Asociación Argentina
de Mecánica Computacional

Mecánica Computacional Vol XXIX, págs. 4419-4430 (artículo completo)
Eduardo Dvorkin, Marcela Goldschmit, Mario Storti (Eds.)
Buenos Aires, Argentina, 15-18 Noviembre 2010

CHARACTERIZATION OF MATERIAL PROPERTIES UNDER FINITE STRAIN USING AN OPTICAL METHOD OF MEASUREMENT

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Keywords: Digital Image Correlation, Parameter identification, Material properties, Finite element method.

Abstract. Numerical simulations need a constitutive model suitable to represent the mechanical behavior of the materials. The material parameters for the model are obtained by means of a parametric identification procedure using data acquired in experimental tests and numeric simulation of the discretized specimen. Non-linear materials, like polymers, present a localized heterogeneous displacement fields under finite strain and then the use of non-contact measurement techniques are the most appropriated. The optical method Digital Image Correlation (DIC) has been demonstrated great acceptability in the material characterization due its capability to measure the entire displacement field over the specimen. The experimental data provided by the testing machine and the DIC method provide useful data for the parametric identification procedure of nonlinear materials under finite strain. A cost function of the minimization problem was constructed based on the weighted least squares of the differences between the experimental and numeric data. The cost function has a multi-objective characteristic since it relates experimental and numerical data of different quantity and magnitudes, i.e., forces and displacements. This work presents a characterization of the material parameters of the polymer Polyvinyl chloride (PVC). The material showed localized heterogeneous strain fields, where the DIC method was capable to obtain information regarding the mechanical behavior of necking to be used in the parametric identification. From the results of the sensibility analysis of this material only the experimental force obtained from a uniaxial test is not enough to find the material parameters, since that different combinations of the parameters can result in a numerical force equivalent to the experimental force but quite different with regard to the experimental and numerical transverse displacement.

1 INTRODUCTION

The numerical simulation of structural components needs suitable constitutive models and appropriated material parameters to describe its mechanical behavior. In order to determine the material parameters, experimental tests are performed in specimens and the experimental conditions are numerically reproduced in a simulation.

The mixed numerical-experimental procedure has been used in inverse modeling in several areas and it is widely applied in parameter identification procedures. Several experimental procedures can be set up to analyze the material mechanical behavior. Tensile tests are commonly used due to its simplicity. In Mahnken (1999) a material characterization is presented for an axisymmetric tensile bar of ferritic steel. A sensitivity analysis is performed and the gradient-based descent methods are used for the minimization of a least-square function. Other example of a possible approach for the inverse problem is found in Gavrus et al. (1996) to characterize a thermal-visco-plastic model for Aluminum by means of resistance torque data of a torsion test.

The presence of finite strain and localizations in nonlinear materials, like metals, rubbers and plastics, turns the identification of material parameters a non trivial task. Frequently experimental uniaxial tests provides insufficient information about the mechanical behavior of the material, especially for some polymeric materials. In Frank and Brockman (2001), the authors notice the formation of a traveling neck in glassy polymers materials under tension, differently of the metallic materials that have a localized necking phenomenon. Second the author, this particular behavior of the necking can mask the mechanical behavior of the real stress-strain curve.

Nonhomogeneous deformation fields, as in the necking region are capable to add extra information regarding of material behavior, as presented in Oomens et al. (1993). In this case, to correctly identify the constitutive parameters is useful to know the full displacement field. Measurement techniques such as clip gages or extensometers have some limitations since they need physical contact, which turn them difficult to use in the presence of finite strains and localizations. in addition, non-contact techniques like optical ones, are capable of providing the full displacement field over a focused region.

The optical method of digital image correlation (DIC) was extensively studied by Sutton et al. (2000), Lu and Cary (2000) and Pan et al. (2009), showing a great acceptability in academic and industrial fields. This optical technique is capable to track predefined points on the specimen surface in a digital image sequence, obtained using video cameras in experimental tests. presently the method has been applied to different engineering problems. In Roux et al. (2008) the DIC method is presented as an alternative to capture the crack progressing of a cyclic tensile test. Parameter identification procedures with the aid of the DIC method have been performed to characterize different constitutive models involving elasticity, plasticity and viscosity. Avril et al. (2008) uses the DIC method in a tensile test for notched steel bars in order to identify the material parameters of an elasto-visco-plastic constitutive model. In Kajberg and Lindkvist (2004) the plastic behavior of thin sheets of two types of hot-rolled steel are analyzed using two models: piecewise linear plasticity model and a parabolic hardening model, where the material parameters are adjusted to achieve a minimum in a objective function by means of a inverse modelling including finite element analysis. The inverse problem including the optical measurement method also demonstrated good results in different experimental tests, as observed in Sutton et al. (2008) and Milani et al. (2009), where the authors performed a parameters identification of a Johnson-Cook model, respectively, for tensile and compressive tests. In Wang and Cuitiño (2002) and Jin et al. (2007), the authors use the DIC method to capture localized and heterogeneous strain fields in order to characterize polymeric foams in compressive tests.

