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THE MACROSCOPIC BEHAVIOR OF POWER-LAW VISCOPLASTIC AND IDEALLY PLASTIC POROUS MATERIALS WITH EVOLVING MICROSTRUCTURES

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Abstract. Theoretical predictions are given for the mechanical behavior of two-dimensional porous media comprised of a power-law viscoplastic or ideally plastic matrix containing a *random* distribution of porosity and subjected to *large* deformations. The predictions follow from a homogenization-based constitutive theory for two-phase nonlinear composites recently proposed by the author (M.I. Idiart, *J. Mech. Phys. Solids* 56:2599-2617, 2008). In view of the preliminary results given in this work, it is conjectured that the new estimates provide more reliable predictions for viscoplastic porous media than standard Gurson-type models.

1 INTRODUCTION

Predicting the ductile failure of metals in terms of void nucleation, growth and coalescence requires a constitutive model for plastic/viscoplastic porous media. Such models must be able to account for the evolution of the microstructure due to the finite changes in geometry induced by the deformation. The most widely accepted model for the plastic behavior of isotropic porous media is that of Gurson (1977). Extensions of this model have been proposed by several authors to include the effects of power-law viscoplasticity (*e.g.*, Leblond *et al.*, 1994; Gărăjeu *et al.*, 2000) and pore anisotropy (*e.g.*, Gărăjeu *et al.*, 2000; Gologanu *et al.*, 1993; Mariani and Corigliano, 2001). Such micromechanical models are all based on approximate analyses of a hollow sphere or ellipsoid subject to axisymmetric loadings, and recover the exact result for a hollow sphere under hydrostatic loading.

In parallel developments, homogenization bounds for power-law porous materials with more realistic microstructures and subject to general loading conditions have been generated by means of 'variational' methods (Ponte Castañeda, 1991; Willis, 1991; Ponte Castañeda and Zaidman, 1994) and Hölder's inequality (Suquet, 1992). At low to moderate stress triaxialities, these bounds are generally 'tighter' than Gurson models, indicating the inadequacy of Gurson models to represent shear-dominated deformation processes. At large stress triaxialities, by contrast, 'variational' bounds can be significantly 'stiffer' than Gurson models, leading to unrealistic predictions for pressure-dominated processes. Consequently, efforts have been concentrated in developing models that satisfy the 'linear comparison' bounds for the entire range of stress triaxiality, and at the same time, that reproduce Gurson models for isotropic materials under hydrostatic loadings. A fairly general model along these lines has been recently introduced by Danas *et al.* (2008a,2008b).

More recently, Idiart (2008a) proposed the use of certain theoretical constructions known as 'sequential laminates', to model the macroscopic behavior of two-phase nonlinear media with random, 'particulate' microstructures, which include porous materials as a special case. The resulting homogenization estimates have the distinctive feature of being *realizable*, and are therefore guaranteed to satisfy all pertinent bounds. In addition, when specialized to isotropic porous materials with a power-law matrix, these estimates reproduce exactly the hydrostatic behavior of a hollow sphere (Idiart, 2007; 2008a; 2008b), and therefore, the hydrostatic limit of Gurson models. Thus, this approach delivers fairly general estimates that meet the requirements mentioned above. The purpose of this paper is to report preliminar comparisons between the new estimates of Idiart (2008b) and the aforementioned models.

The common assumption is made that the matrix phase is a purely viscoplastic material characterized by an incompressible, power-law dissipation potential w, such that the Cauchy stress $\boldsymbol{\sigma}$ and the Eulerian strain rate $\mathbf{D}$ are related by

$$\boldsymbol{\sigma} = \frac{\partial w}{\partial \mathbf{D}}(\mathbf{D}), \quad w(\mathbf{D}) = \begin{cases} \dfrac{\sigma_0 D_0}{1+m}\left(\dfrac{D_e}{D_0}\right)^{1+m} & \text{if } \mathrm{tr}\mathbf{D} = 0, \\ +\infty & \text{otherwise}, \end{cases} \tag{1}$$

where m is the so-called strain-rate sensitivity, such that $0 \leq m \leq 1$, σ_0 is a flow stress, D_0 is a reference strain rate, and the von Mises equivalent strain rate is given in terms of the deviatoric part of the strain-rate tensor by $D_e = \sqrt{(2/3)\mathbf{D}_d \cdot \mathbf{D}_d}$. In turn, the von Mises equivalent stress is defined as $\sigma_e = \sqrt{(3/2)\boldsymbol{\sigma}_d \cdot \boldsymbol{\sigma}_d}$. The limiting values of the exponent $m = 1$ and $m = 0$ correspond, respectively, to linearly viscous and rigid-ideally plastic behaviors, see fig. 1a. In the case of ideal plasticity, the potential w is not differentiable at $\mathbf{0}$, and the derivative in $(1)_1$ should be understood as the subdifferential of convex analysis.

