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LINEAR BENDING ANALYSIS BY THE BOUNDARY ELEMENT METHOD OF BUILDING FLOOR STRUCTURES WITH COLUMNS DEFINED IN THE DOMAIN

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Abstract. In this work, the building floor structure is modelled by a BEM (Boundary Element Method) formulation based on Kirchhoff's hypothesis. The presented BEM formulation to perform linear bending analysis is derived by applying the reciprocity theorem to zoned plates, where the beams are treated as thin sub-regions with larger rigidities. This composed structure is treated as a single body, being the equilibrium and compatibility conditions automatically taken into account. In order to reduce the number of degrees of freedom some kinematics hypothesis are assumed along the beam cross section. Thus the values remain defined on the beam skeleton line instead of its interface. The columns are introduced into the formulation by considering domain points where tractions can be prescribed. Some numerical examples are presented to show the accuracy of the proposed model.

1 INTRODUCTION

The boundary element method (BEM) has already proved to be a suitable numerical tool to deal with plate bending problems. The method is particularly recommended to evaluate internal force concentrations due to loads distributed over small regions that very often appear in practical problems. Moreover, the same order of errors is expected when computing deflections, slopes, moments and shear forces. Shear forces, for instance, are much better evaluated when compared with other numerical methods. They are not obtained by differentiating approximation function as for other numerical techniques.

Bezine (1981) apparently was the first to use a boundary element to analyse building floor structures by considering plates with internal point supports. More recently, several woks using BEM to model stiffened plates have been presented (Sapountzakis and Katsikadelis (2000), Tanaka and Oida (2000), Paiva and Aliabadi (2004)).

Recently Fernandes and Venturini (2002) and (2005) have proposed two numerical models to perform bending analysis of plates reinforced by beams using only a BEM formulation based on Kirchhoff's hypothesis. In these works the building floor is modelled by a zoned plate where the beams are considered as narrow sub-regions with larger thickness for which some kinematic approximations were assumed to reduce the number of degrees of freedom. In Fernandes and Venturini (2002) the authors present a formulation to perform simple bending analysis of building floor structures. Then this formulation is extended in Fernandes and Venturini (2005) to consider the membrane effects.

In this paper the formulation presented in Fernandes and Venturini (2002) is extended to define columns in the stiffened plate domain. Initially are introduced into the formulation domain points where bending tractions can be prescribed. Then the columns reactions over the plate are considered as prescribed tractions in the central point of the column-plate interface. A numerical example is then presented to illustrate the accuracy of the results and the capability of the formulation to analyse complex building floor structures.

2 BASIC EQUATIONS

The Without loss of generality, let us consider the plate depicted in Figure 1a, where t_1, t_2 and t_3 are the thicknesses of the sub-regions Ω_1, Ω_2 and Ω_3, whose external boundaries are Γ_1, Γ_2 and Γ_3, respectively. The total external boundary is given by Γ while Γ_{jk} represents the interface between the adjacent sub-regions Ω_j and Ω_k. In the simple bending analysis all sub-regions are represented by their middle surface, as shown in Figure 1b. For a point placed at any of those plate sub-regions one can define the following equilibrium equations in terms of internal forces:

$$m_{ij},_j - q_i = 0 \qquad i,j=1,2 \tag{1}$$

$$q_i,_i + g = 0 \qquad i=1,2 \tag{2}$$

where g is the distributed load acting on the plate middle surface, m_{ij} are bending and twisting moments and q_i represents shear forces.