In Meuwissen et al. (1998) the author proposes the use of non-standard aluminum specimens, where the force obtained from the testing machine and the displacements measured optically at the clamps region are used as boundary conditions of its numerical simulation, so the objective function is constructed only with the displacements measured at the necking region. This study shown the flexibility provided by the use of measurement optical methods associated to numerical simulation of non-standard geometries of specimens. The DIC method is very important to the identification of the material parameters, because it provide information about the three-dimensional displacement field over the specimen surface while the test is performed (Parker, 2009), i.e., it can capture the formation and propagation of the necking during the tensile test.

Recently, the material characterization with optical methods has demonstrated interesting in the medical community, especially in characterization of biological tissues, not only due to the fact that it allows to test different geometries, but also due to the capability to perform tests *in vivo*. Gambarotta et al. (2005) studied the mechanical characterization *in vivo* of human skin to simulate a reconstructive surgery with the finite element method. In Gundiah et al. (2009) is used a neo-Hookean isotropic model to characterize elastin networks obtained from porcine arteries. The experimental data was captured from biaxial test in order to analyze the structural integrity of blood vessels in cycles of pulsatile motion.

In view of the advantages of including a measurement optical method to a parameter identification procedure, the main objective of the present work is to study a methodology to characterize polymeric materials under finite strains using a optical method and modified standard specimens. The optical method provides extra information concerning the material mechanical behavior during test, mainly in the necking region where the nonhomogeneous fields are noticed.

2 EXPERIMENTAL PROCEDURE

2.1 Specimen definition

The identification of the mechanical behavior of the nonlinear materials under finite strain can be a complex task since they may present a high level of localized strains. The polymers are known by their complex mechanical behavior under finite strains that provide high levels of heterogeneous strain characterized by increasing stiffness due to internal molecular orientation.

The specimens used were manufactured with the glassy polymer Polyvinyl chloride (PVC) because it presents the appreciated complex mechanical behavior that this study is concerned about, and due to the fact that its stress-strain curve have been subject of discussion in Frank and Brockman (2001). Many studies on the mechanical behavior of glassy polymers were published showing an apparent stress drop in the real stress-strain curve right after yield. In Duan et al. (2001) the author show experimental curves for some glassy polymers exhibiting yielding followed by intrinsic strain softening and subsequent orientation hardening. This mechanical behavior is presented in Fig.1. Unlike from the widely accepted softening behavior (stress drop), Frank and Brockman (2001) attribute this behavior to the formation of a traveling neck that masks the actual stress response, claiming that the true stress for most polymers in the glassy regime does not actually decrease after yield.

The geometry of the specimen used in the present study is based on the standard test ASTM $D638$ (ASTMD638-10, 2010). Its geometry was modified by the insertion of a notch at the central region in order to initiate the necking phenomenon at the region of interest (Figure 2).

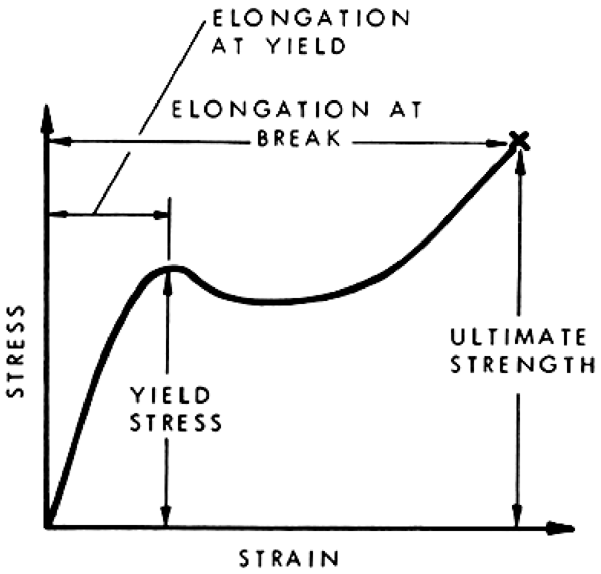

Figure 1: Generalized tensile stress-strain curve for polymeric materials (Blaga, 1973)

