Asociación Argentina de Mecánica Computacional

Mecánica Computacional Vol XXIX, págs. 8571-8578 (artículo completo)
Eduardo Dvorkin, Marcela Goldschmit, Mario Storti (Eds.)
Buenos Aires, Argentina, 15-18 Noviembre 2010

# FINITE ELEMENT MODELING OF HYDRAULIC FRACTURING IN VERTICAL WELLS

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**Keywords:** hydraulic fracture, finite element method, cohesive zone model

**Abstract.** Hydraulic fracturing is one of the techniques employed in reservoir stimulation to maximize production and extend the reservoir's lifetime. In this sense, the prediction of the fracture's geometry and propagation in the formation is crucial to estimate production gains and thus to determine the treatment feasibility. This paper investigates vertical hydraulic fracture propagation through rock formation with finite element models. The vertical fractures are modeled by cohesive elements, making use of a traction-separation law together with a damage model to govern fracture propagation, while internal tangential and normal flows reproduce the fluid pressure. The finite element model is validated with analytical results and then used in a parametric study to analyze the influence of formation configuration, different material properties for the pay zone and barrier and pressure on fracture propagation.

# 1 INTRODUCTION

Reservoir stimulation by hydraulic fracturing is a widely employed treatment to enhance oil production. Modeling of fracture propagation by hydraulic induced fractures is of great interest in order to define the required amount of fluid, injection pressure, and proppant volume and to predict the effectiveness and feasibility of the treatment. Hydraulic fracturing is intrinsically a three dimensional non linear coupled problem, where fluid flow and diffusion into rock formation, fracture propagation, and inelastic rock deformation are mechanisms to be described by the model.

However, most of the hydraulic fracture simulators employed in the oil industry are based on empirical methods or on linear fracture mechanics theories. For hard rocks where brittle fracture mechanism prevails, reasonable results are obtained. For ductile rocks or under highly confined in-situ stress conditions however, these simulators give conservative results. According to the literature (Papanastasiou, 1999) the predictions obtained with conventional simulators, based on the linear elastic fracture mechanics theory (LEFM) or on empirical methods, lead to conservative results. The in field net-pressures can be up to twice as high as the predicted ones. One of the reasons for this is the fact that for ductile rock behavior or for micro-cracking, the fracture process is associated with the development of a region around the crack tip that presents plastic deformations prior to fracture propagation. This region, called the fracture process zone, presents two nonlinear regions: one localized region characterized by softening material behavior; and a subjacent region where perfect plasticity or even plastic hardening takes place. For brittle materials, the plastic process zone is small, so LEFM is applicable. For ductile rocks however, although the softening zone is still small, the plastic process zone is not negligible and calls for the application of elasto-plastic fracture mechanics. In the framework of hydraulic fracturing this was first pointed out in the work of Papanastasiou and Thiercelin, 1993.

In order to take into account the inelastic deformations that arise in the fracture process zone, the cohesive zone method (CZM), originally proposed by Barrenblatt, 1962 can be employed. With this method the deformations at the crack tip prior to fracture propagation are accounted for and energy dissipation occurs in a finite region. Shorter and wider fractures are thus computed with the CZM. The size of this region is a function of the rock material properties (Young’s Modulus and fracture toughness), the fluid’s viscosity and the in-situ stresses. Also, the cumbersome 1/r singularity at the crack tip, inherent to the LEFM, is not present. As cited in (Chen et al., 2009) the cohesive zone method not only gives more realistic results but also simplifies finite element modeling issues, once pre-determination of the crack tip is not required. Crack initiation and propagation are natural outcomes of the solution of the CZM. Recent works in the field of numerical modeling of hydraulic fracturing are presented in (Yao et al., 2010) and (Zhang et al., 2010).

In this work fracture propagation in a hydraulically fractured reservoir sandwiched by barriers is investigated using a two dimensional finite element model with the CZM. Special attention is given to fracture propagation through the barriers, where the changes of both in-situ stresses and fracture toughness can hinder fracture height growth.

