Asociación Argentina
de Mecánica Computacional

Mecánica Computacional Vol XXIX, págs. 6761-6770 (artículo completo)
Eduardo Dvorkin, Marcela Goldschmit, Mario Storti (Eds.)
Buenos Aires, Argentina, 15-18 Noviembre 2010

UNCERTAINTY QUANTIFICATION IN FLOW-STRUCTURE INTERACTION MODEL USING AN ADAPTATIVE SPARSE GRID COLLOCATION METHOD

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Keywords: Uncertainty Quantification, Flow-Structure Interaction, Adaptative Stochastic Method, Parallel Computation

Abstract. The growing use of Computational Simulation in the design and analysis of complex engineering systems, has underscored the need of developing methodologies capable of assessing the unavoidable uncertainty contained in the numerical results. One major issue to be deeper understood and controlled is how uncertainties in the input data impacts the reliability of the results obtained through computer simulations. Specifically in the present work, the focus relies on hydro-ship dynamics in the context of floating offshore structures. Particular emphasis is placed on investigating uncertainty propagation in the nonlinear response of flow-structures interactions, investigating the response of the system to random load, performing a stability analysis of the system. It is important to remind that waves and currents, major agents in the dynamics of the floating structures, are usually modeled as random processes. Therefore, stochastic modeling seems to offer an appropriate framework to handle external forces and uncertainties in the data, like, for instance, damping and boundary conditions. We propose in this work, apply an adaptative sparse grid stochastic collocation method in a prototype problem of a single oscillator excited by means of an interaction stochastic force corresponding to the Morison formula. The velocity and acceleration of the flow are determined using the Pierson-Moskovitz power spectrum that lead to a highly non-linear equation of motion. Even in the presence of nonlinearities, the collocation approach, approximates the solution in the stochastic space using Lagrange polynomial interpolation, requiring only repetitive calls to an existing deterministic solver, just as in sampling-based methods, such as Monte Carlo. Moreover, uncertainty in the system parameters can be also taken into account. Comparisons with non-adaptative Collocation method and Monte Carlo method, taken as reference, are also presented to demonstrate the accuracy and efficiency of the method.

1 INTRODUCTION

The complexity involved in engineering systems has been, frequently, tackled with the use of sophisticated computational models. That, from the decision makers standpoint, requires the use of robust and reliable numerical simulators. Often, the reliability of those simulations is disrupted by the inexorable presence of uncertainty in the model data, such as inexact knowledge of system forcing, initial and boundary conditions, physical properties of the medium, as well as parameters in constitutive equations. These situations underscore the need for efficient uncertainty quantification (UQ) methods for the establishment of confidence intervals in computed predictions, the assessment of the suitability of model formulations, and/or the support of decision-making analysis.

The traditional statistical tool for uncertainty quantification within the realm of Engineering is the Monte Carlo method, (Elishakoff, 2003). This method requires, first, the generation of an ensemble of random realizations associated to the uncertain data, and then it employs deterministic solvers repetitively to obtain the ensemble of results. The ensemble results should be processed to estimate the mean and standard deviation of the final results. The implementation the Monte Carlo is straightforward, but its convergence rate is very slow (proportional to the inverse of the square root of the realization number) and often infeasible due the large CPU time needed to run the model in question. Other technique that has been applied recently is the so called Stochastic Galerkin Method (SG), which employs Polynomial Chaos expansions to represent the solution and inputs to stochastic differential equations, (Babuska and Zouaris, 2005). A Galerkin projection minimizes the error of the truncated expansion and the resulting set of coupled equations is solved to obtain the expansion coefficients. SG methods are highly suited to dealing with ordinary and partial differential equations, even in the case of nonlinear dependence on the random data. The main drawback with SG relies on its need of solving a system of coupled equations that requires efficient and robust solvers and, most importantly, the modification of existing deterministic code. This last issue entails difficulties on using commercial or already in use codes. A non-intrusive method, referred to as Stochastic Collocation (SC), (Dongbin and Hesthaven, 2005), arises towards addressing this point. SC methods are built on the combination of interpolation methods and deterministic solvers, likely Monte Carlo. A deterministic problem is solved in each point of an abstract random space. Similarly to SG methods, SC methods achieve fast convergence when the solution possesses sufficient smoothness in random space.