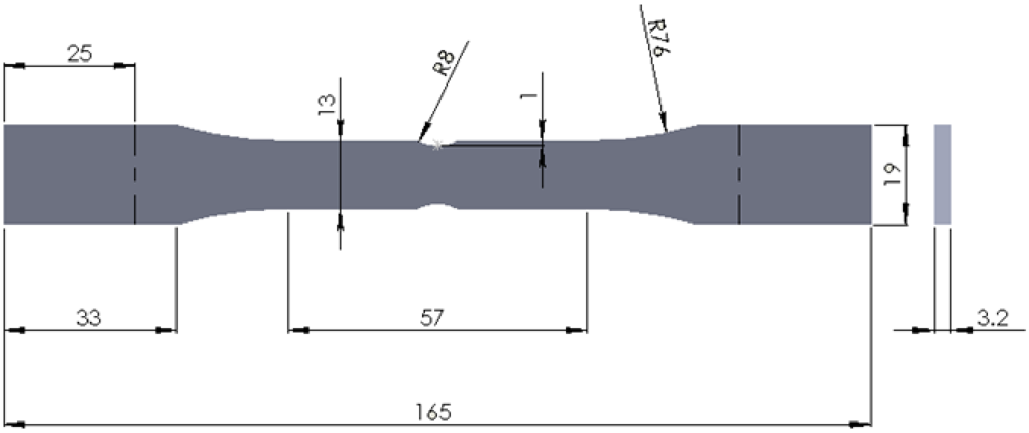

Figure 2: Test sample

2.2 DIC method

The Digital Image Correlation method (DIC) tracks predefined points on the specimen surface in a sequence of digital images obtained by video cameras during an experimental test (Sutton et al., 2000). The method essentially finds the position of points, called markers, in a sequence of images, correlating the gray values of the neighbor pixels. The correlation is made within small squared areas of $(2M + 1)$x$(2M + 1)$ pixels around the marker, called subsets, where the geometric center of the subset is the marker. A wide set of coefficients correlating may be found in the literature (Pan et al., 2009). The one used in this study is based on the

Zero-Normalized Sum Square Differences (ZNSSD) (Pan et al., 2010):

$$C_{ZNSSD} = \sum_{i=-M}^{M} \sum_{i=-M}^{M} \left[\frac{f(x,y) - f_m}{\sqrt{\sum\limits_{i=-M}^{M} \sum\limits_{i=-M}^{M} (f(x,y) - f_m)^2}} - \frac{g(x',y') - g_m}{\sqrt{\sum\limits_{i=-M}^{M} \sum\limits_{i=-M}^{M} (g(x',y') - g_m)^2}} \right]^2 \tag{1}$$

where

$$\begin{aligned} x' &= x + \bar{u} = x + u + u_x \triangle x + u_y \triangle y + u_{xy} \triangle x \triangle y + u_{xx} \triangle x^2 + u_{yy} \triangle y^2, \\ y' &= y + \bar{v} = y + v + v_x \triangle x + v_y \triangle y + v_{xy} \triangle x \triangle y + v_{xx} \triangle x^2 + v_{yy} \triangle y^2, \end{aligned} \tag{2}$$

$f(x,y)$ is the gray value of the reference image at the pixel position (x,y), $g(x,y)$ is the gray value of the target image at the pixel position (x',y'), f_m and g_m are respectively the means gray value of the reference and deformed subset, u and v are respectively the $x-$ and $y-$directional displacement components of the reference subset center, u_x, u_y, v_x, v_y are the first-order displacement gradients and u_{xx}, u_{xy}, u_{yy}, v_{xx}, v_{xy}, v_{yy} are the second-order displacement gradients. Equation 2, by means of the displacement gradients, allows to find the center of a deformed subset. Figure 3 show the reference image with a reference subset of the marker P and its respective target subset (deformed) in the deformed image.

Two modifications proposed in Pan et al. (2010) to deal with deficiencies of this method was implemented: a scanning strategy guided by the correlation coefficients of computed points to ensure the convergence of consecutive points and the use of binary masks that modify the valid subset area in order to allow the computation of the displacement of markers located near or at the boundaries of specimen, where the selected square subsets surrounding these markers may contain unwanted or foreign pixels from background image or other regions.

