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DYNAMIC ANALYSIS OF MODELS FOR SUSPENSION SYSTEMS OF GROUND VEHICLES WITH UNCERTAIN PARAMETERS

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Abstract. Land transportation vehicles have suspension systems to limit the vibrations caused by the movement. The main function of these systems is to maintain the welfare of people (in vehicles or cars) or the integrity and functionality of installed equipment (in robotic platforms). Suspension systems are constituted by damping and elastic components whose properties may have a wide variation due to the geometrical configuration, manufacturing process and assembly protocols. This entails that the dynamic response of the system is not the expected one, which implies a degree of uncertainty although the properties of the components appear to be correctly specified. In this context it is important to characterize the propagation of uncertainty of the response of the system, caused by the uncertainty of its mechanical components: dampers, dash-pots, springs, anti-roll bars, etc. In this paper, the stochastic dynamic response of a suspension platform is analyzed. The mathematical model of the platform, characterized by 7 degrees of freedom, is derived by means of newtonian dynamics of rigid bodies. This mathematical model is deterministic and it is adopted as the mean model for the further stochastic studies. The parameters of the system: mass, springs and dampers are considered uncertain. Second order random variables are defined for every uncertain parameter involved. The probability density functions of the random variables are derived using the Maximum Entropy Principle. Once the probabilistic model is constructed, the Monte Carlo method is employed to perform the statistical simulations. Numerical studies are carried out to show the main advantages of the modeling strategies employed, as well as to quantify the propagation of the uncertainty in the dynamics of suspension systems.

1 INTRODUCTION

The suspension system of a vehicle, such as cars, trucks, SUV, robotic platform or any other ground vehicular platform, is an important part to control the motion. This means that stability, maneuverability or comfort of a vehicular platform is subjected mainly to the features of the components of the suspension system (Gillespie, 1995). Consequently the design, study and complete analysis of the suspension system is quite important. A basic conceptual suspension system is constituted by buffers and/or shock absorbers, pneumatics or tires, springs and/or elastic components such as anti-roll bars, lower and upper arms, bell-cranks, among others. In more advanced systems, like the ones employed in racing cars some extra elements are included such as pull-push rods and rockers among others. Some configurations of suspension systems can have more than one damper or spring/elastic elements, although it is not very common (Dixon, 2009).

There are different approaches to model, to analyze and to simulate the dynamics of a suspension system, such as finite element method, multibody method, and the method of condensed parameters among others (Thomsen and True, 2010; Dixon, 2009). The first two methods are general and highly sophisticated although they are quite expensive in computational time. On the other hand, the condensed parameters method, though simple in concept as it leads to mathematical models of a few degrees of freedom, enables the implementation of various strategies associated with the calculation of the dynamic response when the model parameters have variations or dispersions. Condensed parameter models for suspension systems require many degree of freedom (involving mechanical components) in order to be more representative. In this paper a basic model of seven degrees-of-freedom representing the suspension of a vehicle platform is derived by means of newtonian mechanics (Williams, 1996). The seven degrees-of-freedom are the vertical displacement and two rotations of a rigid platform suspended by four elastic supports at each corner. Each support is composed by a suspended mass (whose vertical displacement adds a degree-of freedom to the system) and the elastic spirals, springs, anti-roll bars and dash-pots as well as the elastic coefficients of the pneumatics.

If the parameters of the suspension system were clearly identified (i.e. the deterministic problem), the calculation and analysis of the suspension dynamics would not have any complexity, even if it is done with highly sophisticated programs in which the several components of the suspension are modeled as elastic elements (springs, anti roll-bars, etc.) and others are modeled as rigid elements (arms, push-rods, etc). However, the components and/or parameters of real systems eventually can have random features (depending on a number of factors such as the manufacturing process, the system assembling, the type of forces involved, etc), which lead to a stochastic problem. The variable parameters of the system are the elastic constants of the spiral, the effective elastic constant of the tires and the constants of the damping elements. These parameters are effective on the hubs, which makes the model independent of the suspension geometry. The elastic/damping coefficients of the tires, parameters not easy to be measured directly, depend strongly on the manufacturing process and the manufacturer occasionally supplies the data of that parameter.

Now, taking into account that the randomness of the system parameters can substantially modify the dynamic response of suspension, then it is important to establish a way to quantify the uncertainty associated with the suspension parameters and its propagation to the dynamic response. The parametric probabilistic approach is a method normally employed to quantify the uncertainty, in which the uncertain parameters of the model or system are associated to random variables characterized by appropriate probability distribution functions (PDF) constructed according to given statistical information such as mean, standard deviation, among others (Soize, 2001; Ritto et al., 2008).

