Asociación Argentina de Mecánica Computacional

Mecánica Computacional Vol XXIX, págs. 5227-5236 (artículo completo)
Eduardo Dvorkin, Marcela Goldschmit, Mario Storti (Eds.)
Buenos Aires, Argentina, 15-18 Noviembre 2010

ELASTOPLASTIC CONSTITUTIVE MODEL FOR CONCRETES OF ARBITRARY STRENGTH

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Keywords: high strength concrete, normal strength concrete, performance parameter, constitutive formulation, plasticity, fracture energy, non associative.

Abstract. In this paper, the main features of a new elastoplastic constitutive formulation for concretes valid for a large spectrum of strengths, including normal to high strength concretes, are presented.
The model, depending on the three stress invariants, has a non associative flow rule, a non uniform hardening law, and an isotropic softening law based on the fracture energy of modes I and II. It considers as material parameters the uniaxial compressive strength, which is a macroscopic property, as well as the performance parameter for concretes. The latter is an index defined by the authors in previous works which takes into account some properties of the concrete mixture. Moreover, the maximum aggregate size is considered in order to evaluate the concrete fracture energy. As result, a constitutive model capable to distinguish different mechanical behaviors of concretes of different qualities is obtained.
Particularly, in this work a description of the constitutive model is presented, with focus on the hardening and softening laws, ending with the presentation of numerical experiments.

1 INTRODUCTION

In what concerns to concrete constitutive formulations as well as failure criteria, usually no restriction about its type or quality is explicitly considered. Nevertheless, experimental evidence demonstrates that for increasing uniaxial compression strength (f'_c), the mechanical properties gradually change but not proportionally to the strength increment. The different behaviors of normal (NSC) and high strength concretes (HSC) have been extensively detailed in the literature (See Xie et al. 1995, Imran and Pantazopoulou 1996; Rashid et al. 2002, Hussein and Marzouk 2000, etc.). HSC are achieved by the incorporation of chemical additives as well as mineral admixtures in the concrete mixture, which considerably change the microstructure of the cement past.

A clear example of the non isotropic variation of the mechanical properties is represented by the ratio between the uniaxial tensile strength (f'_t) and f'_c: although f'_t increases for a greater compressive strength, the ratio between these two strengths f'_t / f'_c decreases.

Ductility is one of the concrete mechanical properties that most varies whit a variation of concrete quality and, therefore, the transition point from ductile to brittle failure modes.

Most of the concrete constitutive laws in the literature were developed for NSC (Willam and Warnke 1974, Ohtani and Chen 1988, etc.). Some of them can be also used for HSC after an appropiate and complex calibration (Oller 1988, Etse and Willam 1994, Kang and Willam 1999, Grassl et al. 2002). Nevertheless, contrarily to the case of NSC, the mechanical behavior of HSC has still many unknown aspects, which are the subject of several ongoing experimental and numerical studies by the international research community. The increasing demand of HSC in civil constructions and structures requires the development of reliable constitutive laws able to accurately predict the different behavior of these high performance concretes.

The concrete constitutive model subject of this paper, so called Performance Dependent Model (PDM) covers the whole spectrum from NSC to HSC in the range 20 to 120 MPa. The main features of the model are described in the next section. Details of the hardening and softening laws and the corresponding plastic potential surface are presented. Finally, some numerical results are plotted and compared against experimental data.

2 PDM MAIN FEATURES

This model is based on the smeared crack approach and the elasto-plastic incremental flow theory enriched with fracture energy concepts in order to achieve a non local formulation. It depends on the three stress invariants.