The material systems of interest here demand an *Eulerian* description of motion. We denote by Ω the domain occupied by the specimen in the *deformed* configuration. The pores are assumed to be *randomly* distributed over the specimen and to have characteristic dimensions that are much smaller than the size of the specimen and the scale of variation of the applied loads. It is further assumed that the distribution of pores is statistically uniform and ergodic. At each step of a deformation process, the spatial distribution of pores within Ω can be formally described by a random indicator function $\chi(\mathbf{x})$ that takes the value 1 if $\mathbf{x}$ is in the porous phase, and 0 otherwise. The ensemble average of $\chi(\mathbf{x})$ represents the one-point probability $p(\mathbf{x})$ of finding a pore at $\mathbf{x}$; the ensemble average of the product $\chi(\mathbf{x})\chi(\mathbf{x}')$ represents the two-point probability $p(\mathbf{x}, \mathbf{x}')$ of finding simultaneously pores at $\mathbf{x}$ and at $\mathbf{x}'$. Higher-order multi-point probabilities can be defined similarly, but will not be considered in this work. Due to the assumed statistical uniformity and ergodicity, the one-point probability $p(\mathbf{x})$ can be identified with the volume fraction of pores f —or porosity— in the deformed configuration. In addition, we make the simplifying assumption that throughout the deformation process the porosity distribution is 'elliptical', in the sense that the two-point probabilities depend on $\mathbf{x}$ and $\mathbf{x}'$ only through the combination $|\mathbf{Z}(\mathbf{x} - \mathbf{x}')|$, where $\mathbf{Z}$ is a second-order tensor describing the elliptical distribution, such that $\mathbf{Z} = \mathbf{I}$ corresponds to statistical isotropy (see Ponte Castañeda and Zaidman, 1994). During a deformation process, the evolution of the microstructure can be characterized by the evolving porosity level f and pore anisotropy tensor $\mathbf{Z}$.

The instantaneous macroscopic response of the porous material is defined as the relation between the volume averages of the Cauchy stress $\overline{\boldsymbol{\sigma}}$ and the Eulerian strain rate $\overline{\mathbf{D}}$ over Ω, and can be characterized by an effective dissipation potential $\overline{w}$, such that (*e.g.*, Ponte Castañeda and Suquet, 1998)

$$\overline{\boldsymbol{\sigma}} = \frac{\partial \overline{w}}{\partial \overline{\mathbf{D}}}(\overline{\mathbf{D}}), \quad \overline{w}(\overline{\mathbf{D}}) = \min_{\mathbf{D}\in\mathcal{K}(\overline{\mathbf{D}})} \frac{1}{|\Omega|}\int_{\Omega} (1 - \chi(\mathbf{x}))\, w(\mathbf{D})\, \mathrm{d}\Omega(\mathbf{x}). \tag{2}$$

In this expression, the overbar denotes averaged quantities, and $\mathcal{K}$ is the set of kinematically admissible strain-rate fields with prescribed volume average $\overline{\mathbf{D}}$.

The estimates for the potential $\overline{w}$ proposed by Idiart (2008a) are given in Section 2; approximate stress-strain-rate relations then follow from $(2)_1$. Although the formulation allows for fairly general porosity distributions and completely arbitrary loading conditions, the preliminar comparisons presented here are restricted to *two*-dimensional porous media with initially *isotropic* porosity, subjected to *biaxial* plane-strain deformations

$$\overline{\mathbf{D}} = \overline{D}_{11}\mathbf{e}_1 \otimes \mathbf{e}_1 + \overline{D}_{22}\mathbf{e}_2 \otimes \mathbf{e}_2 \tag{3}$$

with fixed loading axes. Then, the deformation-induced anisotropy of porosity will have symmetry axes aligned with the loading axes throughout the deformation process, and we can write

$$\mathbf{Z} = l_1^{-1}\mathbf{e}_1 \otimes \mathbf{e}_1 + l_2^{-2}\mathbf{e}_2 \otimes \mathbf{e}_2, \quad \omega = \frac{l_2}{l_1}, \tag{4}$$

where ω is the aspect ratio of the assumed 'elliptical' distribution of porosity, see fig. 2b. The relevant microstructural variables are therefore f and ω.