In this article the Maximum Entropy Principle (Shannon, 1948; Jaynes, 2003) is employed in order to define the PDFs of the random variables. The construction of the PDFs follows the methodology indicated in the works of Soize (2001) and Ritto et al. (2008) provided some statistical measures such as the mean value and the dispersion coefficient of the random variables. Then, the probabilistic model is constructed on the mathematical structure of the deterministic model employing the random variables. The deterministic mathematical model of seven degrees-of-freedom, can be considered the given mean response. The Monte Carlo method is used to simulate the response of a number of realizations to guaranty the convergence of a stochastic response. After that, a statistical analysis is carried out with the outcome in order to evaluate the magnitude of the dispersion in the response and the dispersion of the response provided the features of the random parameters of the vehicular suspension system.

The article is structured in the following form: the first part is devoted to the description and derivation of the deterministic model with 7 degrees-of-freedom that simulates the vehicular platform. After that the probabilistic model is constructed. Then computational studies are made to show the features of the modeling strategies employed, as well as to quantify the propagation of the uncertainty in the dynamics of the suspension systems of a vehicular platform.

2 DETERMINISTIC MODEL OF THE VEHICULAR PLATFORM

Figure 1 shows a basic mechanic model of the vehicular platform. The platform consists of a rigid plate connected to the ground by four subsystems at each corner of the plate. The rigid plate can move vertically and can rotate around x-axis and y-axis. Each suspension subsystem is characterized (Gillespie, 1995) by a suspended mass that can move vertically connected to a pair of springs and a damper: One spring identifies the elastic interaction between the suspended mass and the ground, and the other spring together with the damper alleviate the vibration to the platform. The effective mass, elastic and dashpot components of each subsystem are condensed in the center of the corresponding supporting tires.

The system dynamics is characterized by 7 degrees-of-freedom: the vertical displacement of the four suspended masses, the two rotation angles of the rigid plate and the vertical displacement of the gravity center of the rigid plate.

The equations of motion of the vehicular platform can be derived by means of newtonian mechanics, and they can be written in the following compact matrix form:

$$\begin{gathered}\mathbf{M}\ddot{\overline{\mathbf{U}}} + \mathbf{R}\dot{\overline{\mathbf{U}}} + \mathbf{K}\overline{\mathbf{U}} = \overline{\mathbf{F}} \\ \overline{\mathbf{U}}(0) = \overline{\mathbf{U}}_0 \\ \dot{\overline{\mathbf{U}}}(0) = \dot{\overline{\mathbf{U}}}_0\end{gathered} \tag{1}$$

In Eq. (1), $\mathbf{M}$ is the matrix of effective inertia parameters, $\mathbf{R}$ is the matrix of the damping components, $\mathbf{K}$ is the matrix of effective spring elements, whereas $\overline{\mathbf{F}}$ is the vector of applied forces and $\overline{\mathbf{U}}$ is the vector of kinematic variables, $\overline{\mathbf{U}}_0$ and $\dot{\overline{\mathbf{U}}}_0$ are the initial conditions of the differential system.

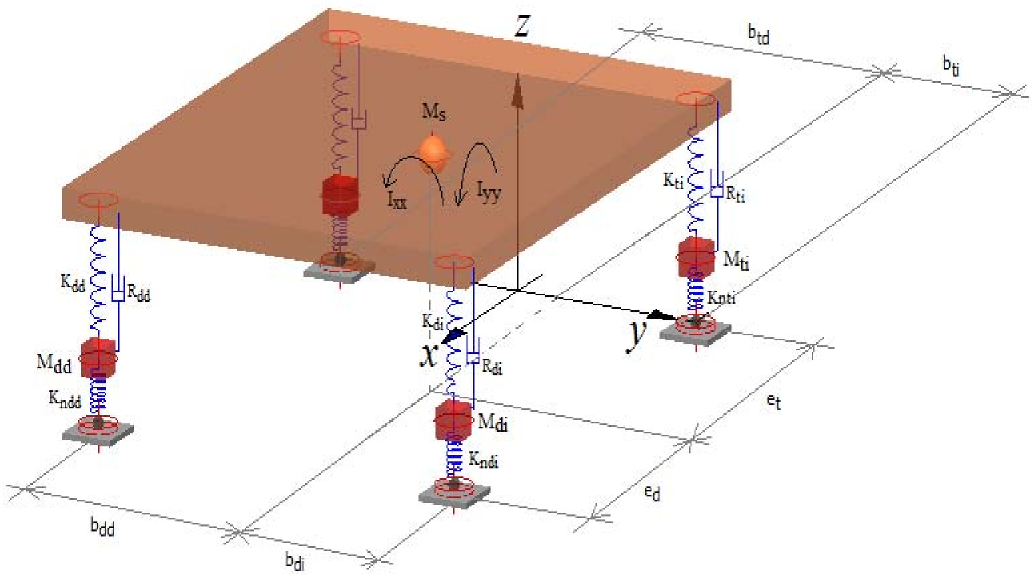


Figure 1: Platform model: basic scheme of the 7 dof model.

