{+2}\right)^2} - \frac{20\lambda^4y^{+2}}{\left(1+\lambda^2y^{+2}\right)^3} - \frac{24\lambda^6y^{+4}}{\left(1+\lambda^2y^{+2}\right)^4}\right) + 2{,}42\left(\frac{3\lambda^3y^{+2}}{\left(1+\lambda^2y^{+2}\right)^2} - \frac{4\lambda^5y^{+4}}{\left(1+\lambda^2y^{+2}\right)^3}\right) + \left(\frac{\lambda^2y^{+2}}{\left(1+\lambda^2y^{+2}\right)^2}\right)$$

Figure 2 shows the behavior of the three different terms in the dimensionless dissipation (ε+) equation above, as a function of dimensionless wall distance (y+). The first term is the inertial transfer (T+), the second is the diffusion term (D+) and the third is the production (P+). The dissipation distribution is also presented. It can be observed that the contribution of the inertial term is very small. The diffusion is large near y+ = 0, but rapidly decays. The major contribution comes from the production term. In Figure 3 the production and the dissipation are compared. The traditional hypothesis for the wall region (Patankar and Spalding, 1970, Mansur et al., 1989, Lai e So, 1990) is that production and dissipation are equal. By examining Figure 3, it can be seen that this is true, for y+ > 7. Closer to the wall, diffusion is important, resulting in an dissipation value around 0.2 in agreement with experimental observations of Patel et al, 1984.

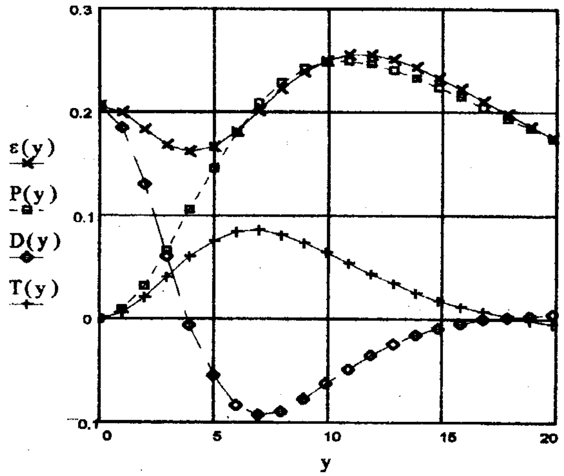

Figure 2. Near-wall behavior of turbulent dimensionless dissipation (ε+), viscous diffusion (D+), production (P+) and inertial transfer (T+), as a function of dimensionless wall distance (y+).

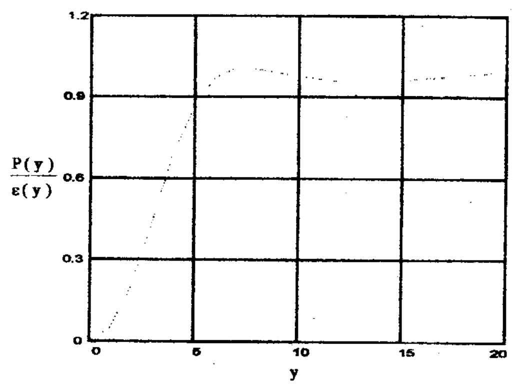

Figure 3. Relationship between dimensionless turbulent production and dissipation as a function of dimensionless wall distance

NUMERICAL PROCEDURE

Some numerical tests were performed to validate the proposed method. The numerical solution are obtained using a control volume based formulation (Patankar, 1980). In this procedure, the domain is discretized by a series of control volume, each containing a grid point. The grid system used is a so-called staggered grid system in which the value of each scalar quantity is associated with every grid node, while the vector quantities are displaced in space relative to the scalar ones. The conservation laws are expressed in an integral manner over the control volume and power profile approximation are considered, leading to a system of algebraic equation solved in an iterative manner by the TDMA algorithm. The pressure field is computed by the SIMPLEC algorithm (Van Doormaal e Raithby, 1984).

TURBULENCE METHOD

The two-equation K-ε differential turbulence model was selected to model the turbulence in the domain. However, this model can only be applied in the regions where $\mu_t >> \mu$. In the region y+ < 11.6, or near-wall region, the eddy viscosity, K and ε are computed with the expression (6), (13) and (21), respectively.