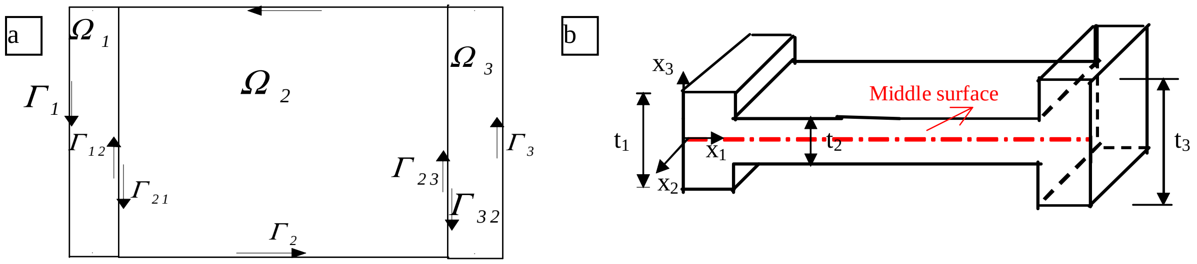

Figure 1. a) General zoned plate domain; b) Plate middle surface view

The plate bending differential equation is given by:

$$w_{,iijj} = g / D \qquad (i, j = 1,2) \tag{3}$$

where $D = Et^3/(1-\nu^2)$ is the flexural rigidity and $w_{,iijj} = \nabla^4 w$, being ∇^4 the bi-harmonic operator.

Finally, the generalised internal force × displacement relations and the effective shear force V_n are defined as follow:

$$m_{ij} = -D\left(\nu\delta_{ij} w_{,kk} + (1-\nu) w_{,ij}\right) \tag{4}$$

$$q_i = -Dw_{,jji} \tag{5}$$

$$V_n = q_n + \partial m_{ns} / \partial s \tag{6}$$

where (n, s) are the local co-ordinate system, with n and s referred to the plate boundary normal and tangential directions, respectively.

The problem definition is then completed by assuming the following boundary conditions over Γ: $u_i = \bar{u}_i$ on Γ_1 (generalised displacements, deflections and rotations) and $p_i = \bar{p}_i$ on Γ_2 (generalised tractions, normal bending moment, effective shear forces), where $\Gamma_1 \cup \Gamma_2 = \Gamma$.

3 INTEGRAL REPRESENTATIONS

In this section, we are going to derive the integral equations for the general case of zoned domain plate problems where the thickness of the plate may vary from one sub-region to another. The equations will be derived by applying the reciprocity theorem to each sub-region and summing them to obtain the reciprocity relations for the whole body. Let us initially consider a single sub-region Ω_m, for which the reciprocity relation can be written in terms of moments and curvatures (see Fernandes and Venturini, 2002). In this work, the Poisson's ration is adopted the same for all sub-regions, so that we can say that moments fundamental values are the same for all sub-regions, i.e. $m^{m*}_{jk} = m^*_{jk}$, while the curvature fundamental solution $w^{m*}_{,ijk}$ can be written in terms of the values $w^*_{,jk}$ and D referred to the sub-region where the load point is placed (see Fernandes and Venturini, 2002). Thus, the following reciprocity relation can be derived for the entire body:

$$\int_{\Omega} w,_{jk}^{*}\, m_{jk}\, d\Omega = \sum_{m=1}^{N_s} \frac{D_m}{D} \int_{\Omega_m} w,_{jk}^{m}\, m_{jk}^{*}\, d\Omega \qquad (7)$$

where $w,_{jk}^{*}$ and m_{jk}^{*} are fundamental solutions with the unit load acting in the direction x3, no summation is implied on m, N_s is the number of sub-regions and D_m is the flexural rigidity in the sub-region Ω_m.

Equation (7) can be integrated by parts to give the deflection representation:

$$K(q)w(q) = -\sum_{m=1}^{N_S} \frac{D_m}{D} \int_{\Gamma_m} \left(V_n^{*} w - M_n^{*} \frac{\partial w}{\partial n} \right) d\Gamma - \sum_{j=1}^{N_{c1}} \frac{D_j}{D} R_{cj}^{*} w_{cj} - \sum_{j=1}^{N_{c2}+N_{c3}} \left(\frac{D_j - D_a}{D} \right) R_{cj}^{*} w_{cj}$$

$$-\sum_{j=1}^{N_{int}} \int_{\Gamma_{ba}} \left(\frac{D_b - D_a}{D} \right) \left(V_n^{*} w - M_n^{*} \frac{\partial w}{\partial n} \right) d\Gamma + \sum_{j=1}^{N_c} R_{cj} w_{cj}^{*} + \int_{\Gamma} \left(V_n\, w^{*} - M_n \frac{\partial w^{*}}{\partial n} \right) d\Gamma + \int_{\Omega_g} g w^{*} d\Omega \qquad (8)$$