The matrix of effective inertia parameters can be written in the following form:

$$\mathbf{M} = \begin{bmatrix} M_{dd} & 0 & 0 & 0 & 0 & 0 & 0 \\ & M_{di} & 0 & 0 & 0 & 0 & 0 \\ & & M_{td} & 0 & 0 & 0 & 0 \\ & & & M_{ti} & 0 & 0 & 0 \\ & & & & M_s & 0 & 0 \\ & sym & & & & I_{xx} & 0 \\ & & & & & & I_{yy} \end{bmatrix} \tag{2}$$

where M_{dd} and M_{di} are the effective suspended masses at the right front wheel and left front wheel respectively; M_{td} and M_{ti} are the effective suspended masses at the right rear wheel and left rear wheel respectively; M_s is the mass of the platform and finally I_{xx} and I_{yy} are the rotary inertias of the platform in their corresponding directions.

The matrix $\mathbf{R}$ of effective damping elements can be written in the following form:

$$\mathbf{R} = \begin{bmatrix} R_1 & 0 & 0 & 0 & -R_{dd} & b_{dd}R_{dd} & e_dR_{dd} \\ & R_2 & 0 & 0 & -R_{di} & -b_{di}R_{di} & e_dR_{di} \\ & & R_3 & 0 & -R_{td} & b_{td}R_{td} & -e_tR_{td} \\ & & & R_4 & -R_{ti} & -b_{ti}R_{ti} & -e_tR_{ti} \\ & & & & R_{ss} & R_{sx} & R_{sy} \\ & sym & & & & R_{xx} & R_{xy} \\ & & & & & & R_{yy} \end{bmatrix} \tag{3}$$

in which the following definitions are introduced:

$$\begin{aligned} R_1 = R_{dd} + R_{ndd},\; R_2 = R_{di} + R_{ndi},\; R_3 = R_{td} + R_{ntd},\; R_4 = R_{ti} + R_{nti}, \\ R_{ss} = R_{dd} + R_{di} + R_{td} + R_{ti},\; R_{xy} = b_{dd}e_dR_{dd} - b_{di}e_dR_{di} - b_{td}e_tR_{td} + b_{ti}e_tR_{ti}, \\ R_{sy} = e_tR_{ti} - e_dR_{dd} - e_dR_{di} + e_tR_{td},\; R_{xx} = b_{dd}^2R_{dd} + b_{di}^2R_{di} + b_{td}^2R_{td} + b_{ti}^2R_{ti} \\ R_{sx} = b_{ti}R_{ti} - b_{dd}R_{dd} + b_{di}R_{di} - b_{td}R_{td},\; R_{yy} = e_d^2R_{dd} + e_d^2R_{di} + e_t^2R_{td} + e_t^2R_{ti} \end{aligned} \tag{4}$$

where, R_{dd} and R_{di} are the damping constants of the right front damper and left front damper, whereas R_{td} and R_{ti} are the damping constants of the right rear damper and left rear damper, respectively. R_{ndd}, R_{ndi}, R_{ntd} and R_{nti} are damping constants associated to the material properties of the wheels, i.e. right front, left front, right rear and left rear wheels, respectively. e_t, e_d, b_{dd}, etc. are distances from the gravity center as it can be seen in Figure 1.

The matrix **K** of effective elastic coefficients and spring constants is written in a similar form as the previous expression, i.e:

$$\mathbf{K} = \begin{bmatrix} K_1 & 0 & 0 & 0 & -K_{dd} & b_{dd}K_{dd} & e_dK_{dd} \\ & K_2 & 0 & 0 & -K_{di} & -b_{di}K_{di} & e_dK_{di} \\ & & K_3 & 0 & -K_{td} & b_{td}K_{td} & -e_tK_{td} \\ & & & K_4 & -K_{ti} & -b_{ti}K_{ti} & -e_tK_{ti} \\ & & & & K_{ss} & K_{sx} & K_{sy} \\ & sym & & & & K_{xx} & K_{xy} \\ & & & & & & K_{yy} \end{bmatrix} \tag{5}$$

in which the following definitions are introduced:

$$\begin{array}{c} K_1 = K_{dd} + K_{ndd},\ K_2 = K_{di} + K_{ndi},\ K_3 = K_{td} + K_{ntd},\ K_4 = K_{ti} + K_{nti}, \\ K_{ss} = K_{dd} + K_{di} + K_{td} + K_{ti},\ K_{xy} = b_{dd}e_dK_{dd} - b_{di}e_dK_{di} - b_{td}e_tK_{td} + b_{ti}e_tK_{ti}, \\ K_{sy} = e_tK_{ti} - e_dK_{dd} - e_dK_{di} + e_tK_{td},\ K_{xx} = b_{dd}^2K_{dd} + b_{di}^2K_{di} + b_{td}^2K_{td} + b_{ti}^2K_{ti} \\ K_{sx} = b_{ti}K_{ti} - b_{dd}K_{dd} + b_{di}K_{di} - b_{td}K_{td},\ K_{yy} = e_d^2K_{dd} + e_d^2K_{di} + e_t^2K_{td} + e_t^2K_{ti} \end{array} \tag{6}$$

where, K_{dd} and K_{di} are the effective spring constants of the right front and left front suspension springs, whereas K_{td} and K_{ti} are the effective spring constants of the right rear and left rear suspension springs, respectively. K_{ndd}, K_{ndi}, K_{ntd} and K_{nti} are spring constants associated to the elastic properties of the wheels, i.e. right front, left front, right rear and left rear wheels, respectively.