Thus when there are steeps gradients or finite discontinuities in the stochastic space, these methods converge very slowly or even fail to converge. In this work, we present an adaptative sparse grid collocation strategy with the aim of obtaining greater accuracy in nonlinear systems analysis. Specifically in the present work, the focus relies on hydro-ship dynamics in the context of floating offshore structures. Particular emphasis is placed on investigating uncertainty propagation in the nonlinear response of fluid-structure interaction, (Dongbin et al., 2002). It is important to remind that waves and currents, major agents in the dynamics of the floating structures, are usually modeled as random processes. Therefore, stochastic modeling seems to offer an appropriate framework to tackle the external forces and uncertainties in the data, like, for instance, damping and boundary conditions. Here, the fluid-structure interaction is modeled in a simple way focusing the assessment of an SC method as an effective tool for uncertainty quantification. The interaction is introduced by means of the Morison's formula, which represents a challenge, despite the simplicity of the model itself, as far as the input is a nonlinear function of the random variables, (Witteveen and Bijl, 2008). Those variables represent the phase angle

which inherent to the time series description of the wave induced motion.

2 THEORY

To quantify the uncertainty in a system of differential equations we adopt a probabilistic approach and define a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Where Ω is the event space, $\mathcal{F} \subset 2^{\Omega}$ is the σ-algebra of subsets in Ω and $\mathcal{P} : \mathcal{F} \rightarrow [0, 1]$ is the probability measure. Utilizing this framework, the uncertainty in a model is introduced by representing the model input data as random field.

2.1 Governing Equations

Consider the general differential equation defined on a d-dimensional bounded domain $\mathcal{D} \subset \mathbb{R}^d$, $(d = 1, 2, 3)$ with boundary $\partial\mathcal{D}$. The problem consists on finding a stochastic function, $\mathbf{u}(\mathbf{x}, \omega) : \Omega \times \mathcal{D} \longrightarrow \mathbb{R}$, such that for $\mathcal{P}$-almost everywhere $\omega \in \Omega$, the following equation holds:

$$\mathcal{L}(\mathbf{x}, \omega; \mathbf{u}) = f(\mathbf{x}, \omega) \qquad \mathbf{x} \in \mathcal{D} \tag{1}$$

$$\mathcal{B}(\mathbf{x}, \omega; \mathbf{u}) = g(\mathbf{x}, \omega) \qquad \mathbf{x} \in \partial\mathcal{D} \tag{2}$$

with $\mathbf{x} = (x_1, \ldots, x_d) \in \mathbb{R}^d$, $d \geq 1$, space coordinates in $\mathbb{R}^d$, $\mathcal{L}$ a linear or non linear differential operator and $\mathbf{u}(\omega) = (u_1(\omega), \ldots u_i(\omega)) \in \mathbb{R}^i$, $i \geq 1$, are unknown solutions. Sometimes, to solve the equations (1) and (2) it is necessary reduce the infinite dimensional probability space $(\Omega, \mathcal{F}, \mathcal{P})$ to a finite dimensional one. This can be accomplished by characterizing the probability space by a finite number of random variables. Thus, employing any truncated spectral expansion it is possible characterize the random inputs by a set of N random variables $\mathbf{Y} = (Y_1(\omega), \ldots, Y_N(\omega))$ and rewrite the random inputs as,

$$\mathcal{L}(\mathbf{x}, \omega; \mathbf{u}) = \mathcal{L}(\mathbf{x}, Y_1(\omega), \ldots, Y_N(\omega); \mathbf{u}), \quad f(\mathbf{x}, \omega) = f(\mathbf{x}, Y_1(\omega), \ldots, Y_N(\omega)), \tag{3}$$

Where, following the Dob-Dynkin lemma, (Oskendal, 1998), the solution of (1) and (2) can be represented by the same set of random variables $\{Y_i(\omega)\}_{i=1}^N$, reducing the infinite dimensional probability space to a N-dimensional space, i.e.,

$$\mathbf{u}(\mathbf{x}, \omega) = \mathbf{u}(, \mathbf{x}, Y^1(\omega), \ldots, Y^N(\omega)) \tag{4}$$