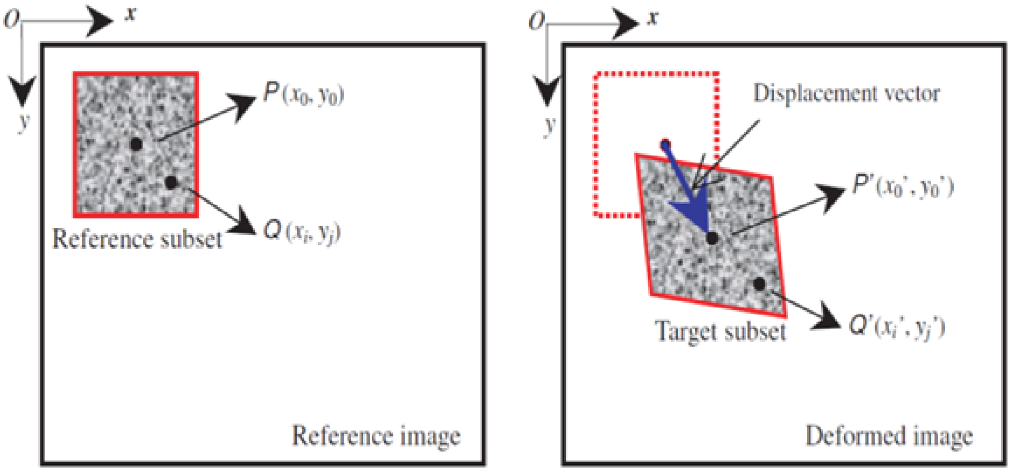

Figure 3: Reference subset and deformed/target subset (Pan et al., 2009).

To properly apply the method, a random speckle pattern is painted with an airbrush on the surface of the specimen. The characteristics of this pattern has important influence on the accuracy of the correlation. According to Chu et al. (1985), it should supply the information needed to correlate the subsets through the test.

2.3 Experimental tests

The experiments were performed in a universal testing machine with a constant displacement rate of $10\ mm/min$. The testing machine is equipped with a load cell and a displacement transducer that provide the reaction force and fixation displacement along time.

The optical information coming from DIC consists on the displacement field of a chosen region of the specimen surface along time. Figure 4 shows the image acquisition system in front of the specimen during the tensile test, while Figure 5 shows the three components X, Y and Z of the displacement at the end of the test.

Figure 4: Cameras arrangement during the test

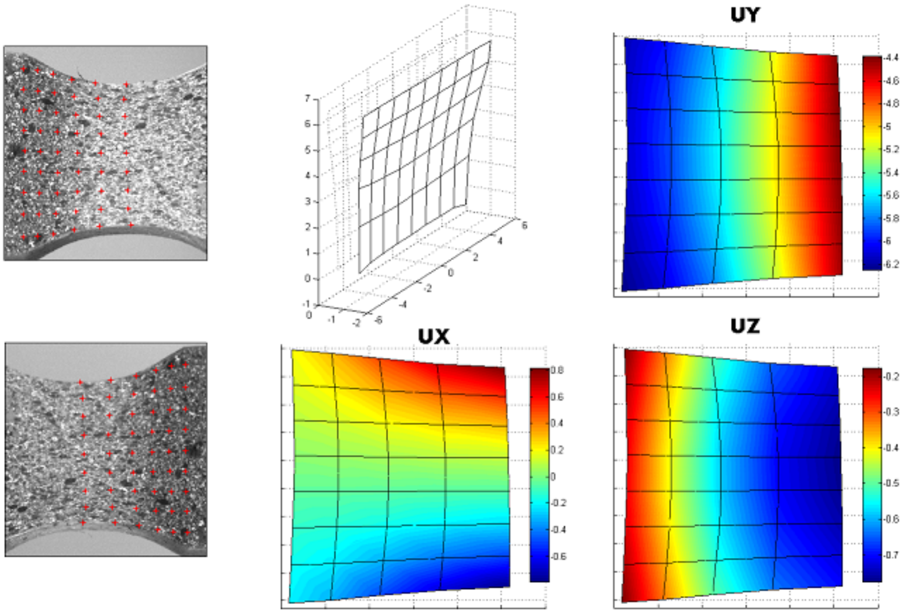

Figure 5: Deformation field in X, Y e Z direction

3 NUMERICAL PROCEDURE

3.1 Geometry

A 3D model of the specimen was generated in order to reproduce the experimental test. Only $^1/_8$ of its geometry was used due to symmetry properties. Also, the region supported by the clamp was eliminated from the model. The final geometry is presented in Fig. 6.