2 HOMOGENIZATION ESTIMATES

2.1 Effective dissipation potential

The central idea behind the estimates proposed by Idiart (2008a) is to approximate the effective dissipation potential by that of an infinite-rank sequential laminate with the same one- and

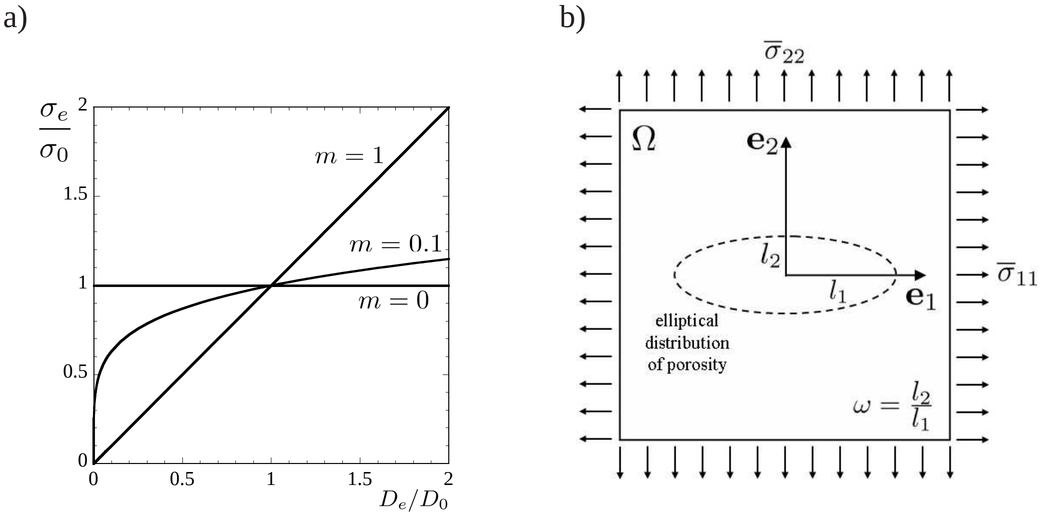

Figure 1: a) Viscous material response for various values of strain-rate sensitivity ($m = 0, 0.1, 1$); b) schematic of the problem.

two-point microstructural statistics as those of the porous material. For the material systems considered here, the resulting estimate requires the solution of the first-order partial differential equation

$$f\frac{\partial \overline{w}}{\partial f} - H\left(\overline{\mathbf{D}}, \overline{w}, \frac{\partial \overline{w}}{\partial \overline{\mathbf{D}}}\right) = 0, \quad \overline{w}|_{f=1} = 0, \tag{5}$$

where

$$H\left(\overline{\mathbf{D}}, \overline{w}, \overline{\boldsymbol{\sigma}}\right) = \overline{w} + \max_{\mathbf{a}(\mathbf{n})} \langle \mathbf{a} \cdot \overline{\boldsymbol{\sigma}}\mathbf{n} - w\left(\overline{\mathbf{D}} + \mathbf{a} \otimes_s \mathbf{n}\right)\rangle. \tag{6}$$

In this last expression, $\mathbf{a}(\mathbf{n})$ is a vector-valued function of the unit vector $\mathbf{n}$, $\otimes_s$ denotes the symmetric part of the tensor product, and

$$\langle \cdot \rangle \doteq \int_{|\mathbf{n}|=1} (\cdot)\nu(\mathbf{n})\mathrm{d}s(\mathbf{n}) \tag{7}$$

denotes an orientational average. The weighing function $\nu(\mathbf{n})$ depends on the angular variation of the pore distribution function $p(\mathbf{x}, \mathbf{x}')$ in the deformed configuration as characterized by the tensor $\mathbf{Z}$. An expression for $\nu(\mathbf{n})$ can be found in Idiart (2008b), see expressions (5) and (6) in that Ref.

Equation (5) constitutes a nonlinear Hamilton-Jacobi equation where the current porosity f and the macroscopic strain rate $\overline{\mathbf{D}}$ play the role of 'time' and 'space' variables, respectively, and the function H plays the role of a Hamiltonian. In this regard, note that the matrix behavior, porosity level f, and pore distribution function $p(\mathbf{x}, \mathbf{x}')$ are contained in the Hamiltonian (6), while the behavior of the porous phase dictates the 'initial' condition —at $f = 1$. A solution strategy to integrate equation (5) numerically has been given by Idiart (2008b).