where q is the collocation point, no summation is implied on n and s that are local normal and shear direction co-ordinates, respectively; the subscripts b and a refers, respectively, to the beam sub-region and its adjacent sub-region, N_{int} is the number of interfaces; c_1, c_2 and c_3 are different kinds of corners (for their definitions and their corresponding free term values (see Fernandes and Venturini, 2002); Ω_g is the plate loaded area; $K(q)=1$, $K(Q)=0.5$ and $K(Q)=0.5(1+Da/D)$, respectively, for internal, boundary and interface points, N_c is the total number of corners.

Note that in Eq. (8) the tractions have been eliminated along interfaces. Although Eq. (8) can be used to solve the bending problem of stiffened plates, we can reduce further the number of degrees of freedom associated with the plate beam interface by assuming, along the beam cross section, linear approximation for deflection and constant approximation for the deflection derivative $w,_n$ with respect to the skeleton line normal direction (see Fernandes and Venturini, 2002). Thus, by adopting these approximations the number of values at each beam skeleton node remains two: the displacements w and w,n. It is important to stress that all values are referred to nodes defined along the beam axis, while the integrals are still performed along the interfaces. Thus, no singular or hyper-singular term is found when transforming the integrals representations into algebraic ones.

To consider the inclusion of columns into the formulation developed previously, initially the column bending reactions (moments $M_{\bar{x}}$ and $M_{\bar{y}}$ and normal force R) over the plate (see Fig. 2 where $\bar{y}$ and $\bar{x}$ indicate the column principal directions) will be written in terms of the normal stress (σ^c) uniformly distributed over the column cross section Ω_c, as follow:

$$\sigma^c = -\frac{M_{\bar{y}}}{I_{\bar{x}}}\bar{y} - \frac{M_{\bar{x}}}{I_{\bar{y}}}\bar{x} \qquad (9)$$

where A_c is the column cross section area, $I_{\bar{x}}$ and $I_{\bar{y}}$ are the moment inertia with respect to directions $\bar{x}$ and $\bar{y}$.

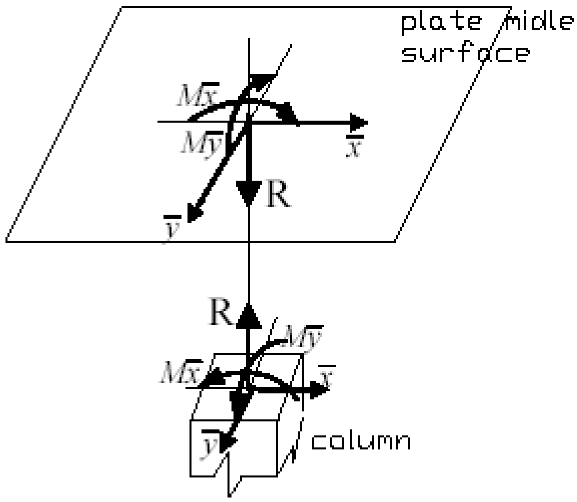

Figure 2 - Columns reactions over the plate

In order to have the final set of equations given in terms of the column generalized displacements we have to write Eq. (9) in terms of displacements. Considering the column stiffness matrix, we can define the following relations for the bending reactions:

$$M_{\bar{y}} = \left(a_c E_c I_{\bar{x}} / L_c\right)\left(w,_{\bar{y}}\right)_c \tag{10a}$$

$$M_{\bar{x}} = \left(a_c E_c I_{\bar{y}} / L_c\right)\left(w,_{\bar{x}}\right)_c \tag{10b}$$

$$R = \frac{\quad}{L_c}\left(E_c A_c / L_c\right) w_c \tag{10c}$$

where a_c=3 for simply supported columns and ac=4 for fixed columns, Ec and Lc are, respectively, the Young's modulus and the column length; w_c, $\left(w,_{\bar{x}}\right)_c$ and $\left(w,_{\bar{y}}\right)_c$ are generalized displacements in the column cross section.