The wall parameter u* is defined as

$$u_* = \left(\frac{\tau_*}{\rho}\right)^{1/2} = \left[\nu\frac{\partial U}{\partial n}\right]^{1/2} \tag{22}$$

can be computed from the mean velocity value U, if the y+ = U+ hypothesis, valid for y+ < 5 (Kline et al. 1967) is considered. u* can be obtained in the following way

$$y^{+2} = \frac{yy^+u_*}{\nu} = \frac{yu^+u_*}{\nu} = \frac{yU}{\nu} = Y^+ \tag{23}$$

Hence, in the near-wall grid point, the Y+ is computed and the u* parameter is recuperated.

Since y+ = U+ for y+ < 5, in the calculation of the wall parameter u*, with the procedure suggested above, it is necessary that the nearest to solid wall grid point must be located in y+ < 5.

NUMERICAL RESULTS

The proposed wall function was tested, at first, for shear flow in the boundary layer of a circular duct and for turbulent flow between parallel plates, with excellent result. Then, it was tested for a more complex turbulent flow, i.e., the turbulent flow over a backward facing step, which is presented here.

Turbulent flow over a backward facing step is one of the standard test case to evaluate performance model. The backward-facing step domain is shown in figure 4. The same situation experimentally investigated by Kim et al (1980) was considered here. The Reynolds number is Re = 132000, based in the inlet centerline mean velocity U_C and the outlet high H_S. The specified aspect ratios was $H/H_S = 1/3$, where H is the step height.

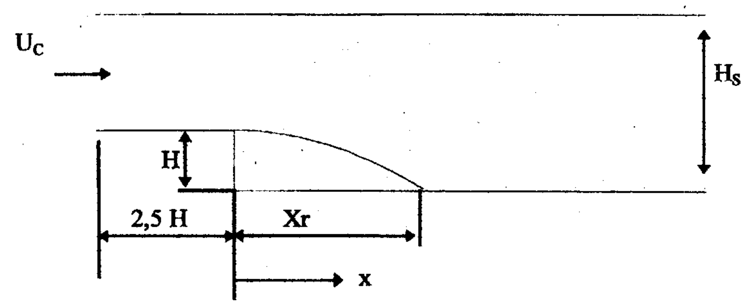

Figure 4. Backward facing step domain.

The reattachment distance was predicted as Xr/H = 6.65 for a mesh of 118 x 82 points in the x and y directions, respectively (Pasinato and Nieckele, 1996). This result presents a difference of 5% in relation to the experimental results of Kim et al., who measured Xr/H = 7. At the present work a finner mesh equal to 200 x 100, was employed and the reattachment point predicted was 7.35. The difference in relation to the experimental result was again of 5%. In this case the recirculating zone was over predicted.

It should be mentioned here, that the present law of the wall is valid for zero pressure gradient, which is a great simplification of the model. However, even with this simplification, reasonable results were obtained.

Figure 5 presents the mean velocity profile along the vertical direction, near the wall, before and after the reattachment point. The negative velocities at x/H = 5.89 and 6.56 indicate the presence of the secondary flow. For x/H = 7.0, the derivative at the wall is almost zero, indicating that this position is very close to the reattachment point. Further downstream, x/H = 7.71, all velocity profile is positive, indicating that the flow field is developing.

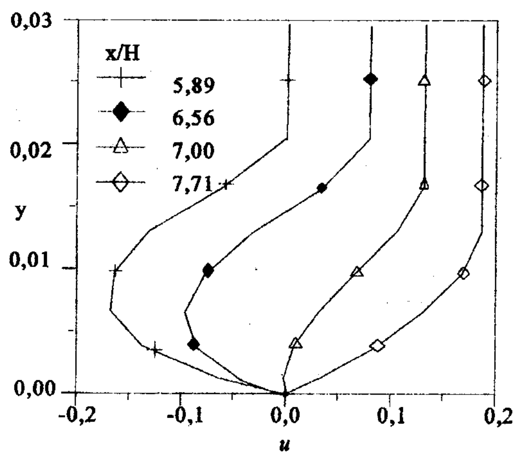

Figure 5. Mean velocity profile next to the reattachment point for the backward facing step problem

CONCLUSION

The present work presented a new law of the wall based on the velocity profile proposed by Haritonidis (1989). The dissipation of turbulent kinetic energy near the wall is obtained, keeping the inertia and diffusion terms. As a consequence the predicted ε value at the wall agreed with experimental observations. The method was tested in the backwards facing step problem. The results obtained can be considered satisfactory, with a good prediction of the reattachment point.

As a final comment, it can be said that the model presented here, although simple, can predict reasonable well the flow field in complex geometries, with a low cost, what is very attractive for technological applications.

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