2.2 Microstructure evolution

When the porous material is subjected to finite deformations, the underlying microstructure evolves as a result of the finite changes in geometry. This evolution depends on the complex interaction between the various microstructural variables, the loading conditions and the instantaneous constitutive response of the porous material. For the material systems considered here,

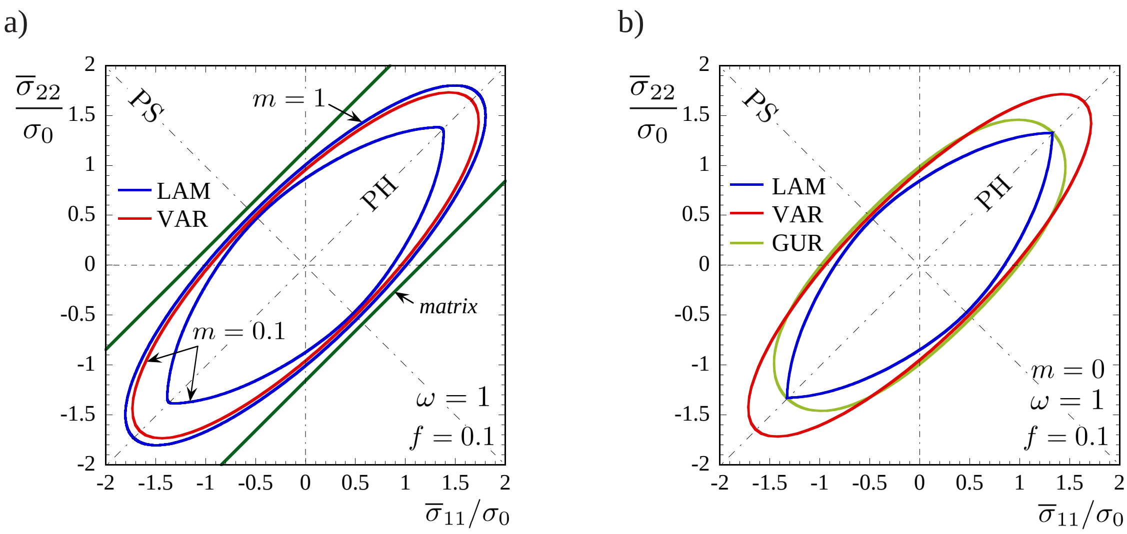

Figure 2: Instantaneous response of isotropic ($\omega = 1$) power-law materials with porosity $f = 0.1$. The new estimates (LAM) are compared with the 'variational' (VAR) outer bound and the Gurson (GUR) predictions: a) gauge surfaces for linear ($m = 1$) and strongly nonlinear ($m = 0.1$) materials; b) yield surfaces for an ideally plastic ($m = 0$) material. Diagonal axes correspond to pure shear (PS) and purely hydrostatic (PH) loadings.

the most relevant microstructural variables are the porosity level f and the aspect ratio ω of the assumed 'elliptical' distribution of porosity. The evolution laws for these variables are (see, for instance, Ponte Castañeda and Zaidman, 1994)

$$\dot{f} = (1 - f)\text{tr}\overline{\mathbf{D}} \quad \text{and} \quad \dot{\omega} = \omega(\overline{D}^{(p)}_{22} - \overline{D}^{(p)}_{11}), \tag{8}$$

where $\overline{D}^{(p)}_{ij}$ denote the components of the average strain rate undergone by the porous phase, relative to the coordinate system shown in fig. 1a. Equation $(8)_1$ follows from simple kinematical arguments, while equation $(8)_2$ requires the additional assumption that, on average, the shape of the pores has the same aspect ratio as the shape of their distribution —as characterized by $\mathbf{Z}$— throughout the deformation process. The reader is referred to Ponte Castañeda and Zaidman (1994) for a discussion on this last assumption.

Within the present formulation, an equation for $\overline{\mathbf{D}}^{(p)}$ can be obtained by following the procedure of Idiart and Ponte Castañeda (2007). The following first-order differential equation is obtained:

$$f\frac{\partial \overline{D}^{(p)}_{ij}}{\partial f} - \frac{\partial \overline{D}^{(p)}_{ij}}{\partial \overline{\mathbf{D}}} \cdot \langle \mathbf{a} \otimes_s \mathbf{n} \rangle = 0, \quad \overline{D}^{(p)}_{ij}|_{f=1} = \overline{D}_{ij}, \tag{9}$$

where the vector function $\mathbf{a}(\mathbf{n})$ is that optimizing the Hamiltonian (6) —and therefore depends on f and $\overline{\mathbf{D}}$, but not on $\overline{\mathbf{D}}^{(p)}$. This expression constitutes a linear Hamilton-Jacobi equation for each component $\overline{D}^{(p)}_{ij}$, which can be solved numerically following the solution strategy employed for equation (5).