# 2 FINITE ELEMENT HYDRAULIC FRACTURE MODEL

A finite element model based on the cohesive zone model for the analysis of vertical well hydraulic fracturing is presented. In this method, the rock softening behavior is included via a scalar damage constitutive model in terms of a traction-separation law. This law defines the propagation criterion. Due to lack of available data a linear softening material is assumed.

This traction-separation law is presented in Figure 1. The area under the stress-displacement curve equals the strain energy release rate, $G_C$ when the size of the cohesive zone is small compared to the crack length. For elastic material behavior, the fracture toughness, $K_{IC}$ is related to $G_C$ through the following relation:

$$K_{IC}^2 = \frac{G_C E}{1-\nu^2} \tag{1}$$

where E is the Young's modulus and ν is the Poisson ratio. For given values of the fracture toughness, $K_{IC}$ and rock tensile strength, $T_{max}$ the stress-displacement law is uniquely defined. The opening displacement, $\delta_f$ for which the fracture traction falls to zero is calculated with the expression

$$\delta_f = \frac{2G_C}{T_{\max}} \tag{2}$$

where the fracture energy release rate, $G_C$ is obtained from Eq. (1). The critical opening displacement to activate material damage, $\delta_0$ is related to $\delta_f$ through a so called separation coeficient, $\alpha$, i.e.

$$\delta_0 = \alpha\delta_f \quad \text{with} \quad 0 < \alpha < 1 \tag{3}$$

To complete the model description, the initial cohesive stiffness, K, is given by

$$K_n = \frac{T_{\max}^2}{2\alpha G_C} \tag{4}$$

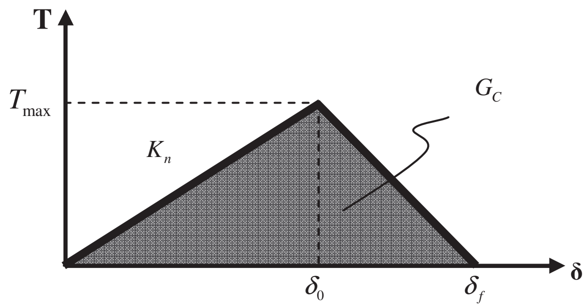



Figure 1 Bi-linear cohesive element traction-separation (T-δ) law

It is worth mentioning that both the critical separation, $\delta_0$ and the damage stiffness, change according to the chosen value of α , $G_C$ kept constant. Small values of α imply that a large fraction of the cohesive energy is consumed in the damage phase, i.e. there is large fracture process zone and the fluid front penetrates significantly into it. On the other hand, for large values of α, most of the energy is stored in the cohesive element as elastic energy. In this case the fracture process zone is small and, correspondingly, so is the penetration distance of the fluid front inside de fracture process zone. A thorough discussion on the effect of the separation coefficient on fracture propagation is presented in (Chen et al., 2009).

The inelastic rock material behavior follows the Mohr-Coulomb flow theory of plasticity for a cohesive frictional dilatant material. Associative behavior with constant dilatation angle is considered. These assumptions are justified by the presence of high confining stresses prior to crack propagation and to a decrease in the initial in-situ mean pressure near the crack tip during propagation.

The model includes the effect of the viscous fluid flow, considering an incompressible, uniform, Newtonian fracturing fluid. The continuity equation and momentum balance will give the lubrication equation relating the pressure gradient to the fracture width. Constant flow rate at the wellbore and zero fluid pressure at the fluid front are the boundary conditions to be enforced in the model. The effect of leak-off from the fracture surface into the rock formation is left to a further study.

## 3 SIMULATION

In this preliminary study plane strain fracture geometry is considered. Therefore, only the material parameters related to the pure normal deformation mode are required. The lithology of the analyzed section is composed of three layers: two barriers interspaced with a reservoir. The vertical fracture is in the center of the model. Figure 2 shows a schematic representation of the section.