Now assuming that $\{Y^i\}_{i=1}^N$ are independent random variables with probability density functions $\rho_i : \Gamma^i \rightarrow \mathbb{R}^+$, and their images $\Gamma^i \equiv Y^i(\Omega)$ bounded intervals in $\mathbb{R}$ for $i = 1, \ldots, N$, the joint probability density of $\mathbf{Y} \equiv (Y^1, \ldots, Y^N)$ hold,

$$\rho(y) = \prod_{i=1}^{N} \rho_i(Y^i) \qquad \forall y \in \Gamma, \tag{5}$$

and the space support,

$$\Gamma \equiv \prod_{i=1}^{N} \Gamma^i \subset \mathbb{R}^N. \tag{6}$$

This allow us to rewrite (1) and (2) as a $(N + d)$ dimensional differential equation as following,

$$\mathcal{L}(\mathbf{x}, \mathbf{Y}; \mathbf{u}) = f(\mathbf{x}, \mathbf{Y}), \qquad (\mathbf{x}, \mathbf{Y}) \in \Gamma \times \mathcal{D} \tag{7}$$

$$\mathcal{B}(\mathbf{x}, \mathbf{Y}; \mathbf{u}) = g(\mathbf{x}, \mathbf{Y}), \qquad (\mathbf{x}, \mathbf{Y}) \in \Gamma \times \partial\mathcal{D} \tag{8}$$

with N dimensionality of the random space Γ and d the dimensionality of the physical space $\mathcal{D}$.

Thus, the original infinite dimensional problem become in a deterministic problem in the physical domain $\mathcal{D}$ and can be solved by a common discretization technique as finite elements for example.

3 STOCHASTIC COLLOCATION METHOD

The idea of this method is approximate the multidimensional stochastic space building a interpolation function on a set of collocation points $\{\mathbf{Y}_i\}_{i=1}^{M}$ in the stochastic space $\Gamma \subset \mathbb{R}^M$. The method, similarly to Monte Carlo methods, requires only the solution of a set of decoupled equations, allowing the model to be treated as a black box and solved it with existing deterministic solvers. The multidimensional interpolation can be built through either full-tensor product of 1D interpolation rule or by the so called sparse grid interpolation based on the Smolyak algorithm. The Smolyak algorithm provides a way to construct interpolations functions based on minimal number of points in multidimensional space (Bungartz and Griebel, 2004). This method is easily extended from the univariate interpolation to the multivariate case by using tensor products.

Hence, considering a smooth functions $f : [-1,1]^N \to \mathbb{R}$, for the $1D$ case $(N = 1)$, f can be approximated by the following:

$$\mathcal{U}^i(f)(y) = \sum_{j=1}^{m_i} f(\mathbf{Y}_j^i) a_j^i, \tag{9}$$

with the set of support nodes

$$X^i = \mathbf{Y}_j^i | \mathbf{Y}_j^i \in [0,1] for j = 1, \ldots, m_i \tag{10}$$

where, $i \in \mathbb{N}$, $a_i(\mathbf{Y}_j^i) \in C[0,1]$ are the interpolation nodal basis functions and m_i is the number of elements of the set X^i. Hence, in the multivariate case, the tensor product formula is:

$$(\mathcal{U}^{i_1} \otimes \ldots \otimes \mathcal{U}^{i_N})(f) = \sum_{j_1=1}^{m_1} \cdots \sum_{j_N=1}^{m_N} f(Y_{j_1}^{i_1} \ldots Y_{j_N}^{i_N}).(a_{j_1}^{i_1} \otimes \cdots \otimes a_{j_N}^{i_N}) \tag{11}$$

which serve as building blocks for the Smolyak algorithm. So, The Smolyak algorithm build the interpolant $\mathcal{A}_{q,N}(f)$ using products of $1D$ functions as given in (Xiang and Zabaras, 2009).