3 SAMPLE RESULTS AND DISCUSSION

3.1 Instantaneous response

The effective dissipation potential of a power-law porous material is also a power-law function of the macroscopic strain rate, with the same strain-rate sensitivity m and reference strain rate D_0 as the matrix phase. It is commonly represented in terms of cross sections of its Legendre transform $\overline{w}^*$. Specifically,

$$\overline{w}^*(\overline{\boldsymbol{\sigma}}) = \frac{\sigma_0 D_0}{1+1/m}, \quad \overline{w}^*(\overline{\boldsymbol{\sigma}}) \doteq \sup_{\overline{\mathbf{D}}} \left[\overline{\boldsymbol{\sigma}} \cdot \overline{\mathbf{D}} - \overline{w}(\overline{\mathbf{D}})\right]. \tag{10}$$

Expression $(10)_1$ constitutes a parametric equation for the so-called *gauge surface* (in stress space) introduced by Leblond *et al.* (1994). Any other cross section of $\overline{w}^*$ represents a homothetic surface to the gauge surface. In the limiting case of ideal plasticity, the gauge surface reduces to the yield surface of the porous material. For the microstructural symmetry and loading conditions considered here, the relevant stress space is $(\overline{\sigma}_{11}, \overline{\sigma}_{22})$.

In fig. 2, the new estimates (LAM) of Section 2 are compared with the 'variational' (VAR) estimates of Ponte Castañeda (1991) and the standard Gurson (GUR) model. The VAR estimates are in fact rigorous 'outer' bounds for the gauge surface of the class of porous materials considered here, and they reduce to the generalized Hashin-Shtrikman (HS) bounds of Willis (1977) when the matrix material is linear. In turn, the GUR model delivers an outer bound for the yield surface of porous materials with a special class of microstructures known as 'composite cylinder assemblages'. However, it is believed that in the hydrostatic limit, the GUR model may actually deliver an outer bound for isotropic porous materials in general (see, for instance, Idiart, 2007).

Fig. 2b shows gauge surfaces for isotropic porous materials with a linear ($m = 1$) and a strongly nonlinear ($m = 0.1$) matrix phase. It is observed that, while the linear LAM and VAR surfaces both coincide with the HS bound, the nonlinear LAM surface is significantly 'softer' than the corresponding VAR surface, especially at large stress triaxialities. In fact, the VAR surface is relatively insensitive to constitutive nonlinearity. It is also noted that the nonlinear LAM surface deviates considerably from an elliptical shape, and develops a strong curvature on the hydrostatic axis. This is in contrast with the VAR estimates, as well as with several other nonlinear estimates proposed in the literature (see, for instance, Michel and Suquet, 1992), which obey the equation of an ellipse for all values of the nonlinearity. The relevance of the gauge surface shape stems from the fact that its normal dictates the direction of flow of the porous material.

The limiting case of ideal plasticity is considered in fig. 2c. The LAM surface is seen in this case to lie not only within the VAR surface but also within the GUR surface. Moreover, the LAM surface coincides with the GUR surface for hydrostatic loadings, as anticipated. By contrast, the GUR surface lies outside the VAR surface at low to moderate triaxialities, while the VAR surface is significantly 'stiffer' than the GUR surface at large triaxialities. More interestingly, however, is the fact that the LAM surface develops a *corner* on the hydrostatic axis, in contrast to the VAR and GUR smooth surfaces. It then follows that the direction of flow at hydrostatic stresses as predicted by the LAM estimates is not uniquely determined; instead, it is confined by the cone of normals. For a more detailed discussion on this point the reader is referred to Idiart (2008b).

3.2 Overall response under finite deformations

We now consider a porous material deformed quasi-statically with a constant strain rate $\overline{\mathbf{D}}$ and up to a prescribed macroscopic strain. At each step in the deformation process, the effective potential $\overline{w}$ and the average strain rate in the pores $\overline{\mathbf{D}}^{(p)}$ are obtained by integrating equations (5) and (9) with the current value of aspect ratio ω and up to the current value of porosity f.