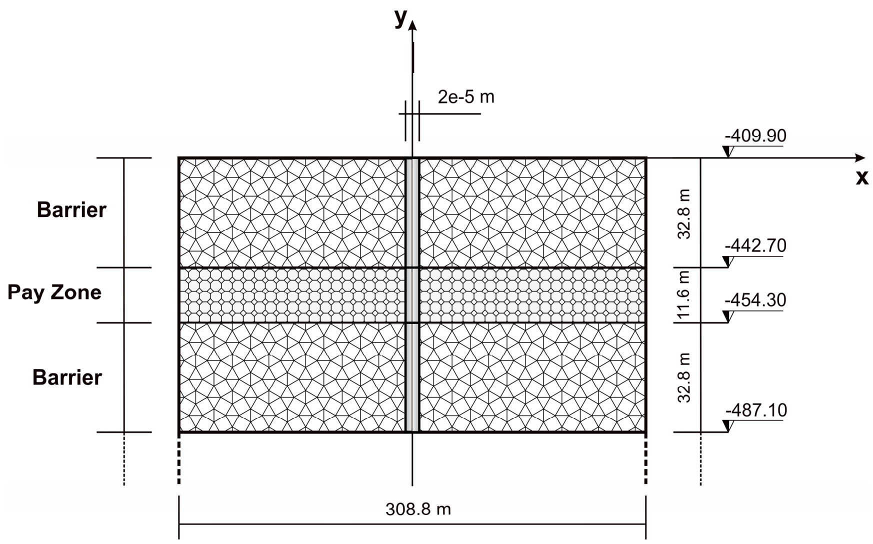



Figure 2 Schematic representation of the formation

The finite element software ABAQUS/Standard was used in the analysis. The mesh is composed of two dimensional four-node quadrilateral elements for the rock formation and of six-node cohesive elements having both displacements and pore pressure as degrees of freedom. Altogether the finite element mesh has 42837 elements and 43486 nodes. A detail of the mesh in the fracture region is shown in Figure 3.

The analysis was carried out in two load stages. Initialization of the in-situ stresses with the geostatic load, followed by fluid flux, prescribed along the fracture. In the hydraulic fracturing stage, flux is prescribed along the whole pay zone. Fracture fluid is injected at the center of the fracture with a constant rate of $0.001 m^3/s$. To allow initial flow the cohesive elements at the injection point are preset as initially open.

Two sets of analysis were carried out. In the first set the effect of different fracture toughness between pay zone and barrier is investigated. By the second set the effect of different in-situ stresses between layers is studied. For this purpose the lateral earth coefficient $K_0$ in the reservoir is varied. Table 1 shows the values of the parameters adopted in the parametric analysis of both sets. These parameters were taken from (Chen et al., 2009) and are shown in Table 1 and Table 2.

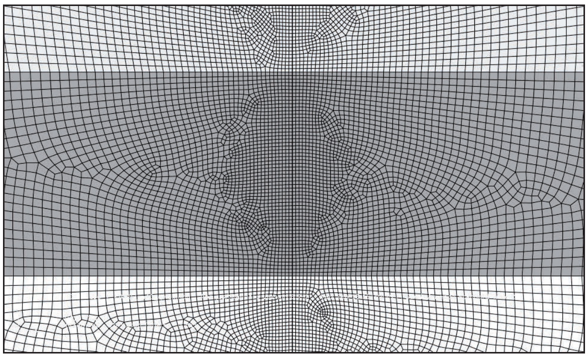


Figure 3 Mesh retail along the fracture

In the first set of analysis fracture toughness of the barriers was varied, assuming values ranging from $K_{IC}$=1,0 to $K_{IC}$=1,5. For the pay zone the fracture toughness a constant toughness, $K_{IC}$=1,0, was used. The stress-fracture opening displacement curves obtained at point B in the barrier are presented in Figure 4. The corresponding pore pressure evolution curves are presented in Figure 5.