Replacing (10) into (9), the normal stress σ^c in terms of displacements is given by:

$$\sigma^c = \left[-\left(a_c E_c / L_c\right)\left(w,_{\bar{x}}\right)_c\right]\bar{x} + \left[-\left(a_c E_c / L_c\right)\left(w,_{\bar{y}}\right)_c\right]\bar{y} - E_c w_c / L_c \tag{11}$$

Considering now σ^c as additional distributed load acting on the plate sub-region Ω_c and the generalized displacements constant over the column cross section Ω_c, one obtains the integral representation of deflection for the collocation point q:

$$K(q)w(q) = -\sum_{m=1}^{N_{Sub}} \frac{D_m}{D} \int_{\Gamma_m} \left(V_n^* w - M_n^* \frac{\partial w}{\partial n}\right) d\Gamma - \sum_{j=1}^{N_{c1}} \frac{D_j}{D} R_{cj}^* w_{cj} - \sum_{j=1}^{N_{c2}+N_{c3}} \left(\frac{D_j - D_a}{D}\right) R_{cj}^* w_{cj}$$

$$+ \int_{\Gamma} \left(V_n w^* - M_n \frac{\partial w^*}{\partial n}\right) d\Gamma - \sum_{j=1}^{N_{\text{int}}} \int_{\Gamma_{ba}} \left(\frac{D_b - D_a}{D}\right)\left(V_n^* w - M_n^* \frac{\partial w}{\partial n}\right) d\Gamma$$

$$+ \sum_{j=1}^{Ncol} \left[\left(-\frac{a_j E_j}{L_j}\left(\frac{\partial w}{\partial \bar{x}}\right)_j\right) \int_{\Omega_{ci}} \bar{x} w^* d\Omega_{cj} + \int_{\Omega_g} g w^* d\Omega + \sum_{j=1}^{N_c} R_{cj} w_{cj}^* + \right.$$

$$+\left(-\frac{a_j E_j}{L_j}\left(\frac{\partial w}{\partial \bar{y}}\right)_j\right)\int_{\Omega_{ci}} \bar{y} w^* d\Omega_{cj} - \frac{E_j}{L_j} w_j \int_{\Omega_{ci}} w^* d\Omega_{cj} \Bigg] \tag{12}$$

where N_{col} is the columns number.

Considering this scheme three new values remain as unknowns on the column-plate interface: w, $w,_{\bar{x}}$ and $w,_{\bar{y}}$. Note that the integral representations of $w,_n$, can be easily obtained by differentiating Eq. (12). To obtain the curvature integral representations one has to differentiate once more Eq. (12). Then, bending and twisting moment integral representations are obtained by simply applying the definition given in Eq. (4). To obtain the shear force integral representation, completing the internal force values at internal points, one can differentiate the curvature equation once to apply the definition given in Eq.(5).

4 ALGEBRAIC EQUATIONS

The integral representations are transformed into algebraic expressions after discretizing the boundary and beam axes into geometrically linear elements, where quadratic shape functions were adopted to approximate the variables.

The corresponding boundary nodal values remained in the algebraic system are: one deflection w and its normal derivative $w,_n$, the moment M_n normal to the boundary and the effective shear force V_n. Thus, for each boundary node we define two collocation points, where the deflection representation is written: the first point is the node itself or another point placed along the adjacent element when boundary value discontinuity is assumed; the second collocation is an external point very near the boundary. In each corner are defined two values: the deflection and the corner reaction. Thus, in this case, we have chosen to write the deflection representation at each corner. The skeleton nodal values maintained in the algebraic system are: one deflection w and one deflection derivative $w,_n$ with respect to the skeleton line normal direction, being the counterpart values along interfaces eliminated. Thus, for each beam skeleton node we write one deflection relation and one slope relation at collocations defined along the skeleton line. They are coincident with the node when variable continuity is assumed or defined at skeleton element internal point when variable discontinuity is required. In the central points of the column-plate interfaces are defined the following generalized displacements: w, $w,_{\bar{x}}$ and $w,_{\bar{y}}$. Thus we write the corresponding three displacement relations in each one of these points to complete the necessary number of equations to solve the problem. After selecting the recommended collocation points and writing the corresponding algebraic relation for all of them, one obtains the following set of equations:

$$HU = GP + T \tag{13}$$

where U contains the generalized displacement nodal values defined in the columns, along the boundary and along skeleton lines, P contains boundary nodal tractions, T is the independent vector due to the applied loads.