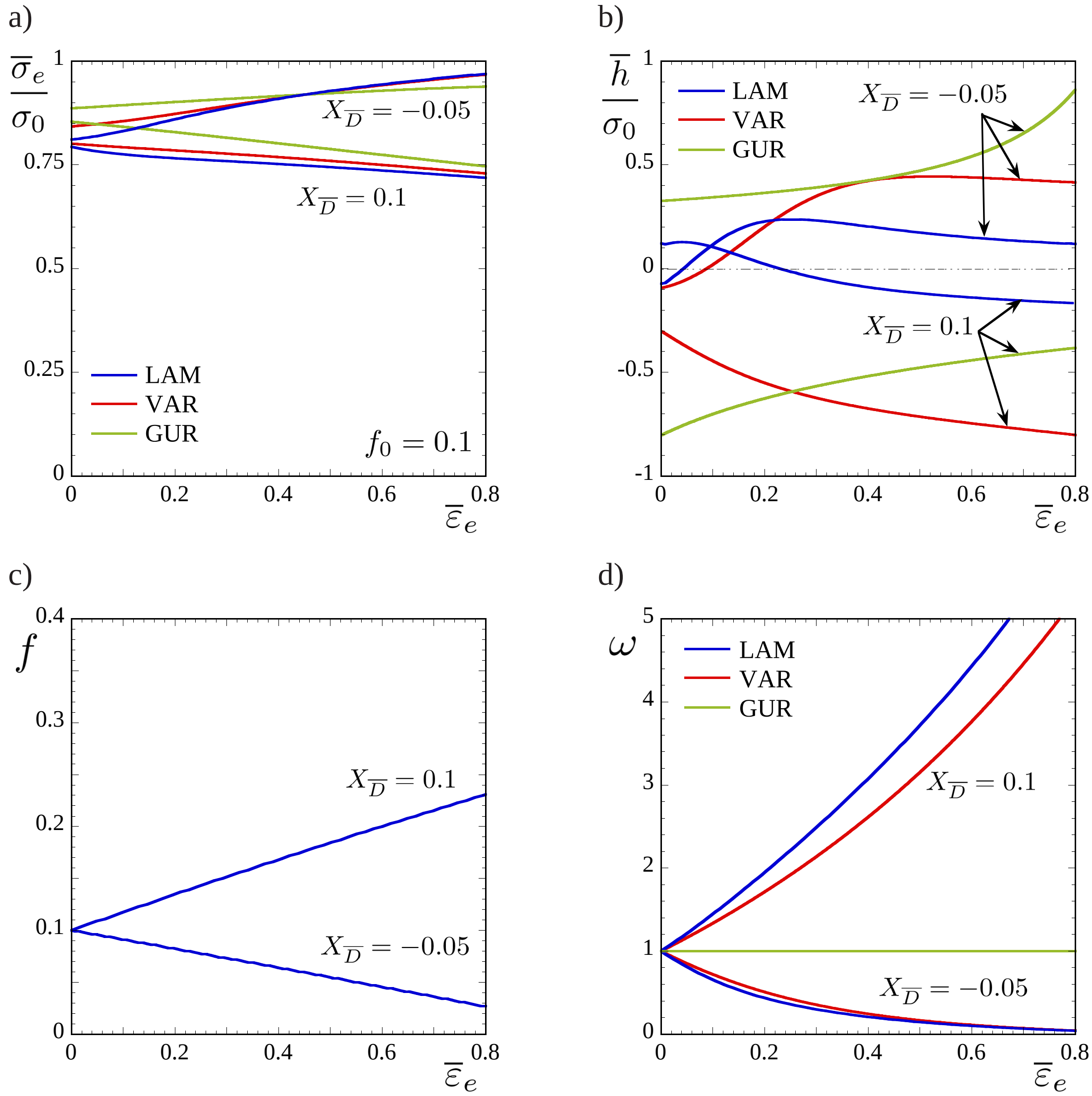

Figure 3: Overall response of an isotropic ($\omega = 1$) ideally plastic ($m = 0$) porous material with initial porosity $f_0 = 0.1$, subjected to finite deformations with strain-rate triaxialities $X_{\overline{D}} = 0.1$ and $X_{\overline{D}} = -0.05$. The new estimates (LAM) are compared with the 'variational' (VAR) and the Gurson (GUR) predictions: a) stress-strain response; b) hardening rate; c) porosity; d) void aspect ratio.

The corresponding value of Cauchy stress $\overline{\boldsymbol{\sigma}}$ is obtained by evaluating the derivative $(2)_1$ at the applied $\overline{\mathbf{D}}$. Expressions (8) are then invoked to update the microstructural variables f and ω by a forward-Euler scheme, and the procedure is repeated until the desired value of macroscopic strain is reached.

For simplicity, results are only reported here for the limiting case of ideal plasticity. It is convenient to define macroscopic loading parameters

$$\overline{D}_e = \frac{|\overline{D}_{11} - \overline{D}_{22}|}{\sqrt{3}}, \quad \overline{D}_m = \frac{\overline{D}_{11} + \overline{D}_{22}}{2}, \quad X_{\overline{D}} = \frac{\overline{D}_m}{\overline{D}_e}, \tag{11}$$

representing the standard equivalent and mean strain rate, and the strain-rate triaxiality, respectively. The macroscopic strain is then defined as $\overline{\varepsilon}_e \doteq \int_0^t \overline{D}_e \mathrm{d}t$. Note that $\overline{D}_e$ is essentially a time variable, and that the results for ideally plastic materials are independent of $\overline{D}_e$ since the model is rate-independent. Comparisons are shown in fig. 3 for a porous material with an initial porosity level $f_0 = 0.1$, subjected to two different loading conditions:

$$\text{loading A } (\overline{D}_m > 0) \quad : \quad \frac{\overline{D}_{11}}{\overline{D}_e} = X_{\overline{D}} - \frac{\sqrt{3}}{2}, \quad \frac{\overline{D}_{22}}{\overline{D}_e} = X_{\overline{D}} + \frac{\sqrt{3}}{2}, \quad X_{\overline{D}} = 0.1, \qquad (12)$$

$$\text{loading B } (\overline{D}_m < 0) \quad : \quad \frac{\overline{D}_{11}}{\overline{D}_e} = X_{\overline{D}} + \frac{\sqrt{3}}{2}, \quad \frac{\overline{D}_{22}}{\overline{D}_e} = X_{\overline{D}} - \frac{\sqrt{3}}{2}, \quad X_{\overline{D}} = -0.05. \qquad (13)$$

Note that loading A represents a shear deformation with superposed isotropic *dilation*, while loading B represents a shear deformation with superposed isotropic *contraction*. Note also that $|\overline{D}_{22}| > |\overline{D}_{11}|$ in both loadings.

We begin by noting that the stress-strain curves predicted by the LAM estimates do not differ significantly from those predicted by the 'variational' and Gurson estimates, see fig. 3a. However, substantial differences are found between corresponding predictions for the associated hardening rate[1] $\overline{h}(\overline{\varepsilon}_e)$, shown in fig. 3b. In this connection, it is recalled that for the loadings considered here, the condition $\overline{h}(\overline{\varepsilon}_e) = 0$ signals the onset of macroscopic shear-band instabilities (Rice, 1977). Thus, according to the LAM estimates the porous material becomes unstable at $\overline{\varepsilon}_e \approx 0.24$ for loading A, and at $\overline{\varepsilon}_e \approx 0.04$ for loading B. In contrast, the VAR estimates predict that the porous material is always stable for loading A, and becomes unstable at the larger strain $\overline{\varepsilon}_e \approx 0.087$ for loading B, while the GUR estimates predict a stable response for both loadings. These significantly different predictions can be explained in terms of the associated microstructure evolution.

The various predictions for the evolution of porosity and pore aspect ratio are shown in figs. 3c and 3d, respectively. All estimates give the same evolution of porosity, which is completely dictated by the kinematics of the problem. On the other hand, different predictions are obtained for the evolution of pore aspect ratio. On the one hand, the Gurson estimate predicts a constant aspect ratio $\omega = 1$ —*i.e.*, isotropic porosity— throughout the deformation process, which is a well-known limitation of this model. On the other hand, the LAM and VAR estimates are able to capture the expected pore anisotropy induced by the applied deformation. The evolution of porosity and pore anisotropy play competing roles in the macroscopic hardening rate of the porous material, and the relative importance of each mechanism varies as the deformation proceeds.

Under loading A, the porosity increases due to the imposed isotropic dilation, and as a result the material should soften. At the same time, however, the pores elongate in the 2-direction —*i.e.*, $\omega \geq 1$— due to the fact that $\overline{D}_{22} > 0$ and $\overline{D}_{11} < 0$. As a result, the material should harden since the larger axis of the pores is aligned with the direction of maximum load. In the LAM theory, the hardening mechanism overcomes the softening mechanism initially, giving a positive hardening rate. The situation reverses, however, for sufficiently large strains, and consequently the hardening rate vanishes at a certain critical strain ($\overline{\varepsilon}_e \approx 0.24$) at which the predicted behavior becomes unstable. In the VAR theory, the softening mechanism overcomes the hardening mechanism thoughout the deformation process, the hardening rate is thus always negative, and the predicted behavior remains stable.

Under loading B, the role of each mechanism reverses. Indeed, the porosity decreases due to the imposed isotropic contraction, and as a result the material should harden. At the same time,

[1] The hardening rate is calculated via the consistency condition of the plastic porous solid. The VAR and LAM results follow from the yield surface defined by $(10)_1$, while the GUR results follow from the standard expression of Gurson's two-dimensional yield surface as given by expression (3.19) in Gurson (1977).

however, the pores elongate in the 1-direction —*i.e.*, $\omega \leq 1$— due to the fact that $\overline{D}_{11} > 0$ and $\overline{D}_{22} < 0$. As a result, the material should soften, since the larger axis of the pores is aligned with the direction of minimum load. In the LAM and VAR theories, the softening mechanism overcomes the hardening mechanism initially so that the hardening rate is negative, but the situation reverses at relatively small strains. Consequently, the hardening rate vanishes at a certain critical strain at which the predicted behavior becomes unstable.