$$\mathcal{A}_{q,N}(f) = \sum_{q-N+1 \le |i| \le q} (-1)^{q-|i|} \binom{N-1}{q-|i|} (\mathcal{U}^{i_1} \otimes \ldots \otimes \mathcal{U}^{i_N}) \tag{12}$$

with $q \geq N$, $\mathcal{A}_{N-1,N} = 0$ and where the multi-index $i = (i_1, \ldots, i_N) \in \mathbb{N}^N$ and $|i| = i_1 + \cdots + i_N$. Here $i_k, k = 1, \ldots, N$, is the level of interpolation along the $k-th$ direction. The Smolyak algorithm builds the interpolation function by adding a combination of $1D$ functions of order i_k with the constraint that the sum total $(|i| = i_1 + \ldots + i_N)$ across all dimensions is between $q - N + 1$ and q. Therefore, the Smolyak interpolation $\mathcal{A}_{q,N}$ is given by;

$$\mathcal{A}_{q,N}(f) = \sum_{|i| \le q} (\triangle^{i_1} \otimes \ldots \otimes \triangle^{i_N}) = \mathcal{A}_{q-1,N}(f) + \sum_{|i|=q} (\triangle^{i_1} \otimes \ldots \otimes \triangle^{i_N}) \tag{13}$$

To compute the interpolant $\mathcal{A}_{q,N}(f)$ is necessary to compute the function at the nodes covered by the sparse grid $\mathcal{H}_{q,N}$:

$$\mathcal{H}_{q,N}(f) = \bigcup_{q-N+1 \leq |i| \leq q} (X^{i_1} \times \cdots \times X^{i_N}) \tag{14}$$

The construction of the algorithm allows to utilizing all the previous results generated to improve the interpolation. By choosing the appropriate points for interpolating the $1D$ function, it is possible ensure that the sets of points are nested $X^i \subset X^{i+1}$. Where to extend the interpolation from level $i-1$ to i, one only has to evaluate the function at grid points that are unique to X^i. Hence, to go from an order $q-1$ to q in N dimensions, one only needs to evaluate the function at the differential nodes:

$$\triangle\mathcal{H}_{q,N}(f) = \bigcup_{|i|=q} (X^{i_1} \otimes \cdots \otimes X^{i_N}) \tag{15}$$

Finally after a choice of collocation points and the nodal basis functions, any function $u \in \Gamma$ can be approximated by;

$$u(x, \mathbf{Y}) = \sum_{|i| \leq q} \sum_{j \in B_i} w_j^i(x) a_j^i(\mathbf{Y}) \tag{16}$$

This equation is a simple weighted sum of the value of the basis functions for all collocations points in the sparse grid, being an approximation to the solution of the equations (7) and (8). From this equation, it is possible calculate easily the useful statistics of the solution for example, the mean of the random solution can be evaluated as follow:

$$\mathbb{E}[u(x)] = \sum_{|i| \leq q} \sum_{j \in B_i} w_j^i(x). \int_\Gamma a_j^i(\mathbf{Y}) d\mathbf{Y} \tag{17}$$

where denoting $\int_\Gamma a_j^i(\mathbf{Y}) d\mathbf{Y} = I_j^i$ we can write

$$\mathbb{E}[u(x)] = \sum_{|i| \leq q} \sum_{j \in B_i} w_j^i(x).I_j^i \tag{18}$$

the mean is an arithmetic sum of the product of the hierarchical surpluses and the integral weight at each interpolation point. To obtain the variance of the random solution we can be calculate first:

$$u^2(x, \mathbf{Y}) = \sum_{|i| \leq q} \sum_{j \in B_i} v_j^i(x) a_j^i(\mathbf{Y}) \tag{19}$$

and then

$$\mathbb{V}ar[u(x)] = \mathbb{E}[u^2(x)] - (\mathbb{E}[u(x)])^2 = \sum_{|i| \leq q} \sum_{j \in B_i} v_j^i(x).I_j^i - (\sum_{|i| \leq q} \sum_{j \in B_i} w_j^i(x) I_j^i)^2 \tag{20}$$

The method allows us to obtain an approximation of the solution dependent random variables and also easily extract the mean and variance analytically as well its probability density function (PDF)by simple sampling of this function, leaving only the interpolation error (Dongbin and Hesthaven, 2005).

4 ADAPTATIVE SPARSE GRID COLLOCATION METHOD

When the the smoothness condition in the stochastic space is not fulfilled it is possible to use adaptative strategies to improve de interpolation function in the stochastic space. The basic idea here is to use hierarchical surpluses $w_j^i(x)$ as an error indicator to detect the smoothness of the solution and refine the grid around the discontinuity region and less points in the region of smooth variation. This method proposed in (Xiang and Zabaras, 2009), automatically detect the discontinuity region in the stochastic space and refine the collocation points in this region.