The influence of the difference in the in-situ stresses from the pay zone to the barriers is studied through a parametric analysis of the lateral earth coefficient $K_0$=0,1 to $K_0$=0,5 in the pay zone. In the cap rock this coefficient is left constant with the value $K_0$= 0,5. The bi-linear material law in the pay zone at point A is shown in Figure 6, for the different values of $K_0$. Table 3 compares the in-situ stress contrast and the pore pressure contrast for these analyses.

| **Layer** | **E (GPa)** | **ν (-)** | **K0 (-)** |
|---|---|---|---|
| Barrier | 30.00 | 0.20 | 0.5 |
| Pay zone | 30.00 | 0.20 | 0.5 |

(*) $\gamma = 12.56$ kN/m$^3$

Table 1 Parameters of solid elements

| **Layer** | **E (GPa)** | **ν (-)** | **K0 (-)** | **Tmax (MPa)** | **$K_{IC}$ (MPa.m$^{1/2}$)** |
|---|---|---|---|---|---|
| Barrier | 30.00 | 0.20 | 0.50 | 2.00 | (**) |
| Pay zone | 30.00 | 0.20 | 0.50 | 2.00 | 1.00 |

(*) $\mu$ = 1cp (0.001 Pa·s)
(**) $K_{IC}$=1.0, 1.1, 1,2, 1.3, 1.4 and 1.5 MPa·m$^{1/2}$

Table 2 Parameters of cohesive elements

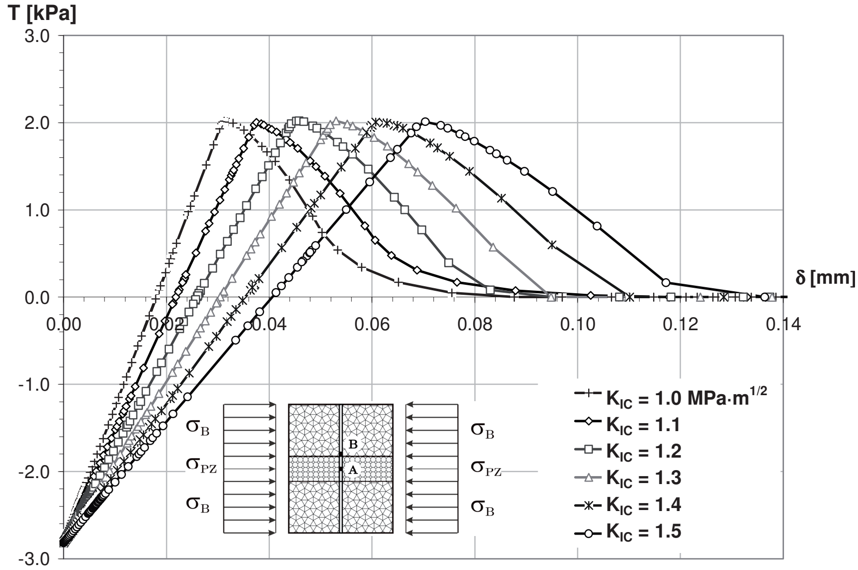
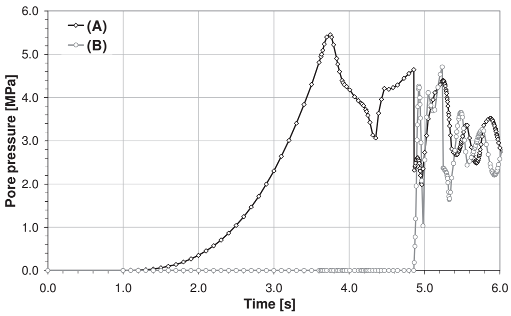



Figure 4 Bi-linear cohesive law at point B in the barrier for different $K_{IC}$ values