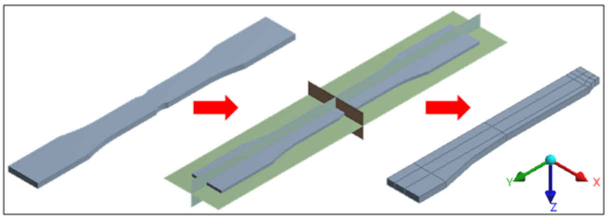

Figure 6: CAD model of the specimen

3.2 Finite Element Model

The discretization in finite elements was done using the package ANSYS Workbench 12.1 (ANSYS, Inc., 2010). The mesh generated is shown in Fig. 7. The longitudinal displacement experimentally obtained from the testing machine are applied at the extreme of the discretized specimen, while symmetry conditions are applied on the three symmetry planes.

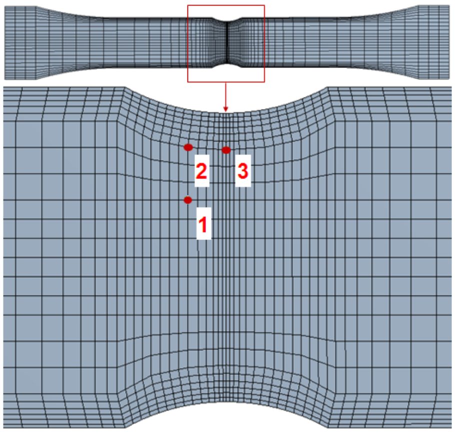

Figure 7: Finite element mesh

In order to simulate the mechanical behavior of the material, A first test was performed using a multi-linear elasto-plastic model presented in Fig. 8 was chosen. This constitutive model was used due to its simplicity and capability to reproduce nonlinear elasto-plastic behaviors, despite of its limitation in to reproduce only stress-strain curves with positive tangent slopes. The material model has 2 parameters for the elastic part: elastic modulus E, Poisson ratio ν; and 6 parameters for the multi-linear plastic part: the yield stress (σ_y), two stress increment (Δ), three tangent modulus (ϕ, H and I, where the last represent the orientation hardening of the molecules) and the hardening plastic strain (ε_p).The results of the numerical analysis provide the force-time curve and displacement-time curves of X, Y and Z directions for each marker of interest.

Since the true stress-strain curve of the PVC may shown negative tangent slopes with a significant softening region (Fig. 1), this material model may not be able to reproduce THE mechanical behavior of the specimen. On other hand, if the true stress-strain curve of the PVC actually is in accordance to the assumption of Frank and Brockman (2001), It is expected that the model will be able to achieve reasonable good solutions.

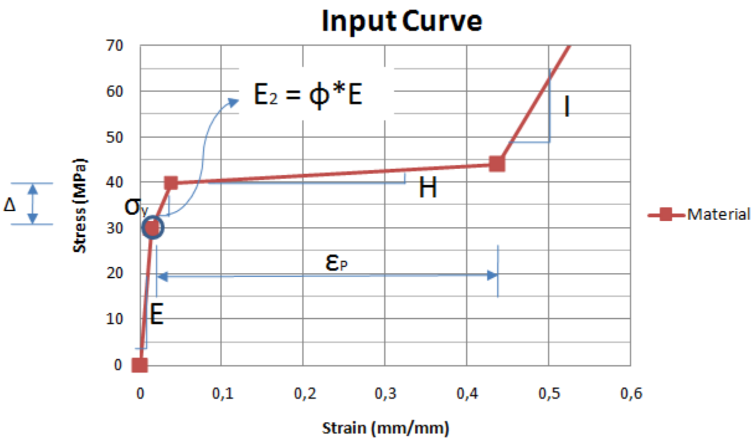

Figure 8: Constitutive model

4 OPTIMIZATION PROCESS

The basic idea is searching for the material parameters capable to better reproduce the mechanical behavior of the experimental test. To this aim a cost function Ω is defined based on the weighted least squares of the difference between experimental and numerical data:

$$\Omega = \sum_{i=1}^{N} \left[w^F (F^{num}(\mathbf{p})_{(i)} - F^{exp}{}_{(i)})^2 + \sum_{j=1}^{M} w^{UX(j)} (UX^{num}(\mathbf{p})^{(j)}_{(i)} - UX^{exp}{}^{(j)}_{(i)})^2 \right] \quad (3)$$

where N is number of data acquired during the test, M is the number of markers analyzed, w^F is the weight of the force, $F^{num}{}_{(i)}$ is the $i-$th component of the numerical force, $F^{exp}{}_{(i)}$ is the $i-$th component of the experimental force, $w^{UX(j)}$ is the weight of the displacement of the $j-$th marker, $UX^{num}{}^{(j)}_{(i)}$ is the $i-$th component of the numerical displacement of the $j-$th marker, $UX^{exp}{}^{(j)}_{(i)}$ is the $i-$th component of the experimental displacement of the $j-$th marker and **p** represent the vector with the material parameters:

$$\mathbf{p} = \left[\begin{array}{cccccccc} \mathrm{E} & \nu & \sigma_y & \Delta & \phi & H & I & \varepsilon_p \end{array} \right]. \quad (4)$$