The force vector $\overline{\mathbf{F}}$ and displacement vector $\overline{\mathbf{U}}$ introduced in Eq. (1) are given in the following expressions:

$$\overline{\mathbf{F}} = \left\{ \begin{array}{c} F_{dd}(t) - M_{dd}g + K_{ndd}\left[L_{ndd} - z_{0dd}(t)\right] - K_{dd}L_{dd} \\ F_{di}(t) - M_{di}g + K_{ndi}\left[L_{ndi} - z_{0di}(t)\right] - K_{di}L_{di} \\ F_{td}(t) - M_{td}g + K_{ntd}\left[L_{ntd} - z_{0td}(t)\right] - K_{td}L_{td} \\ F_{ti}(t) - M_{ti}g + K_{nti}\left[L_{nti} - z_{0ti}(t)\right] - K_{ti}L_{ti} \\ F_{ss}(t) - M_sg + K_{dd}L_{dd} + K_{di}L_{di} + K_{td}L_{td} + K_{ti}L_{ti} \\ M_{xx}(t) - b_{dd}K_{dd}L_{dd} + b_{di}K_{di}L_{di} - b_{td}K_{td}L_{td} + b_{ti}K_{ti}L_{ti} \\ M_{yy}(t) - e_dK_{dd}L_{dd} - e_dK_{di}L_{di} + e_tK_{td}L_{td} + e_tK_{ti}L_{ti} \end{array} \right\}, \overline{\mathbf{U}} = \left\{ \begin{array}{c} u_{dd} \\ u_{di} \\ u_{td} \\ u_{ti} \\ u_{ss} \\ \phi_{xx} \\ \phi_{yy} \end{array} \right\} \tag{7}$$

where u_{dd}, u_{di}, are the vertical displacements of the right front and left front wheels, u_{td}, u_{ti}, are the vertical displacements of the right rear and left rear wheels, u_{ss} is the vertical displacement of the platform, ϕ_{xx} and ϕ_{yy} are the rotations of the platform. L_{ndd}, L_{ndi}, L_{ntd} and L_{nti} are the equivalent free length of springs related to the elastic properties of the wheels, whereas L_{dd}, L_{di}, L_{td} and L_{ti} are the equivalent free length of springs of the elastic elements that support the platform. The functions $z_{0dd}(t)$, $z_{0di}(t)$, $z_{0td}(t)$ and $z_{0ti}(t)$ are related to disturbances in the ground that perturb the motion of the suspended masses and consequently the whole platform.

Recall that the differential system given Eq. (1) can be split into a static system and a dynamic counterpart, by substituting $\overline{\mathbf{U}} = \overline{\mathbf{U}}_S + \overline{\mathbf{U}}_D$ in Eq. (1) and handling the remaining equations appropriately (Lee et al., 2008), where $\overline{\mathbf{U}}_S$ and $\overline{\mathbf{U}}_D$ are the static and dynamic parts that comprise the solution. Depending on the case to be analyzed the whole differential system is used, i.e. Eq. (1) or only the dynamic part.

3 PROBABILISTIC MODEL

The Maximum Entropy Principle (Jaynes, 2003) is used to construct the probabilistic models taking into account the uncertain parameters. The Maximum Entropy Principle is used to derive the probability density function of the random variables associated to the uncertain parameters. The deterministic model developed in the previous section has many parameters that can be uncertain, however the most relevant for the suspension system are the effective spring coefficients and effective damping coefficients of each suspension subsystem in the plate corners. Let introduce $\{V_1, V_2, V_3, V_4\}$ as the random variables associated to the parameters $\{K_{dd}, K_{di}, K_{td}, K_{ti}\}$; $\{V_5, V_6, V_7, V_8\}$ the random variables associated to the parameters $\{K_{ndd}, K_{ndi}, K_{ntd}, K_{nti}\}$; $\{V_9, V_{10}, V_{11}, V_{12}\}$ the random variables associated to the parameters $\{R_{ndd}, R_{ndi}, R_{ntd}, R_{nti}\}$ and $\{V_{13}, V_{14}, V_{15}, V_{16}\}$ the random variables associated to the parameters $\{R_{dd}, R_{di}, R_{td}, R_{ti}\}$. Thus the probability functions of the random variables are obtained solving the following optimization problem:

$$p_V^* = \arg \max_{p_V \in C^p} S\left(p_V\right) \tag{8}$$

where p_V^* is the optimal probability density function such that $\forall\ p_V \in C^p$, $S(p_V^*) \geq S(p_V)$, which is the measure of entropy and C^p is the set of the admissible probability density functions that satisfy the information available. The entropy is defined as (Shannon, 1948):

$$S\left(p_V\right) = \int_\Lambda p_V \ln\left(p_V\right) dv \tag{9}$$

where Λ is the support of the probability distributions. In the case in which scarce information is available about the dependency among random variables, the Maximum Entropy Principle states that the random variables involved have to be independent.

The available information to construct consistently the probability density functions is the following: (a) random variables should be positive, consequently the support is $\Lambda =]0, \infty[$, (b) the mean value of each random variable $\mathcal{E}\{V_i\} = \underline{V}_i$ is known and it coincides with the nominal value of the corresponding parameter, (c) V_i, $i = 1, ..., 16$ should be second order random variable to guarantee finite dispersion in the displacements (Ritto et al., 2008). Under this context and employing the Maximum Entropy Principle the following expression of the probability density function can be derived:

$$p_{V_i}\left(v_i\right) = \mathbb{1}_{]0,\infty]}\left(v_i\right) \frac{1}{\underline{V}_i} \left(\frac{1}{\delta_{V_i}^2}\right)^{1/\delta_{V_i}^2} \frac{1}{\Gamma\left(1/\delta_{V_i}^2\right)} \left(\frac{v_i}{\underline{V}_i}\right)^{1/\delta_{V_i}^2 - 1} \exp\left(-\frac{v_i}{\delta_{V_i}^2 \underline{V}_i}\right) \tag{10}$$

where $\mathbb{1}_{]0,\infty]}\left(v_i\right)$ is the support, δ_{V_i} and $\underline{V}_i$ are the dispersion parameter and the mean value of the random variable V_i, whereas $\Gamma(z)$ is the gamma function defined for $z > 0$. The Matlab function $gamrnd(1/\delta_{V_i}^2, \underline{V}_i \delta_{V_i}^2)$ can be used to generate realizations. It should be noted that the dispersion coefficient is constrained to be $\delta_{V_i} \in [0, 1/\sqrt{3}]$ due to the condition that V_i is a second order random variable.

The probabilistic model can be constructed from Eq. (1) by employing Eq. (10) in the assembly of damping and stiffness matrices, and the dynamic systems becomes stochastic as:

$$\begin{aligned} \mathbf{M}\ddot{\overline{\mathbb{U}}} + \mathbb{R}\dot{\overline{\mathbb{U}}} + \mathbb{K}\overline{\mathbb{U}} = \overline{\mathbf{F}} \\ \overline{\mathbf{U}}(0) = \overline{\mathbf{U}}_0 \\ \dot{\overline{\mathbf{U}}}(0) = \dot{\overline{\mathbf{U}}}_0 \end{aligned} \quad (11)$$

In Eq. (11) the math-blackboard font is used to identify the random entities, that is matrices $\mathbb{R}$ and $\mathbb{K}$ are random due to the stochastic nature of the parameters involved in them, and vector $\overline{\mathbb{U}}$ and its time derivatives are the random response.

The Monte Carlo method is used the simulate the stochastic dynamics, which implies the integration of a deterministic system for each independent realization of random variables V_i, $i = 1, ..., 16$. In order to control the quality of the simulation, the convergence of the stochastic response $\overline{\mathbb{U}}$ is evaluated appealing to the following function:

$$conv\left(N_{MS}\right) = \sqrt{\frac{1}{N_{MS}} \sum_{j=1}^{N_{MS}} \int_{t_0}^{t_1} \left\|\overline{\mathbb{U}}_j\left(t\right)\right\|^2 dt} \quad (12)$$

where N_{MS} is the number of Monte Carlo samplings.