Gurson's model, in contrast, accounts for one mechanism only: the change in porosity. Consequently, this model cannot capture the subtle interplay between the changes in porosity and pore anisotropy, and the GUR estimates thus predict stable behaviors under both loading conditions. An extension of Gurson's two-dimensional model that incorporates the effect of pore anisotropy was proposed by Mariani and Corigliano (2001). Comparisons between the LAM theory and Gurson's extended model will be reported elsewhere.

4 CONCLUDING REMARKS

In view of the above preliminary results, it is conjectured that the estimates derived in this work provide more reliable predictions for the macroscopic behavior of porous materials than standard Gurson models. The validity of this conjecture, however, should be confirmed by thorough comparisons with full-field numerical simulations. An admittedly unfavourable criticism against the new approach is that it may involve considerable computations in more general contexts, especially at low levels of porosity. In that case, the alternative approach proposed by Danas *et al.* (2008a,2008b), which is based on a variational approximation, may result more appropriate. The relative merits of these two approaches will be assessed in future work.

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REFERENCES

[1] Danas K., Idiart M.I., Ponte Castañeda P. A homogenization-based constitutive model for two-dimensional viscoplastic porous media. *C. R. Mecanique* 336:79–90, 2008a.

[2] Danas K., Idiart M.I., Ponte Castañeda P. A homogenization-based constitutive model for isotropic viscoplastic porous media. *Int. J. Solids Struct.* 45:3392–3409, 2008b.

[3] Gărăjeu M., Michel J.C., Suquet P. A micromechanical approach of damage in viscoplastic materials by evolution in size, shape and distribution of voids. Comput. *Methods Appl. Mech. Engrg.* 183:223–246, 2000.

[4] Gologanu M., Leblond J.B., Devaux J. Approximate models for ductile metals containing non-spherical voids–case of axisymmetric prolate ellipsoidal cavities. *J. Mech. Phys. Solids* 41:1723–1754, 1993.

[5] Gurson A.L. Continuum theory of ductile rupture by void nucleation and growth: Part I – Yield criteria and flow rules for porous ductile media. *J. Engng. Mat. Tech.* 99:2–15, 1977.

[6] Idiart M.I. Nonlinear sequential laminates reproducing hollow sphere assemblages. *C. R. Mecanique* 335:363–368, 2007.

[7] Idiart M.I. Modeling the macroscopic behavior of two-phase nonlinear composites by infinite-rank laminates. *J. Mech. Phys. Solids* 56:2599–2617, 2008a.

[8] Idiart M.I. The macroscopic behavior of power-law and ideally plastic materials with elliptical distribution of porosity. *Mech. Res. Comm.* 35:583–588, 2008b.

[9] Idiart M.I., Ponte Castañeda P. Field statistics in nonlinear composites. I.Theory. *Proc. R. Soc. Lond. A* 463:183–202, 2007.

[10] Leblond J.B., Perrin G., Suquet P. Exact results and approximate models for porous viscoplastic solids. *Int. J. Plasticity* 10:213–235, 1994.

[11] Mariani S., Corigliano A. Anisotropic behaviour of porous, ductile media. *Int. J. Solids Struct.* 38:2427–2451, 2000.

[12] Michel J.C., Suquet P. The constitutive law of nonlinear viscous and porous materials. *J. Mech. Phys. Solids* 40:783–812, 1992.

[13] Ponte Castañeda P. The effective mechanical properties of nonlinear isotropic composites. *J. Mech. Phys. Solids* 39:45–71, 1991.

[14] Ponte Castañeda P., Suquet P. Nonlinear composites. *Adv. Appl. Mech.* 34:171–302, 1998.

[15] Ponte Castañeda P., Zaidman M. Constitutive models for porous materials with evolving microstructure. *J. Mech. Phys. Solids* 42:1459–1497, 1994.

[16] Rice J. The localization of plastic deformation. *Proc. 14th International Congress of Theoretical and Applied Mechanics*, W.T. Koiter, Ed., North-Holland Publishing Co., 207–220, 1977.

[17] Suquet P. On bounds for the overall potential of power law materials containing voids with an arbitrary shape. *Mech. Res. Commun.* 19:51–58, 1992.

[18] Willis J.R. Bounds and self-consistent estimates for the overall moduli of anisotropic composites. *J. Mech. Phys. Solids* 25:185–202, 1977.

[19] Willis J.R. On methods for bounding the overall properties of nonlinear composites. *J. Mech. Phys. Solids* 39:73–86, 1991.