Then, considering the interpolation level of a grid point Y as the depth of the tree $D(Y)$. After denote the father of a grid point as $F(Y)$, where the father of the root 0.5 is itself. Thus, the conventional sparse grid in the N-dimensional random space Equation 14 can be reconsidered as:

$$\mathcal{H}_{q,N}(f) = \mathbf{Y} = Y_1 \dots Y_N \sum_{i=1}^{N} D(Y_i) \leq q \tag{21}$$

Where we call their sons of a grid point $\mathbf{Y} = (Y_1 \dots Y_N)$ by:

$$Sons(\mathbf{Y}) = \mathbf{S} = (S_1, S_2, \dots, S_N) | (F(S_1), S_2, \dots, S_N) = \mathbf{Y} \tag{22}$$

or

$$(S_1, F(S_2), \dots, S_N) = \mathbf{Y}, \dots, or (S_1, S_2, \dots, F(S_N)) = \mathbf{Y} \tag{23}$$

From this definition, is noted that in general for each grid point here there are two sons in each dimension, therefore, for a grid point in a N-dimensional stochastic space, there are 2N sons. Therefore, by adding the neighbor points, we actually add the support nodes from the next interpolation level, so that the magnitude of the hierarchical surplus satisfies $|w_j^i \geq \varepsilon|$. If this criterion is satisfied, one only add the $2N$ neighbor points of the current point to the sparse grid. It is noted that the definition of level of the Smolyak interpolation por the ASGC method is the same as that of the conventional sparse grid even if not all point are included. A more detailed explanation of the method and algorithm can be found in, (Xiang and Zabaras, 2009).

5 APPLICATION

All numerical result present in this section were obtained using algorithms developed for high-level programming language in parallel environment. To run such calculations on a parallel computer were used sixteen cores of a cluster SGI Altix ICE 8200 of Supercomputer Center of UFRJ (NACAD). With the aim of illustrate the methods developed in the preceding sections, we are going to consider the stochastic response of a single-degree-of-freedom structure exited by a random Morison's force and a restoring force expressed by a cubical polynomial term. This equation can be considered as an idealizing model of one offshore system,

$$\ddot{y} + 2\varsigma_0\omega_0\dot{y} + \omega_0\Big[y + By^2 + Cy^3\Big] = \frac{C_e}{M}F(t, \varphi_i) \tag{24}$$

where $\{\varphi_i\}_{i=0}^n$ are stochastic variables and the others parameters are presented in Figure 2. It is a first approximation to a more complex model of structure with internal damping and stiffness to describe a fluid-structure interaction, (Floris and Pulega, 2002).

The effect of the fluid on the system has been modeled via the force term given by

$$p_d(z, \varphi_i, t) = K_d\Big[U(z, \varphi_i, t)|U(z, \varphi_i, t)|\Big] + K_M\Big(\dot{U}(z, \varphi_i, t)\Big) \tag{25}$$

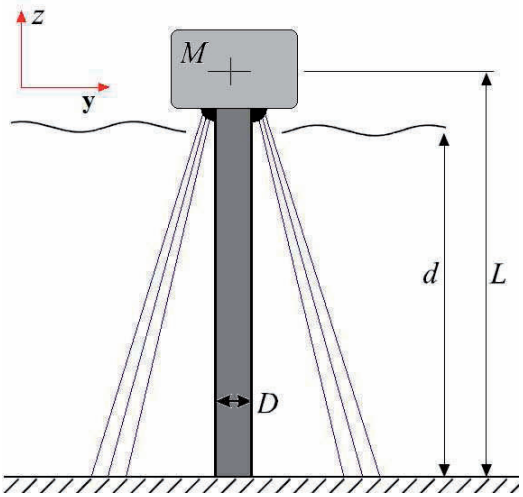

Figure 1: Offshore tower scheme

k	damper factor	0.4024
ω_1	natural frequency	0.356 rad/s
c_e	efficacious dynamic force	0.1
M	structural mass	5e6 Kg
B	nonlinear parameter	0.419074
C	nonlinear parameter	0.321047
L	platform height	80 m
d	water depth	60 m
D	diameter	2 m
$U_{19.5}$	wind velocity	25 m/s
K_d	drag coefficient	2785 kg/sm^2
K_M	inertia coefficient	48734 kg/m