4 COMPUTATIONAL STUDIES

For the computational studies of uncertainty quantification in the dynamics of the platform, the parameters of the model shown in Fig. 1 have the following expected values: $M_{dd} = M_{di} = 350\ Kg$, $M_{td} = M_{ti} = 350\ Kg$, $M_s = 16000\ Kg$, $I_{xx} = 10000\ Kg.m^2$, $I_{yy} = 16000\ Kg.m^2$, $R_{ndd} = R_{ndi} = 16500\ Ns/m$, $R_{ntd} = R_{nti} = 16500\ Ns/m$, $R_{dd} = R_{di} = 12566\ Ns/m$, $R_{td} = R_{ti} = 12566\ Ns/m$, $K_{ndd} = K_{ndi} = 1.207\ 10^6\ N/m$, $K_{ntd} = K_{nti} = 1.207\ 10^6\ Ns/m$, $K_{dd} = K_{di} = 157913\ N/m$, $K_{td} = K_{ti} = 157913\ N/m$, $b_{dd} = b_{di} = 1\ m$, $b_{td} = b_{ti} = 1\ m$, $e_d = 0.8\ m$, $e_t = 1.2\ m$, $L_{ndd} = L_{ndi} = 0.2\ m$, $L_{ntd} = L_{nti} = 0.2\ m$, $L_{dd} = L_{di} = 0.5\ m$, $L_{td} = L_{ti} = 0.5\ m$, $g = 9.81\ m/s^2$. For these studies no perturbation in the ground is involved, that is: $z_{0dd}\left(t\right) = 0$, $z_{0di}\left(t\right) = 0$, $z_{0td}\left(t\right) = 0$ and $z_{0ti}\left(t\right) = 0$.

A previous check of the quality of the deterministic model is performed by comparing its transient response with the one calculated in a commercial multibody dynamics system. The initial conditions are $\overline{\mathbf{U}}(0) = \overline{\mathbf{0}}$ and $\dot{\overline{\mathbf{U}}}(0) = \overline{\mathbf{0}}$, which implies the release of the platform from a situation with uncompressed springs, non active dampers and non unbalanced platform. Also the damping effect of the lower components is neglected, i.e. $R_{ndd} = R_{ndi} = R_{ntd} = R_{nti} = 0$.

Fig. 2 shows the comparison of the dynamic responses of the present deterministic model and a computational model constructed in the Multibody dynamic analysis system called Working Model 3D. This type of programs do not have the possibility to perform stochastic dynamic analysis.

A few preliminary studies are made in order to check the convergence of the Monte Carlo Method. Several dispersion coefficients have been tested. Fig. 3 shows an example of the convergence pattern, which is similar to all the preliminary cases tested. It is possible to see that a good convergence is obtained with $N_{MS} = 1000$ or more. However with $N_{MS} = 500$ or even less it is possible to reach an acceptable solution convergence. Fig. 4 shows histograms of some parameters employed for this calculation.

In the following paragraphs a study of the dynamic stochastic response of the platform is performed. As a first study a low dispersion is used in all parameters, that is $\delta_{V_i} = 0.05$. The

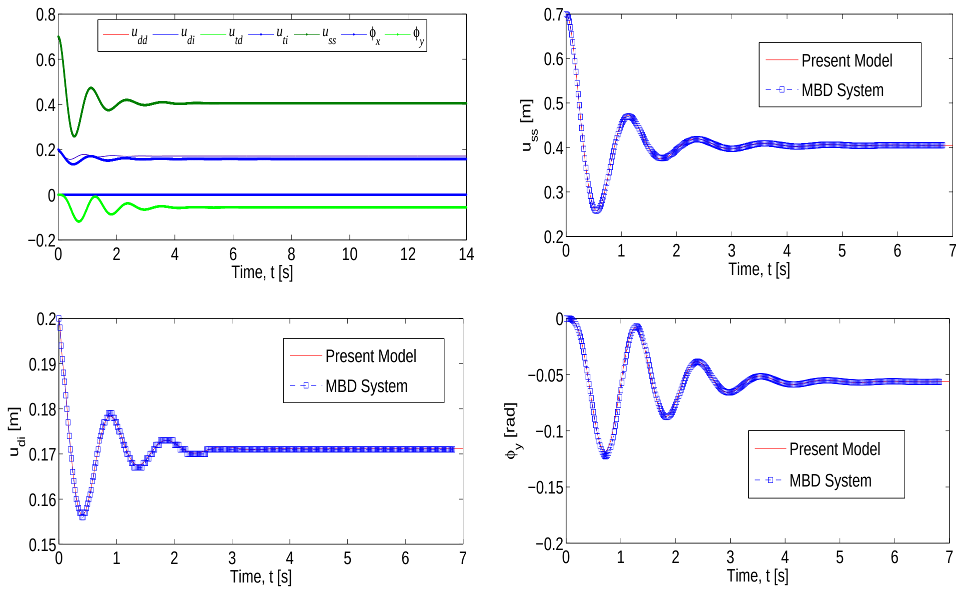


Figure 2: Comparison of the deterministic model with a 3D multibody dynamics analysis system.

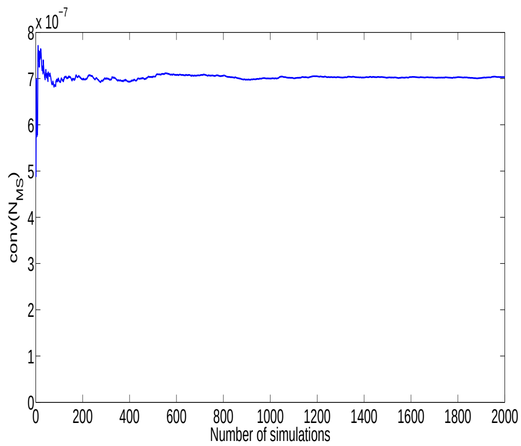


Figure 3: Example of the convergence of the Monte Carlo Method.

platform is loaded with a sudden unitary force at each suspension subsystem on the right side, and applied in the corresponding concentrated masses. The results of this test are presented in frequency response functions (FRF).