The schematic representation of the parameter identification is shown in Fig. 9. An initial guess value of the material parameters **p** are used as input data. The numerical analysis computes the reaction force and displacements over the surface of the specimen that together with the experimental equivalent values compose the cost function Ω to be minimized. In this study only the $x-$displacement of three markers (see Fig. 7) were used as a first attempt for the optimization procedure.

The optimization process was performed in the commercial software modeFRONTIER (modeFRONTIER, 2010). Since the minimization involves 10 unknown parameters, a genetic optimization was used to obtain the global minimum candidates, and then, these candidates are used as initial guess of a SIMPLEX minimization. This strategy was used to avoid local minima.

5 RESULTS

Different tests were carried out varying the weight of the force w^F and the weight of the displacements w^{UX} in the expression 3 in order to obtain an adequate fitting of the curves. When weights are set as w^F»w^{UX} good results were achieved for the force but the displacement

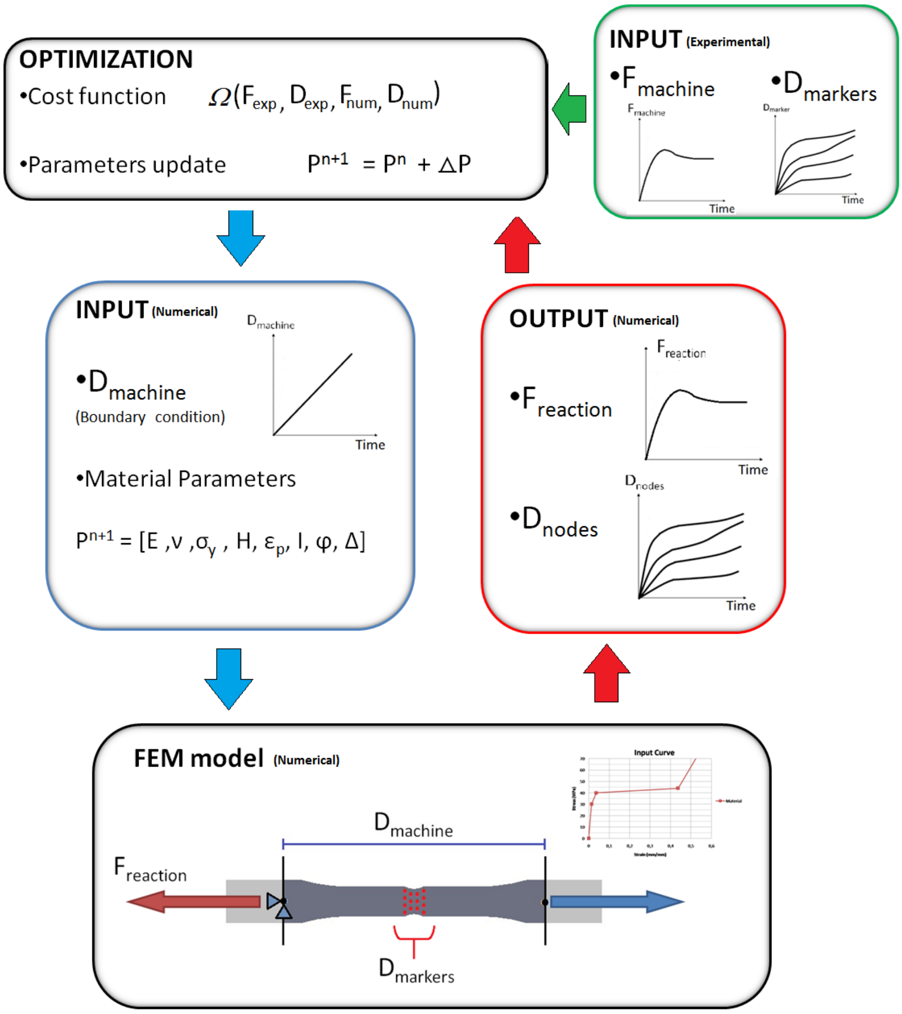

Figure 9: Optimization routine

over the surface shown considerable errors. Conversely, when w^F«w^{UX} good results were achieved only for the displacement over the surface of the specimen. The material parameters obtained in the parameter identification procedure are shown in Tab. 1 and the numerical and experimental curves are shown in Fig. 10.