Figure 2: Numerical and physical parameters of the simulations

being

$$F(t,\varphi_i) = \int_0^{-d} p_d(z,\varphi_i,t)dz \tag{26}$$

In the presence of a mean current $\overline{U}(z,\varphi_i,t)$, the water particle velocity is

$$U(z,\varphi_i,t) = \overline{U}(z,t) + u(z,\varphi_i,t) \tag{27}$$

assuming $\overline{U}(z,t)$ current mean equal to zero by the linear wave theory, also called Airy wave theory we can obtain the linear wave profile and velocity in terms of a spectral density,

$$u_y(z,\varphi_i,t) = \sum_{i=1}^{n} \omega_i \frac{cosh(k_i z)}{sinh(k_i d)} cos(\omega_i t - k_i - \varphi_i)\sqrt{2\mathcal{S}_{\eta\eta}(\omega_i)\Delta\omega_i} \tag{28}$$

and the acceleration

$$\dot{u}_y(z,\varphi_i,t) = \sum_{i=1}^{n} -(\omega_i)^2 \frac{cosh(k_i z)}{sinh(k_i d)} sin(\omega_i t - k_i - \varphi_i)\sqrt{2\mathcal{S}_{\eta\eta}(\omega_i)\Delta\omega_i} \tag{29}$$

where wave number assumed as $k_i = \omega_i^2/g$ and the velocity and acceleration are completely characterized in a statically sense by the wave height spectrum $\mathcal{S}_{\eta\eta}$. In general, ocean waves spectrums models are semi-empirical formulas, derived mathematically where the formulation requires one or more experimentally determined parameters. In this example it has been adopted the Pierson-Moskowitz's spectrum. This is the most extensively used spectrum for representing a fully developed sea, where the sea severity can be specified in terms of the wind velocity,

$$\mathcal{S}_{\eta\eta}(\omega_i) = \frac{8.1\times10^{-3}g^2}{2\omega_i^5} exp\left(-0.74\left(\frac{g}{U_{w,19.5}}\right)^4 \omega_i^{-4}\right) \tag{30}$$

where $\omega_i$is the frequency g the gravity acceleration and $U_{w,19.5}$ the wind speed at a height of 19.5m above the still water. Assuming tree waves frequencies as $\omega_1 = 0.3$, $\omega_2 = 0.45$, $\omega_3 = 0.8$ for the Pierson-Moskowitz's spectrum.

A excitation force was supposed with three random components each one associated to the frequencies (.6, .8 and 1.0) rad/s. The random components are the angles φ_i supposed as

standard uniform random variables between $[-\pi, \pi]$ values. Therefore, using the parameters presented in Table 1 the Equation 24 was integrated over the interval $[0, 120]s$ with time step $dt = 0.01$ using a conventional Runge-Kutta method. Figures 3 and 4 show the statistics of the response obtained for Monte carlo simulation over 1000000 experiments as well as the solutions obtained using Conventional and Adaptative Sparse Grid Method, for the last using $(\varepsilon \geq 1e{-}5)$ as surplus error limit. To allow the refinement of the Sparse Grid, with Smolyak algorithm, was used the Newton-Cotes abscissas that have equidistant support nodes and allows to refine the grid locally easily.