Only infinitesimal strains are considered. Elastic-plastic coupling is neglected, accepting the additive Prandtl-Reuss decomposition of the infinitesimal strain rate tensor into its elastic and plastic parts as

$$\dot{\boldsymbol{\varepsilon}}_{ij} = \dot{\boldsymbol{\varepsilon}}_{ij}^{e} + \dot{\boldsymbol{\varepsilon}}_{ij}^{p} \tag{1}$$

The elastic constitutive response is defined by the generalized Hooke law

$$\dot{\boldsymbol{\sigma}}_{ij} = \boldsymbol{E}_{ijkl}\, \dot{\boldsymbol{\varepsilon}}_{kl} \tag{2}$$

In the above equation $\dot{\boldsymbol{\sigma}}_{ij}$ is the Cauchy stress rate tensor, $\boldsymbol{E}_{ijkl}$ the fourth order elasticity tensor depending on the material Young's modulus E_c and on the Poisson's ratio υ.

A yield surface denoted as $f\left(\underline{\underline{\sigma}};\kappa\right)=0$, limits the elastic range, which size and shape depend on a set of state variables ${}^{t}\kappa$.

Inelastic material response is governed, by the following non associated flow rule

$$\dot{\boldsymbol{\varepsilon}}_{ij}^{p} = \dot{\lambda}\ \boldsymbol{m}_{ij} \text{ where } \boldsymbol{m}_{ij} = \frac{\partial g}{\partial \boldsymbol{\sigma}_{ij}} \tag{3}$$

Being $g\left(\underline{\underline{\sigma}};\kappa\right)=0$ the plastic potential surface, which does not coincide with the yield surface f, and $\dot{\lambda}$ the plastic parameter. For the determination of the latter, the consistency condition is applied.

The evolution of the state variables ${}^{t}\dot{\kappa}$ is defined by the hardening/softening laws.

2.1 The failure criterion and the performance parameter

The failure criterion applied in this formulation is the performance dependent failure criterion (PDFC) for concretes of arbitrary strength previously proposed by the authors (See Folino et al. 2009), which covers the entire spectrum of concrete quality from NSC to HSC. It is defined in the Haigh Westergaard stress space in terms of the normalized stress coordinates (with respect to f'_c) $\overline{\xi}$, $\overline{\rho}$ and θ. The two first ones are functions of the first and second invariants to the stress and deviatoric tensors, respectively. The third coordinate is represented by the Lode angle θ function of the third invariant of the deviatoric tensor.

According to this criterion, concrete failure occurs when the normalized 2nd Haigh Westergaard stress coordinate $\overline{\rho}$ reaches the normalized shear strength $\overline{\rho*}$

$$F = \frac{\overline{\rho}}{\overline{\rho*}} - 1 = 0 \tag{4}$$

In the deviatoric plane, the elliptic interpolation between the compressive $\overline{\rho_c^*}$ and the tensile $\overline{\rho_t^*}$ meridians by Willam & Warnke (1974) is followed

$$\forall\ 0^0 \leq \theta \leq 60^0 \ \Rightarrow\ \overline{\rho}* = \frac{\overline{\rho}_c^*}{r} \tag{5}$$

Being r the ellipticity factor

$$r = \frac{4(1-e^2)\cos^2\theta + (2e-1)^2}{2(1-e^2)\cos\theta + (2e-1)\sqrt{4(1-e^2)\cos^2\theta + 5e^2 - 4e}} \tag{6}$$

and e the eccentricity $\overline{\rho_t^*} / \overline{\rho_c^*}$. The failure surface is represented by the following equation

$$F = A.r^2.\overline{\rho*}^2 + B_C.r.\overline{\rho*} + C.\overline{\xi} - 1 = 0 \tag{7}$$

In the previous equation the coefficients A, B_C, B_t, and C depend on four material parameters: the uniaxial compressive strength f'_c, the uniaxial tensile strength f'_t (through uniaxial strength ratio $\alpha_t = f'_t / f'_c$), the biaxial compressive strength f'_b (through the biaxial compressive strength ratio $\alpha_b = f'_b / f'_c$), and a parameter m representing the friction defined as the tangent to the compressive meridian on the peak stress's shear component corresponding to the uniaxial compression test. Therefore, contrary to other failure criteria, these coefficients change when these four material material change, leading to different failure surfaces. This feature permits to adequately capture the failure of both NSC and HSC. (See Fig. 1)