5 NUMERICAL APPLICATIONS

In this section two numerical examples are presented, where the results are compared to either an ANSYS analysis or a model proposed by Paiva (1987), where the beams and columns are modelled by finite elements and the plate by boundary elements. It is important to stress that the structural systems modelled by ANSYS, by the model presented In Paiva (1987) and by the proposed formulation are not exactly the same and therefore the results can be only similar, we do not expect the same results with the these different numerical analysis. For the ANSYS analysis finite solid elements have been used to discretize the slabs, beams and columns. In the proposed model we have used plate elements and we have treated the whole body as a solid, therefore without splitting the plate and the beams; beams are inclusions in the whole body. It is also important to comment that for the meshes considered in the examples the results convergence had been achieved.

The first example consists of analysing a square plate, whose length side (between external beam axis) is adopted equal to 9m, reinforced by several internal and external beams and supported on four columns, as depicted in Fig. (3), where the distance between beam axis is equal to 3m. A distributed load 10kN/m2 is applied over all stiffened plate surface. For the columns, external and internal beams, the cross section dimensions are, respectively: $0.3x0.3m^2$, $0.3x0.8m^2$ and $0.2x0.8m^2$. For the slabs and beams we have adopted elastic modulus E=15x106kN/m2 and Poisson's ratio ν=1/6, while for the columns has been considered E=21x106kN /m2. The plate thickness was assumed equal to 0.08m and the columns length equal to 4m.

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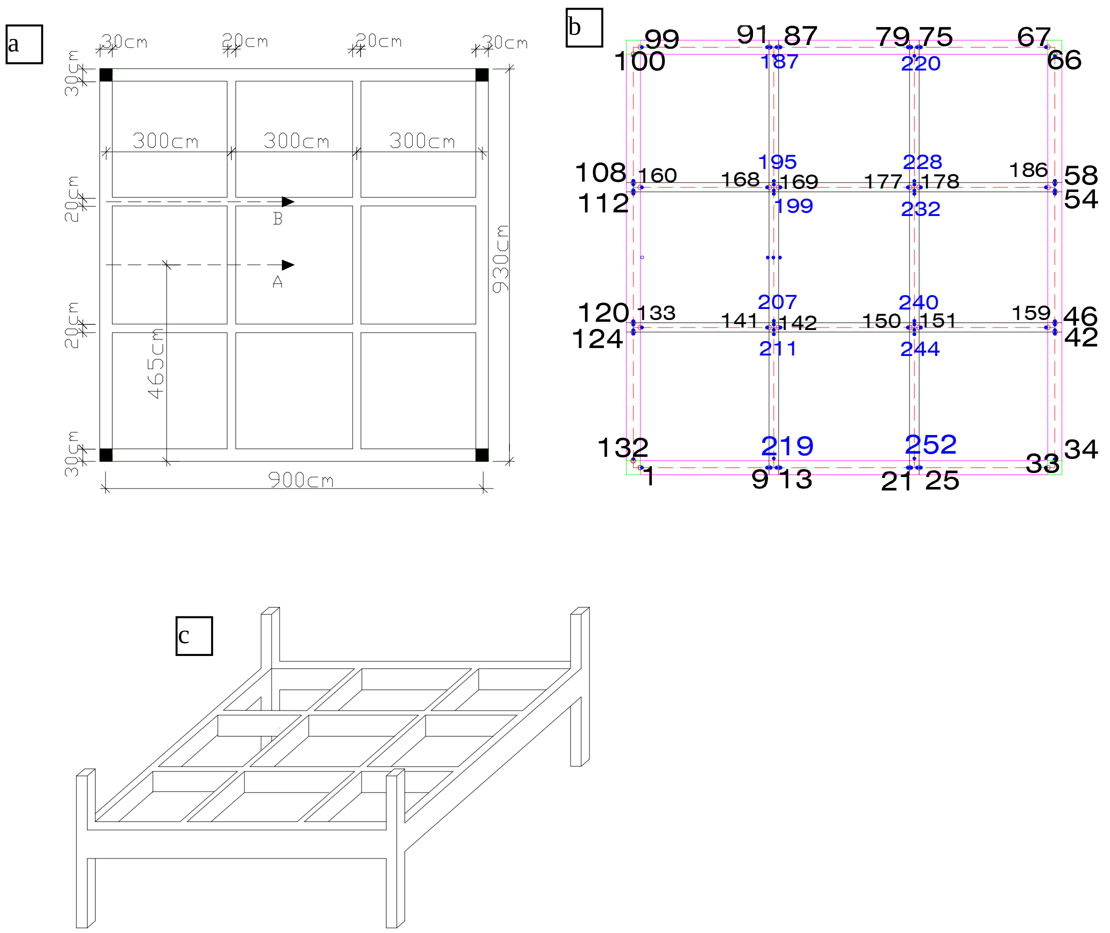