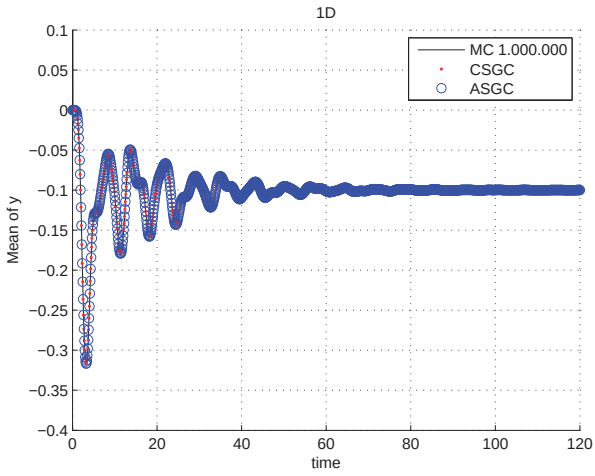

Figure 3: Mean

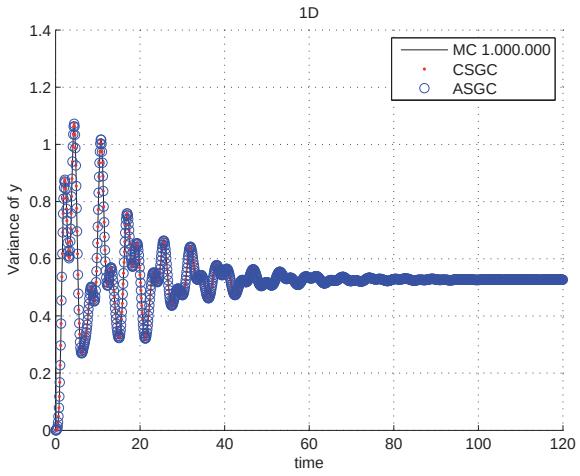

Figure 4: Variance

The mean and variance of the solutions obtained by the conventional and adaptive method, assuming 1D, 2D and 3D stochastic dimensions, was equally well as compared with the reference of the Monte-Carlo. Already in Figures 5, 7, 9 grids obtained are shown for the convergence value as the number of collocation point in Figures 6, 8, 10 for increasing levels of interpolation.

Number of collocations Point dor CSGC and ASGC						
Level	1D/CSGC	1D/ASGC	2D/CSGC	2D/ASGC	3D/CSGC	3D/ASGC
0	1	1	1	1	1	1
1	3	3	5	5	7	7
2	5	5	13	13	25	25
3	9	9	29	29	69	69
4	17	17	65	65	177	177
5	33	33	145	145	441	441
6	65	65	321	289	1073	1009
7	129	129	705	537	2561	2209
8	257	257	1537	931	6017	4601
9	513	511	3329	1493	13953	9217
10	1025	1003	7169	2477	32001	18011
11	2049	1803	15361	4081	72705	33167
12	4097	2717	32769	5909	163841	58687
13	8193	2805	69633	6085	263841	101403