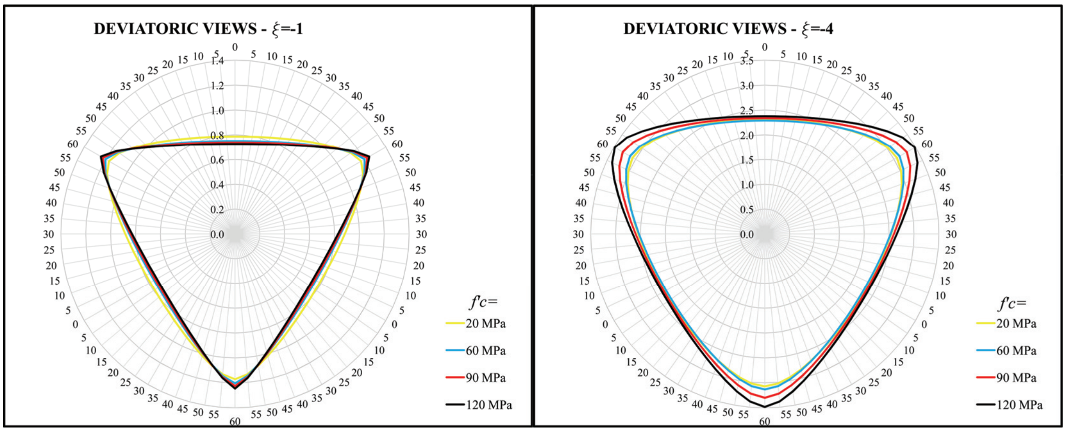


Figure 1: PDFC – Deviatoric views – Normalized plots for different concretes and for two different confinement levels

Considering on one hand that three of the material properties needed to define the coefficients are obtained from non standard and complex experimental tests, and that those properties are defined by the concrete composition, a quality index so-called the performance parameter β_P was formulated for the purpose of the PDFC as

$$\beta_P = \frac{1}{1000}\frac{f'_c}{(W/B)} \qquad f'_c \text{ in } [MPa] \tag{8}$$

where W/B is the water/binder ratio, considered as a fundamental property of the concrete mix controlling the material performance. Also, for the case where the W/B ratio is unknown, empirical functions of a maximum and a minimum value of β_P were proposed. With the performance parameter and f'_c, the other three material properties considered in this failure criterion are evaluated by internal functions. Further details may be found in Folino et al. 2009.

Summarizing, this failure criterion depends on the three stress invariants, it has curve meridians and considers as material parameters the uniaxial compressive strength, which is a macroscopic property, as well as the performance parameter for concretes depending on some properties of the concrete mixture.