Figure 3 – a) Plate middle surface view b) Plate discretization c) Building floor geometry

The results for displacements and moments will be computed along the local axis A and B defined in Figure 3a. The adopted discretization is shown in Fig 3b, where are defined 252 nodes resulting into108 elements along the beams axis (4 elements on each side where the beam has an interface with a slab and 1 element where one beam cross another one). Some necessary elements at beams ends are not shown in the discretization, because they are automatically generated by the code. In Figure 4 are displayed the displacement w along the axis A-A and B-B, where the results are compared to a finite element analysis presented in Paiva (1987) and the model proposed by Paiva (1987), where the building floor analysis is obtained by coupling BEM with FEM. As can be observed in Fig. 4b, the results along the internal beam axis (axis B-B) compare very well with the finite element analysis and the model proposed by PAIVA (1987). On the other hand, along the slab middle axis (Fig 4a) the displacements obtained with the proposed model are bigger than the ones obtained with the other two models. This can be explained by the fact that near to the internal beam the displacements decrease strongly, evidencing the increasing of rigidity due to the beam. This does not happen in the FEM analysis. As can be observed, we have obtained bigger curvatures along the plate which has flexure rigidity much smaller than the beams. The model behaves as the slabs were partially supported on the beams. In the model presented by Paiva [30], the beams are considered as inclusions into the plate formulation. Note that the plate and the beams present the same curvature; it seems that the beams increase the plate flexure rigidity decreasing the displacements along the plate.