PARAMETER	VALUE
E	3000 MPa
ν	0.43
σ_y	36.67 MPa
ϕ	0.22
Δ	15.96 MPa
H	3.19 MPa
ε_p	0.38
I	109.68 MPa

Table 1: Material parameters results

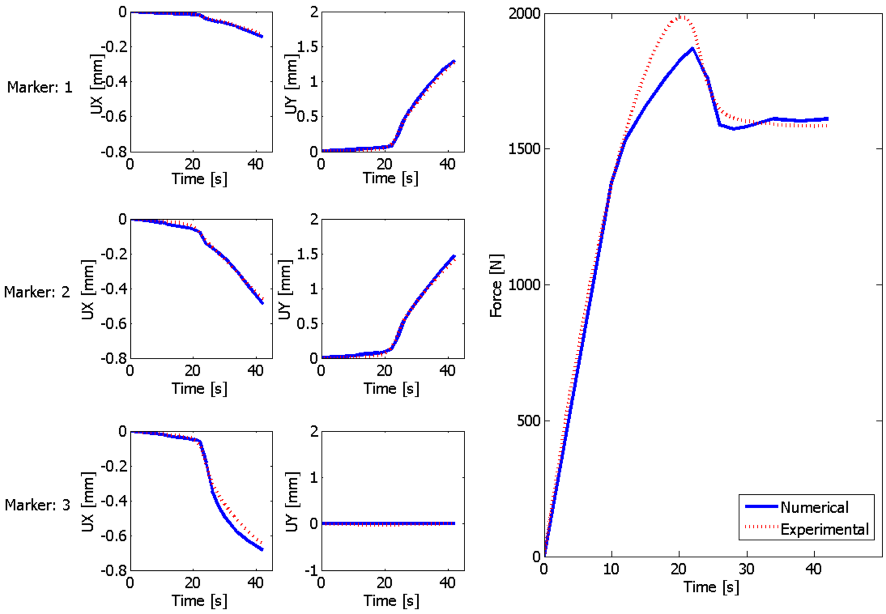

Figure 10: Numerical and experimental curves

6 CONCLUSION

In this preliminary study, the parametric identification using data acquired from a optical measuring system was investigated.

The PVC material provided significative heterogeneous strain fields and the DIC method was capable to obtain important information regarding the mechanical behavior of material (traveling neck), mainly at the surface of the necking region where the control points were analyzed.

Although only three markers were effectively used within the cost function, they seem to provide significative information of the traveling neck. The addition of other intermediate points and displacement in other directions is under investigation.

The constitutive model chosen to represent the mechanical behavior of PVC was the elasto-plastic multi-linear. As expected, the numerical results obtained with this model showed evident differences from experimental data, mainly at the elastic-plastic transition, also observed by Meuwissen et al. (1998). Other non-linear elastic-plastic and elastic-viscoplastic more appropriate models are subject of future investigations. It is also important to note the possible real tensile drop of the material contested by Frank and Brockman (2001) but present in many studies. This softening can not be represented with the material model presently chosen.

The cost function used in this study has a multi-objective characteristic since it relates experimental and numerical data of different quantity and magnitudes. In order to find the similar magnitude for different degrees of freedom present in the objective function, as displacement and force, a weighting method is required, as observed by Milani et al. (2009). This weighting method was fitted through a sensitivity analysis of the parameters involved and is capable to change the convergence region of objective function.

From the results of the sensibility analysis of this material only the experimental force is not enough to find the material parameters, since that different combinations of the parameters can result in a numerical force equivalent to the experimental force but quite different with regard to the experimental and numerical transverse displacement.

Despite of the many performed simplifications, like the use of a simple material model and small number of tracking points, we can conclude that the use of DIC provided significative information to the inverse problem of material parameter characterization. In this context the objective proposed with preliminary study was successfully achieved and opens a promising field of further improvements.

REFERENCES

ANSYS, Inc. *User guide for revision 12.1.* ANSYS, Inc., Pittsburgh, 2010. http://www.ansys.com.

ASTMD638-10. Standard test method for tensile properties of plastics. American Society for Testing and Materials D638-10, 2010.

Avril S., Pierron F., Sutton M.A., and Yan J. Identification of elasto-visco-plastic parameters and characterization of lüders behavior using digital image correlation and the virtual fields method. *Mechanics of Materials*, 40(9):729 – 742, 2008.

Blaga A. Properties and behaviour of plastics. Report 157, NRC Institute for Research in Construction; National Research Council Canada, 1973.