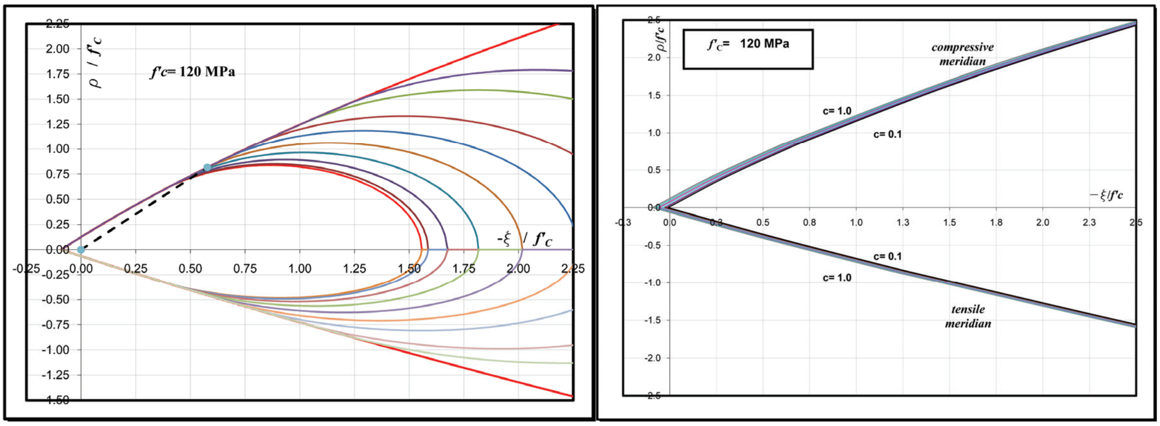
2.2 The hardening

The pre peak regime is described by a non uniform hardening. An intermediate hardening stage is represented by a loading surface that limits the actual elastic range. This surface is composed by two different surfaces. The first one, representing the cone, is the failure surface defined in Equation 7. The second one is a compressive cap. Both surfaces have a common deviatoric plane defined by the hydrostatic coordinate of a point denoted as “P1” where they present a C^1 type continuity. During the hardening process, “P1” will continuously change. (See Folino and Etse 2008). Therefore, the yielding criterion of this model belongs to the type denoted as “cap-cone models”.

Therefore, there exists a zone between the equitriaxial tension point and the deviatoric plane in correspondence with “P1” where no hardening takes place and the peak load is reached without plastic strains. The first Haigh Westergaard coordinate of this point “P1” will depend on the concrete quality and on the hardening level.

The compressive cap is represented in a meridian plane by ellipses centered over the hydrostatic axis and tangents to the failure surface at the first Haigh Westergaard coordinate of point “P1”. The semi-axes ratio remain constant. The loading surfaces result

$$f = \begin{cases} f_{cone} = A.r^2.\overline{\rho*}^2 + B_C.r.\overline{\rho*} + C.\bar{\xi} - 1 = 0 & \text{if } \bar{\xi} \le \bar{\xi}_{1(k)} \\ f_{cap(k)} = \dfrac{\left(\bar{\xi} - x_{cen(k)}\right)^2}{a_{(k)}^2} + \dfrac{r^2.\bar{\rho}^2}{b_{(k)}^2} - 1 = 0 & \text{if } \bar{\xi} > \bar{\xi}_{1(k)} \end{cases} \tag{9}$$



(a) Hardening regime (b) Softening regime

Figure 2

In the left side of Fig. 2, the corresponding yielding surfaces for a HSC may be observed.

In the equations above, k indicates the hardening level parameter, which univocally defines each of the yielding surfaces. The initial hardening parameter k_o is defined by the initial position of Point P1 as

$$k_o = \frac{\bar{\rho}_{cP1o}}{\sqrt{2/3}} \tag{10}$$

Analogously, the hardening level parameter k is defined by the successive positions of this Point P1, characterizing the successive ellipses by

$$k = \frac{\bar{\rho}_{cP1}}{\sqrt{2/3}} \tag{11}$$

It should be noted that the minimum value of k is k_o, but its maximum value is not equal one ($k_o \le k \le \infty$)

The actual hardening parameter is defined in terms of a normalized measure of the developed work hardening κ_h by an elliptic function varying between a minimum and a maximum value as follows

$$k = k_o + \left(k_{(\bar{\xi})}^{\max} - k_o\right)\sqrt{\kappa_h\left(2 - \kappa_h\right)} \tag{12}$$

where $k^{\max}_{(\bar{\xi})}$ corresponds to the maximum possible hardening level associated with a certain level of confinement and the state variable is defined by

$$ {}^{t}\dot{\kappa}_h = \frac{\dot{\omega}^P_a}{W^P_t} = \frac{\underline{\underline{\boldsymbol{\sigma}}} : \underline{\underline{\dot{\boldsymbol{\varepsilon}}}}^P}{W^P_t} \tag{13} $$

In the above equation, the numerator is the actual developed work hardening since the initialization of the plastic process and the denominator is the total work hardening capacity for a given concrete quality and for the level of confinement under consideration. The definition of this parameter and its calibration, have resulted of crucial importance in the formulation of this model.