Table 1: Number of collocations Points for CSGC and ASGC methods

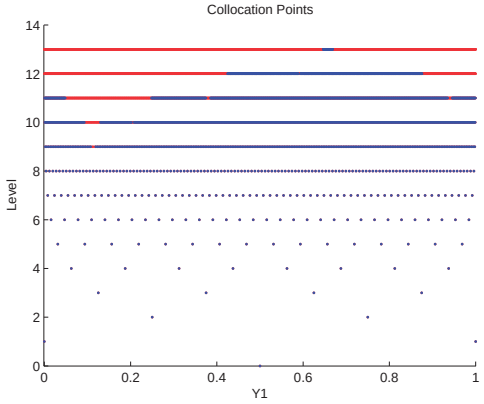

Figure 5: 1D - Conventional and Adaptative Sparse Grids from Level 0 to 13

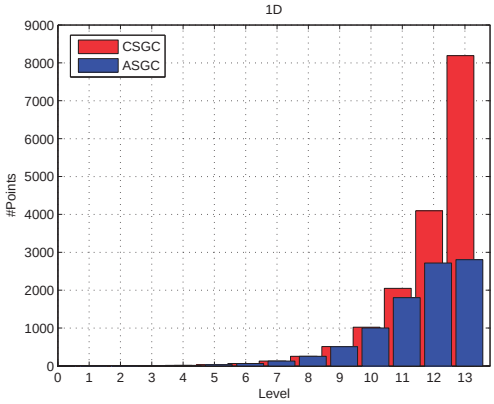

Figure 6: 1D - Number of point for Conventional and Adaptative Sparse Grids

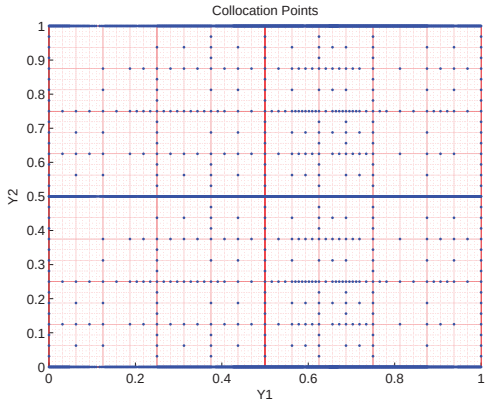

Figure 7: 2D - Conventional and Adaptative Sparse Grids, Level 13

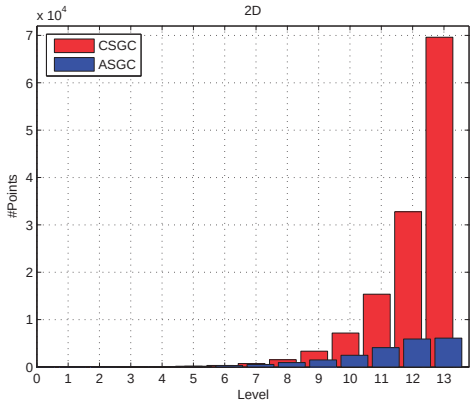

Figure 8: 2D - Number of point for Conventional and Adaptative Sparse Grids

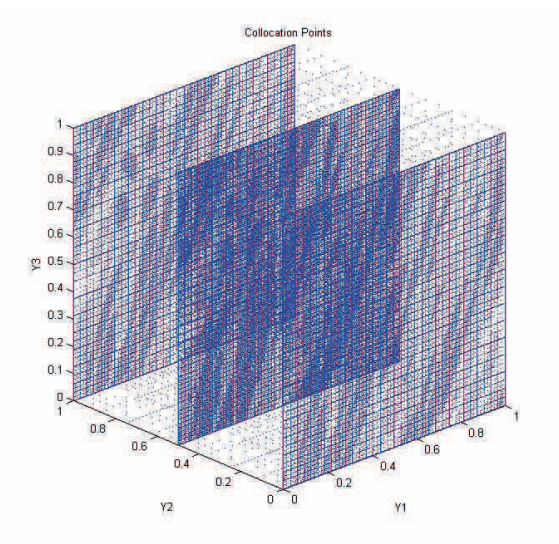

Figure 9: 3D - Adaptative Sparse Grids, Level 13

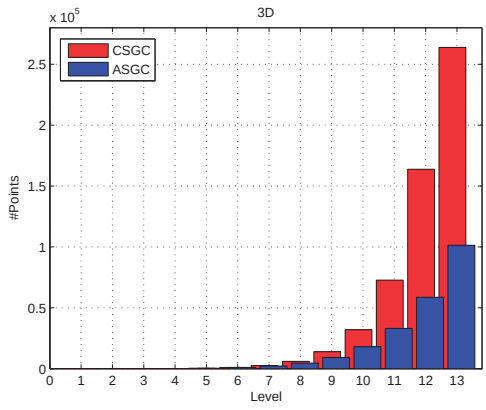

Figure 10: 3D - Number of point for Conventional and Adaptative Sparse Grids

6 CONCLUSIONS

Like the Monte Carlo method, the Sparse Grid Stochastic Collocation method leads to the solution of uncoupled deterministic problems and, as such, it is simple to implement and parallelize. These non-intrusive methods, allow convert any deterministic code into a code that solves the corresponding stochastic problem. Compared with the Monte Carlo Simulation method, the Sparse Grid Stochastic Collocation method presents a significative reduction in the number of experiments required to achieve the same level of accuracy.On the other hand, the results obtained, comparing the Conventional Sparse Grid Collocation method and an adaptative strategy, show that it is possible refine the grid locally identifying automatically non smooth regions in the stochastic space achieving the same accuracy and reducing significatively the cost by the use of less collocations points in smooth regions of the stochastic space.

Due to that the majority of engineering problems varying rapidly in only some dimensions, remaining much smoother in other dimensions and in general it have more stochastic dimensions. Future work of this research will include the study high-dimensional methods mixed with Adaptative Sparse Grid Stochastic Collocation methods, in high performance computer environment, aiming to obtain tools to solve real problems of interest in Engineering

Acknowledgements

The authors would like to thank the financial support of PETROBRAS Research Center, FAPERJ, Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro and CNPq, Conselho Nacional de Desenvolvimento Científico e Tecnológico.

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