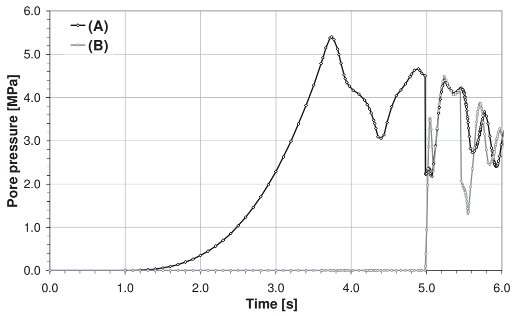
## 4 DISCUSSIONS

Fluid injection along the fracture leads to fracture propagation at a pore pressure of p=3,8 MPa in the pay zone, as can be detected by the first pore pressure drop. This value is not affected by the fracture toughness of the barrier, as expected. The time at which the fracture propagation initiates is slightly changed, growing with the increase in the barrier's fracture toughness. The fracture pore pressure shows, however, a higher dependency on the material's fracture toughness. For the analyzed values, the fracture propagation pressure varied from p=4,2MPa for $K_{IC}$=1,0, to p=5,4MPa for $K_{IC}$=1,5.

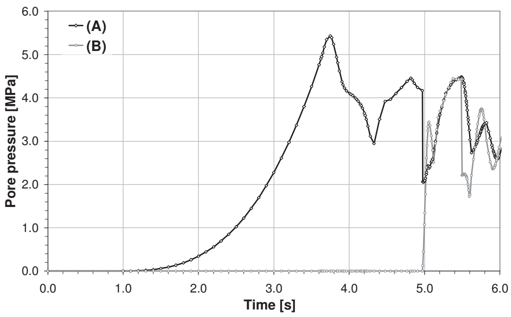
For the analysis of the effect of the in-situ stress variation (Table 3), stress values in the pay zone lower than those in the barrier are associated with an increase of the fracture propagation pressure as the stress difference grows. In this case the barriers have a confining effect on the fracture.