With the purpose of the calibration of the main parameter in the hardening law described above, the pre peak energy resulting from different experimental test results available in the literature has been analyzed, considering different levels of confinement and different concretes. This energy varies with the confinement level and with the concrete quality. It presents a strong rate of increment of the energy as a function of the first invariant from the beginning of hardening to approximately $\bar{\xi} = -1$ and a medium increment rate for greater levels of confinement. The current internal function that evaluates this parameter is the following, where f_i are variable coefficients depending on concrete quality (See Fig. 3)

$$ W^P_t \begin{cases} = E^{wpt}_{(\beta_P, f'_c)} \left(\bar{\xi} - \bar{\xi}_{P1o}\right)\left[f_1 + f_2 \left(\bar{\xi} - \bar{\xi}_{P1o}\right)^6 \right] & \bar{\xi} \geq \bar{\xi}_{\lim} \\ = f_3\, \bar{\xi}^2 + f_4\, \bar{\xi} + f_5 & \bar{\xi} < \bar{\xi}_{\lim} \end{cases} \tag{14} $$

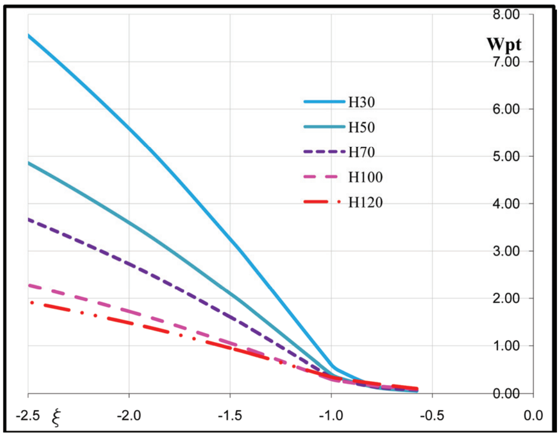


Figure 3: Internal Function of the Prepeak Energy (W^P_t) for different concretes

2.3 The softening

After the maximum strength is reached, a progressive softening is defined in terms of the softening level parameter denoted as *c* which constitutes the state variable in softening and represents the de-cohesion. In the right side of Fig. 2, the corresponding isotropic softening surfaces for a HSC may be observed.

This softening parameter level *c* defines the degradation of the tensile strength during softening.

$$1 \geq c_{(\kappa_s)} = \frac{\sigma_{t(\kappa_s)}}{f_t^{'}} = \exp\left(\frac{-\delta\ \kappa_s}{u_r}\right) \geq 0 \tag{15}$$

This parameter is defined in terms of the softening state variable, represented by the crack opening displacement u_f. The index δ determines the shape of the decay function, and in this constitutive model, it is not a constant value and depends on the maximum size of the coarse aggregates, which is an input datum. The parameter u_r represents the rupture displacement and in this model it is derived once the fracture energy is determined. The softening state variable is represented by

$$\dot{\kappa}_s = \dot{u}_f = l_c\ {}^t\dot{\varepsilon}_f = l_c\ \dot{\lambda}\ \left\| \langle \underline{\underline{m}} \rangle \right\| \tag{16}$$

where ${}^t\dot{\varepsilon}_f$ is the equivalent tensile fracture strain, involving only the tensile components of the plastic potential gradient, and l_c is an internal characteristic length regularizing the softening behavior. This last parameter may be interpreted as the crack spacing in tension h_t factored by the actual ratio of the fracture energies in mode I and II depending on the confinement level.

$$l_c = \frac{G_f^{I}}{G_f^{II}}\ h_t \tag{17}$$

In the implementation of the PDM, an internal function evaluates the fracture energy in mode I in terms of the performance parameter β_P and the maximum size of the coarse aggregates. Another internal function calibrates l_c according to the concrete quality, defined by f'_c and β_P and in terms of the confinement level.

An example of the resulting degradation of the decohesion parameter c for f'_c =70MPa may be observed in Fig. 4.