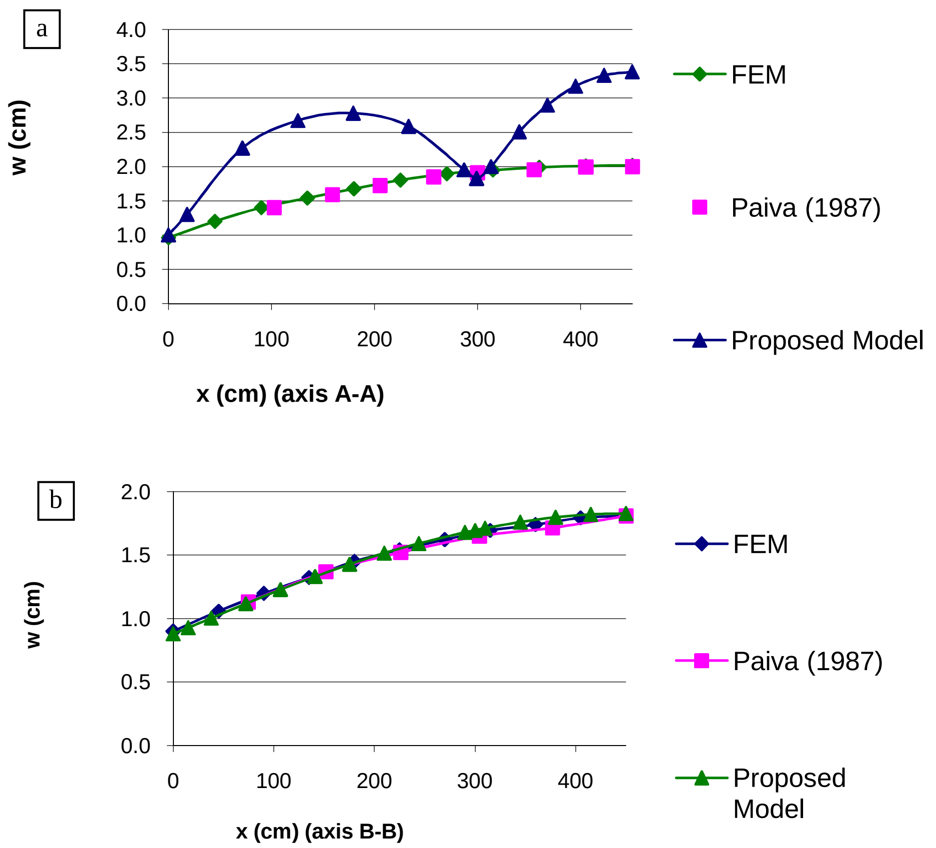

Figure 4: a)Displacement w along the axis A-A b)Displacement w along the axis B-B

In the second example a plate reinforced by external beams along all its boundaries and supported by four columns defined on the corners is analysed (see Figure 5a). The Young's modulus, the Poisson's ratio, the plate and beams thicknesses adopted to analyse this structure are, respectively: $E=25.0x10^6 kN/m^2$, $\nu=0.2$, $t_p=0.1m$ and $t_b=0.3m$ A distributed load of

20kN/m^2 is applied on the whole surface of the structure and all external beams axes have been assumed free. For the columns, which are assumed fixed on their bases, the following data are adopted: length Lc=3m and square cross section with sides equal to $L_{\bar{y}} = L_{\bar{x}} = 0.2$m.

In the ANSYS discretization we have used solid elements (solid brick 8 node 45) whose sides have been adopted equal to 10 cm (see Figure 5b). For the proposed model the finer discretization used to solve this problem, shown in Figure 5c, contains 220 nodes with 104 quadratic elements along the beam axes (including 24 nodes and 8 elements used for the beam intersections which are automatically generated by the code).