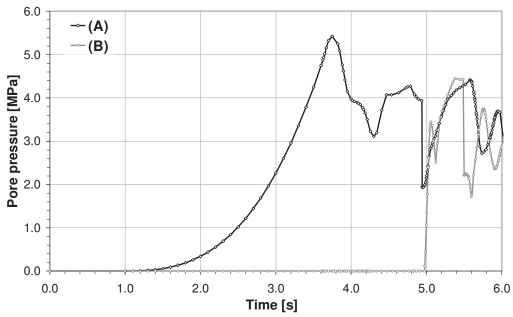

a) $K_{IC} = 1.0$ MPa·m$^{1/2}$



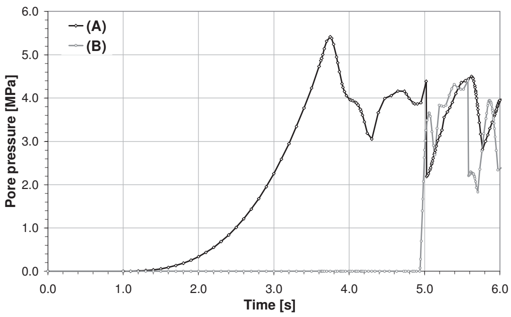

b) $K_{IC} = 1.1$ MPa·m$^{1/2}$



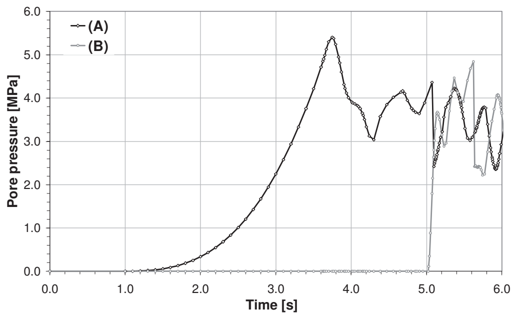

c) $K_{IC} = 1.2$ MPa·m$^{1/2}$




d) $K_{IC} = 1.3$ MPa·m$^{1/2}$




e) $K_{IC} = 1.4$ MPa·m$^{1/2}$




f) $K_{IC} = 1.5$ MPa·m$^{1/2}$

Figure 5 Pore pressure evolution curves for different $K_{IC}$ values

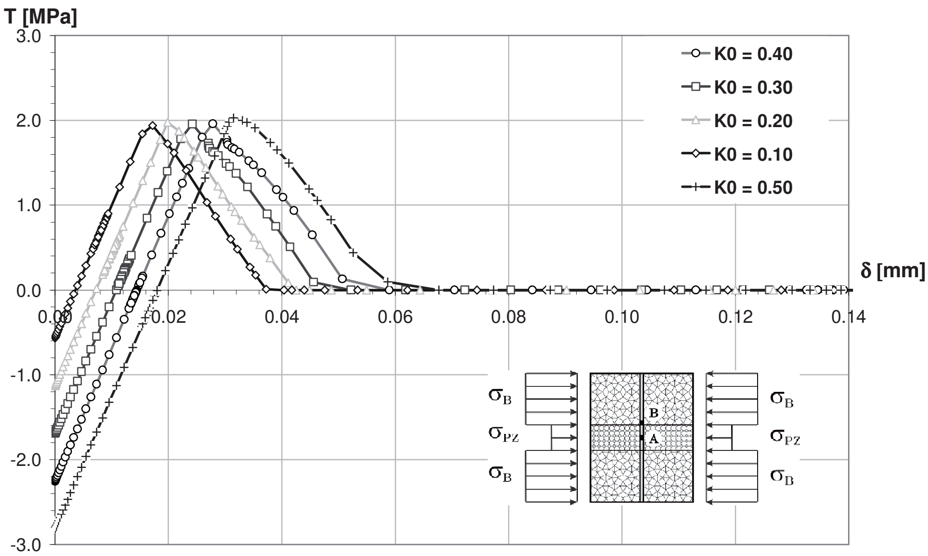



Figure 6 Bi-linear cohesive law at point A in the pay zone for different $K_0$ value

| K0 | Δσ (kPa) | ΔP (kPa) |
|---|---|---|
| 0.05 | 2,534.9 | 2,554.2 |
| 0.10 | 2,253.3 | 2,275.0 |
| 0.20 | 1,689.9 | 1,698.1 |
| 0.30 | 1,126.6 | 1,134.5 |
| 0.40 | 563.3 | 557.3 |
| 0.45 | 281.7 | 277.9 |
| 0.50 | 0.0 | 0.0 |

Table 3 Effect of in-situ stresses contrast and pore pressure contrast

**Acknowledgements** This research was carried out in the Tecgraf/PUC-Rio research laboratory and was funded by Petrobras and CNPq.

## REFERENCES

Papanastasiou, P., The effectice fracture toughness in hydraulic fracturing. *International Journal of Fracture*, 96, 127-147, 1999.

Papanastasiou, P., Thiercelin, M., Influence of inelastic rock behavior in hydraulic fracturing, *International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts*, 30, 1241-1247, 1993.

Barrenblatt, G. I., The mathematical theory of equilibrium of cracks in brittle fracture. *Advances in Applied Mechanics,* 7, 55-129, 1962.

Chen, Z., Bunger, A. P., Zhang, X., Jeffrey, R. G., Cohesive zone finite element-based modeling of hydraulic fractures. *Acta Mechanica Solida Sinia*, Vol.22, No.5 ,443-452, 2009.

Yao, Y., Gosavi, S. V., Searles, K. H., Ellison, T. K., Cohesive fracture mechanics based analysis to model ductile rock fracture. *Proceedings of the 44th US Rock Mechanics Symposium*, Salt Lake City, 2010.

Zhang, G. M., Liu, H., Zhang, J., Wu, H. a., Wang, W. W., Three dimensional finite element simulation and parametric study for horizontal well hydraulic fracture. *Journal of Petroleum Science and Engineering,* 72, 310-317, 2010.