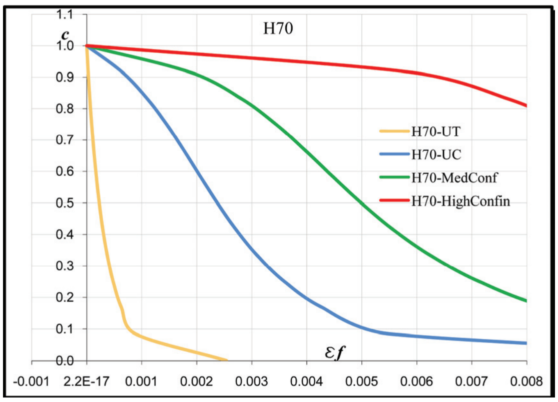
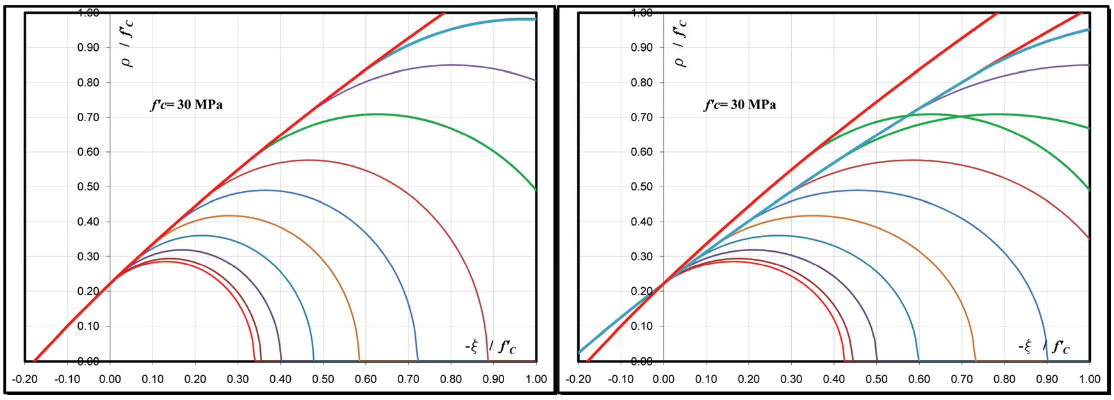


Figure 4: Decohesion for f'_c =70MPa and different confinement levels

2.4 The non associative factor

The limitation of the volumetric dilatancy during plastic range, mainly in low confinement, is introduced by affecting the maximum strength surface by a non associative factor which acts over the volumetric term. Departing from this reduced maximum strength surface, the plastic potential surfaces during hardening and softening are derived. In Fig. 5 this process may be observed for a f'_c =30 MPa concrete.



(a) Associative (b) Non associative

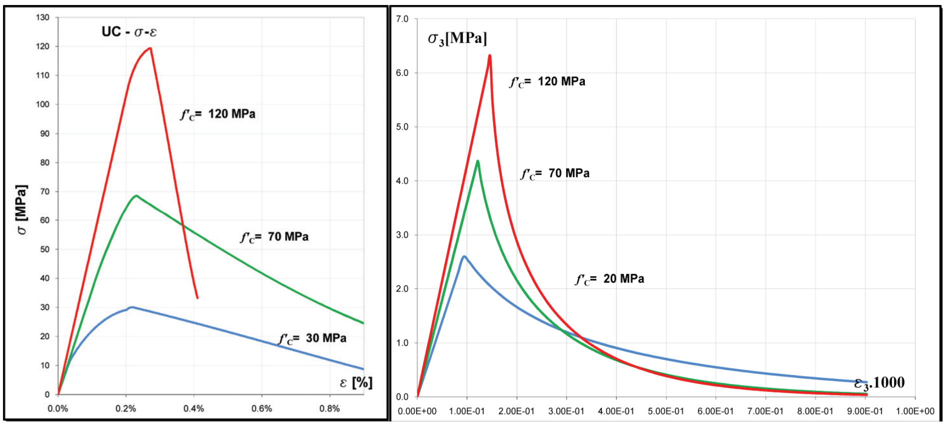
Figure 5: Plastic Potential – Compressive meridian - f'_c =30MPa

3 NUMERICAL RESULTS

The Performance Dependent Model for concretes of arbitrary strength focus of this paper, has been implemented in a finite element code, entirely developed at the LMNI, laboratory of numerical methods in engineering at the Buenos Aires University.