In Fig. 5(a) and Fig. 5(b) the response of the front suspension subsystems are depicted. In Fig. 6 the response of the mass center of the platform is shown. As expected the influence of the uncertainty is larger in the loaded suspension subsystem. The response of the platform is not influenced sensibly by this condition of uncertainty in the lower part of the suspension subsystems, although there is a increase of the size of the confidence interval in the band of 4 Hz to 20 Hz.

Fig. 7 shows the response of the right front suspension subsystem with the same type of excitation adopted for the previous case, but for maximum possible level of uncertainty in the random variables associated to the damping coefficients of the lower part of the suspension subsystems, that is $\delta_{V_i} = 1/sqrt3$, $i = 13, ..., 16$, and the remaining random variables have

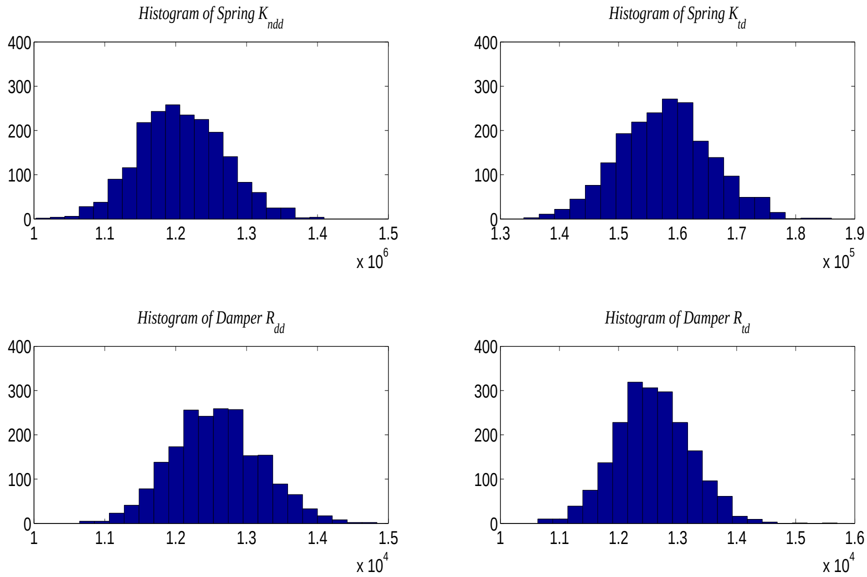
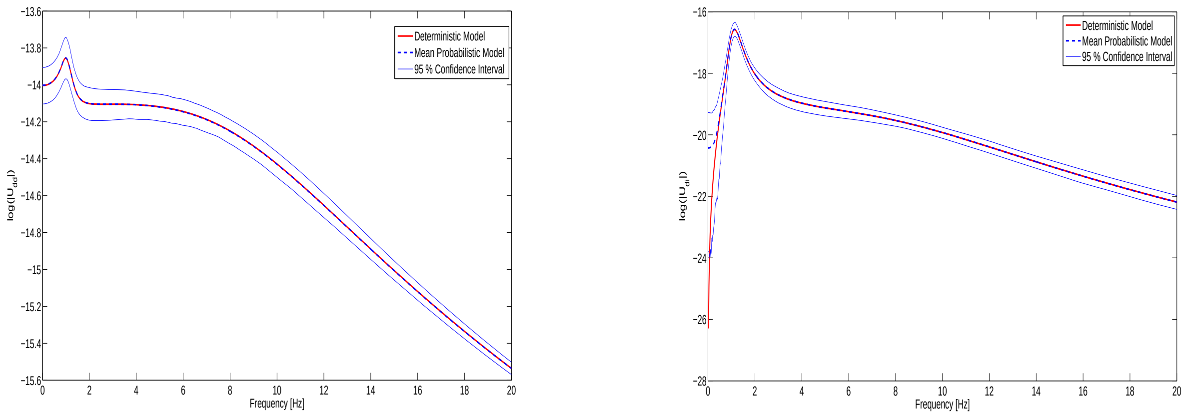


Figure 4: Histograms of some parameters.



(a) (b)

Figure 5: FRFs Frequency response function suspension subsystems of the front: (a) u_{dd} (b) u_{di}.

the same dispersion parameter $\delta_{V_i} = 0.05$. Fig. 8 shows the response of the mass center of the platform.