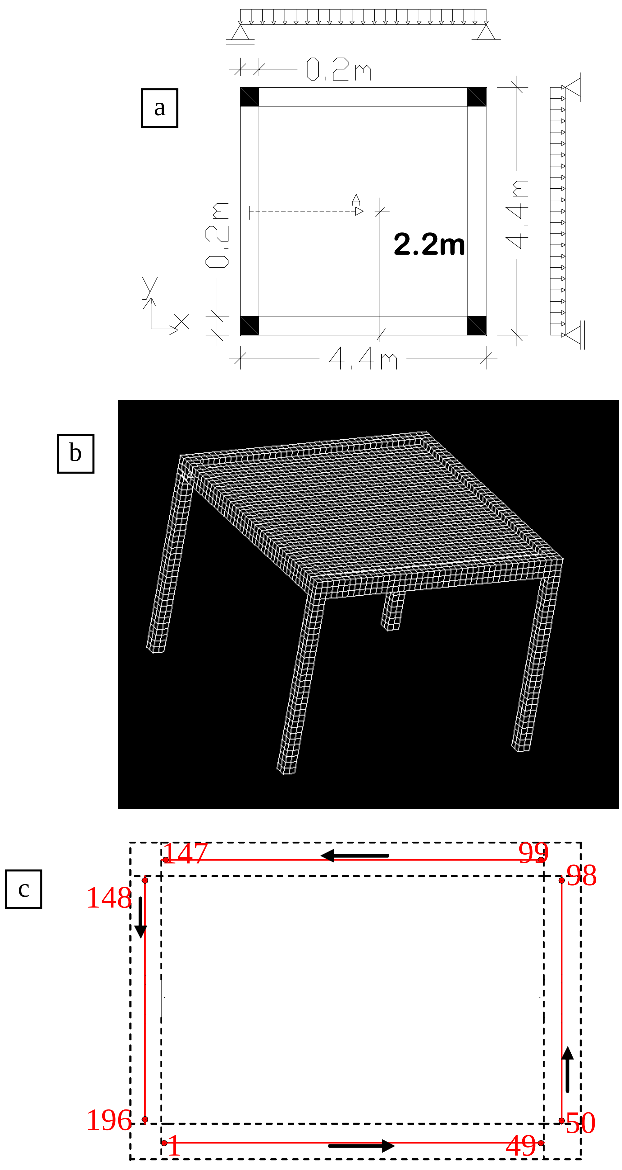

Figure 5 – a) View of the plate reinforced by external beams b)ANSYS discretization c)Discretization for the proposed model

Figure (6) shows, the displacements computed along the beam axis and the A-A axis. As can be observed the results related to the proposed formulation are smaller than those referred to the commercial pack ANSYS, but they are similar.

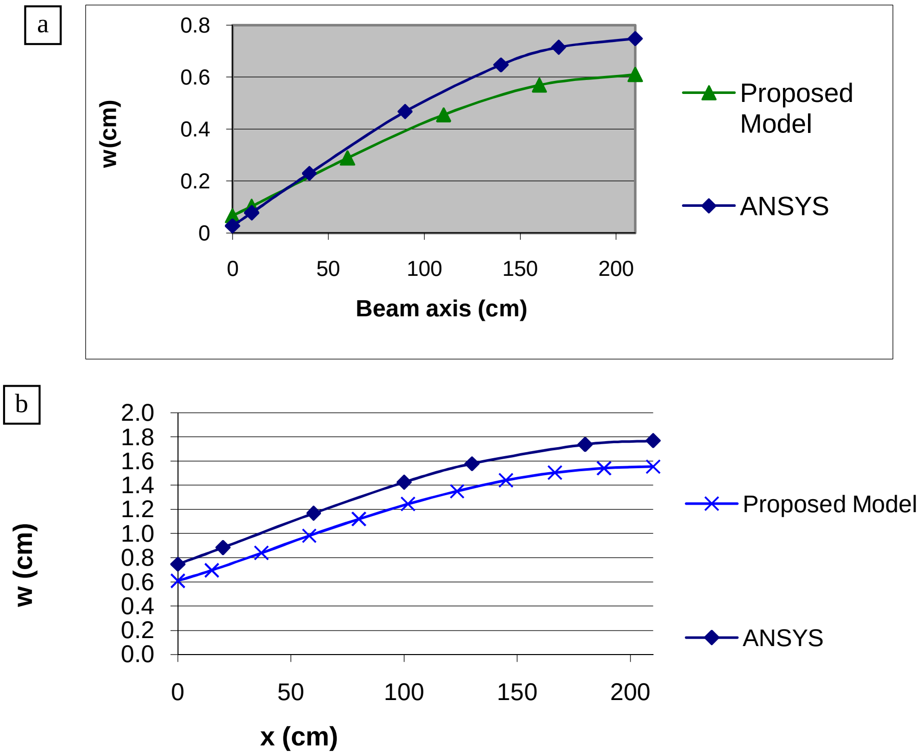

Figure 6 – a)Displacements along the beam axis b)Displacements along the axis A

6 CONCLUSIONS

A BEM formulation for bending analysis of plates reinforced by beams has been extended to consider columns inside the plate domain. The beams are considered as thin sub-regions with larger thickness, being the displacements approximated along the beam cross section to reduce the number of degrees of freedom. Equilibrium and compatibility conditions are automatically guaranteed by the global integral equations. The columns are introduced into the formulation by considering domain points where tractions can be prescribed. The performance of the proposed formulation has been confirmed by comparing the results with solutions obtained by using other numerical models.

7 ACKNOWLEDGEMENTS

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