The main features of the implementation may be summarized as follows: developed in fortran, backward Euler method followed, direct scheme leading to a single iteration level, full consistency was used for the determination of the plastic multiplier and consistent tangent operator was determined.

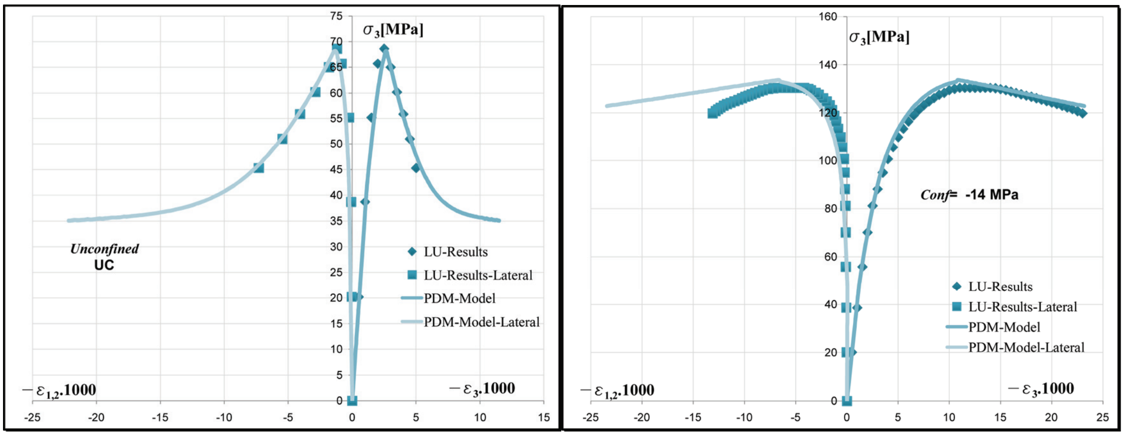
First, in Fig. 6 the numerical predictions obtained with the PDM for uniaxial compression and uniaxial tensile tests are presented. It may be observed that the model is able to capture the different behaviors of both NSC and HSC.



(a) Uniaxial Compression (b) Uniaxial Tensile

Figure 6

In Fig. 7 two numerical predictions obtained with the PDM are compared with experimental test results performed by Lu (2005). It may be observed that it is able to adequately capture the pre and post peak behavior, as well as to predict the peak itself. Currently, the model is still being calibrated and optimized.



(a) Uniaxial Compression (b) Medium Confinement

Figure 7: Comparison of numerical experiments versus Lu (2005) experimental results

4 CONCLUSIONS

In this work, the main features of the Performance Dependent Model for Concretes of Arbitrary Strengths are presented. This model, based on the mathematical framework of the flow theory of plasticity has the following particularities: Depends on the three stress invariants, considers as failure criterion the performance dependent failure criterion previously developed by the authors which is sensitive to the concrete quality, based on the definition and application of the performance parameter for concretes, presents a non uniform hardening represented by cap-cone surfaces with C^1-continuity, presents isotropic softening, non-associated flow rule, and softening formulation includes fracture mechanics concepts incorporating a characteristic length allowing regularization of post-peak behavior.

The constitutive formulation has been implemented in a finite element code. The numerical experiments presented in this paper may conclude that it is capable of capturing the different behaviors of both normal and high strength concretes, as well as the increment in ductility with increasing confinement.

5 ACKNOWLEDGMENTS

The authors acknowledge partial financial supports for this work by ANPCyT through Grant PICT1232/6 and by University of Buenos Aires through Grant UBACYT 2006-2009 Project I813.

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