Chu T., Ranson W., and Sutton M. Applications of digital-image-correlation techniques to experimental mechanics. *Experimental Mechanics*, 25:232–244, 1985.

Duan Y., Saigal A., Greif R., and Zimmerman M.A. A uniform phenomenological constitutive model for glassy and semicrystalline polymers. *Polym Eng Sci*, 41(8):1322–1328, 2001.

Frank G.J. and Brockman R.A. A viscoelastic-viscoplastic constitutive model for glassy polymers. *International Journal of Solids and Structures*, 38(30-31):5149–5164, 2001.

Gambarotta L., Massabò R., Morbiducci R., Raposio E., and Santi P. In vivo experimental testing and model identification of human scalp skin. *Journal of Biomechanics*, 38(11):2237–2247, 2005. ISSN 0021-9290.

Gavrus A., Massoni E., and Chenot J.L. An inverse analysis using a finite element model for identification of rheological parameters. *Journal of Materials Processing Technology*, 60(1-4):447 – 454, 1996. Proceedings of the 6th International Conference on Metal Forming.

Gundiah N., Ratcliffe M.B., and Pruitt L.A. The biomechanics of arterial elastin. *Journal of the Mechanical Behavior of Biomedical Materials*, 2(3):288–296, 2009. ISSN 1751-6161.

Jin H., Lu W.Y., Scheffel S., Hinnerichs T.D., and Neilsen M.K. Full-field characterization of mechanical behavior of polyurethane foams. *International Journal of Solids and Structures*, 44(21):6930 – 6944, 2007.

Kajberg J. and Lindkvist G. Characterisation of materials subjected to large strains by inverse modelling based on in-plane displacement fields. *International Journal of Solids and Structures*, 41(13):3439 – 3459, 2004.

Lu H. and Cary P. Deformation measurements by digital image correlation: Implementation of a second-order displacement gradient. *Experimental Mechanics*, 40(4):393–400, 2000.

Mahnken R. Aspects on the finite-element implementation of the gurson model including parameter identification. *International Journal of Plasticity*, 15(11):1111–1137, 1999. ISSN 0749-6419.

Meuwissen M.H.H., Oomens C.W.J., Baaijens F.P.T., Petterson R., and Janssen J.D. Determination of the elasto-plastic properties of aluminium using a mixed numerical-experimental method. *Journal of Materials Processing Technology*, 75(1-3):204 – 211, 1998.

Milani A., Dabboussi W., Nemes J., and Abeyaratne R. An improved multi-objective identification of johnson-cook material parameters. *International Journal of Impact Engineering*, 36(2):294 – 302, 2009.

modeFRONTIER. *The Multi-Objective Optimization and Design Enviroment. Manual do*

Usuário. Esteco srl. Trieste, Itália., 2010. http://www.modefrontier.com.

Oomens C., Ratingen M., Janssen J., Kok J., and Hendriks M. A numerical-experimental method for a mechanical characterization of biological materials. *Journal of Biomechanics*, 26(4-5):617 – 621, 1993.

Pan B., Qian K., Xie H., and Asundi A. Two-dimensional digital image correlation for in-plane displacement and strain measurement: a review. *Measurement Science and Technology*, 20(6):062001–, 2009.

Pan B., Wang Z., and Lu Z. Genuine full-field deformation measurement of an object with complex shape using reliability-guided digital image correlation. *Opt. Express*, 18(2):1011–1023, 2010.

Parker J.W. *Development And Implementation Of A Low Cost Image Correlation System To Obtain Full-field In-plane Displacement And Strain Data*. Master of science in mechanical engineering, Montana State University, Bozeman, Montana, 2009.

Roux S., Réthoré J., and Hild F. Recent progress in digital image correlation: From measurement to mechanical identification. *Journal of Physics: Conference Series*, 135(1):012002, 2008.

Sutton M., McNeill S., Helm J., and Chao Y. Advances in two-dimensional and three-dimensional computer vision. *Photomechanics*, pages 323–372, 2000.

Sutton M., Yan J., Avril S., Pierron F., and Adeeb S. Identification of heterogeneous constitutive parameters in a welded specimen: Uniform stress and virtual fields methods for material property estimation. *Experimental Mechanics*, 48(4):451–464, 2008.

Wang Y. and Cuitiño A.M. Full-field measurements of heterogeneous deformation patterns on polymeric foams using digital image correlation. *International Journal of Solids and Structures*, 39(13-14):3777 – 3796, 2002. doi:DOI:10.1016/S0020-7683(02)00176-2.