As it is possible to see the propagation of the uncertainty associated to the damping coefficients of the lower part of the suspension system increases dramatically in the band of frequencies $[4, 20]$ when Figs. 5(a) and 7 and Figs. 6 and 8 are compared. However near the peak response of the platform, there is no sensible influence of the parameters uncertainty to promote an instability in the platform. The control of this type of aspects are of major concern in the dynamics of platforms.

5 CONCLUSIONS

This article has presented a preliminary study of the stochastic dynamics of the suspension system in a vehicular platform modeled with a 7 degree-of-freedom ordinary differential equations system. The stochastic aspects are related to the uncertainty of some parameters of the

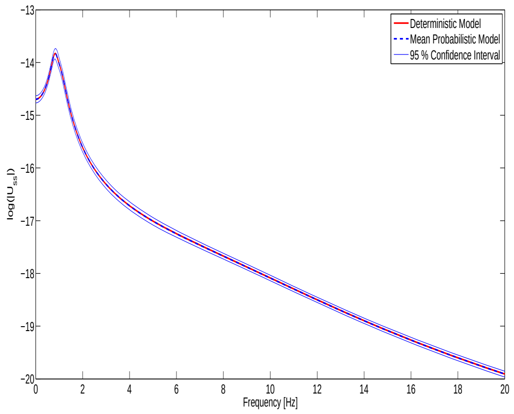


Figure 6: Frequency response function of u_{ss}.

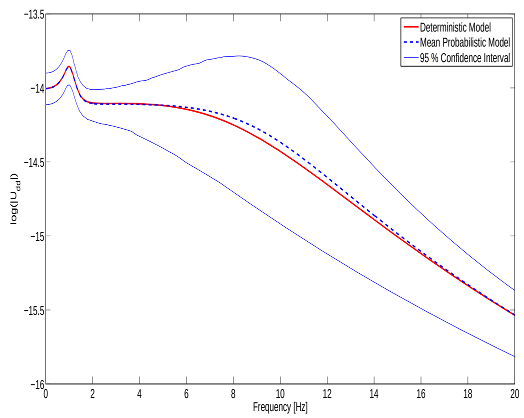


Figure 7: Frequency response function of u_{dd} for a higher uncertainty in the damping coefficients of the lower part of the suspension subsystems.

model such as the effective spring coefficients or the effective damping coefficients. The Maximum Entropy Principle has been used to derive the probability density functions of the uncertain parameters which are needed to construct the probabilistic model taking into account the deterministic model as the mean model. Then the Monte Carlo Method is employed to simulate the stochastic response.

The influence of the uncertainty of many parameters, on the stochastic dynamics of the vehicular platform has been tested. Although some parameters evaluated in this paper appear to scarcely affect the platform dynamics, there is no evidence to state the same for every parameter involved in the model. More studies should be done to evaluate and characterize the risky physical parameters in the dynamics of vehicular platforms.

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REFERENCES

Dixon J. *Suspension geometry and computation*. Wiley and Sons LTD, 2009.

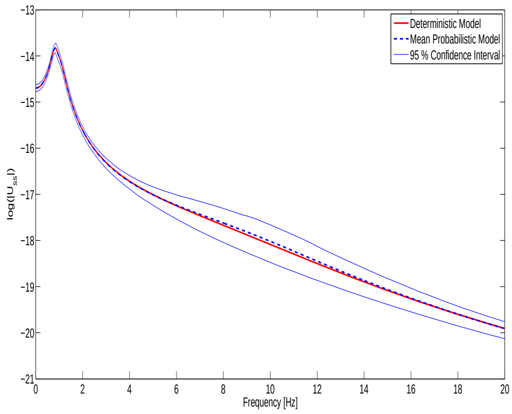

Figure 8: Frequency response function of u_{ss} for a higher uncertainty in the damping coefficients of the lower part of the suspension subsystems.

Gillespie T. *Fundamentals of Vehicle Dynamics*. Society of Automotive Engineers, 1995.

Jaynes E. *Probability Theory: The logic of Science, Vol.1*. Cambridge University Press, Cambridge, U.K., 2003.

Lee S., Sheu J., and Lin S. In-plane vibrational analysis of rotating curved beam with elastically restrained root. *Journal of Sound and Vibration*, 315(4), 2008.

Ritto T., Sampaio R., and Cataldo E. Timoshenko beam with uncertainty on the boundary conditions. *Journal of Brazilian Society of Mechanical Sciences and Engineering*, 30:295–303, 2008.

Shannon C. A mathematical theory of communication. *Bell System Tech*, 28:379–423 and 623–659, 1948.

Soize C. Maximum entropy approach for modeling random uncertainties in transient elastodynamics. *Journal of the Acoustical Society of America*, 109(5):1979–1996, 2001.

Thomsen P. and True H. *Non-smooth problems in Vehicle systems dynamics*. Springer-Verlag, Berlin-Heidelberg, 2010.

Williams J. *Fundamentals of Applied Dynamics*. Wiley and Sons INC, 1996.