Asociación Argentina de Mecánica Computacional

Mecánica Computacional Vol XXXIII, págs. 3-5 (artículo completo)
Graciela Bertolino, Mariano Cantero, Mario Storti y Federico Teruel (Eds.)
San Carlos de Bariloche, 23-26 Setiembre 2014

STRESSES AS UNKNOWNS IN COMPUTATIONAL SOLID AND FLUID MECHANICS

Ramon Codina

Universitat Politècnica de Catalunya, Jordi Girona 1-3, Building C1, 08034 Barcelona, Spain, ramon.codina@upc.edu, http://codina.rmee.upc.edu

Keywords: Mixed methods, stabilization, nonlinear solid mechanics, localization, quasi-Newtonian flows, viscoelastic flows

Abstract.

In this work we provide a summary of our work in the design and analysis of finite element methods for formulations involving the stresses as unknowns in computational mechanics. There are several reason for choosing a formulation of this type, that can be essentially classified into two groups: cases in which some additional stress accuracy is required and cases in which the stress itself is solution of a partial differential equation that needs to be approximated numerically.

Starting with the linearized elasticity problem, it can be written as

$$-\nabla \cdot \boldsymbol{\sigma} = \mathbf{f} \tag{1}$$

$$\mathbf{C}^{-1} : \boldsymbol{\sigma} - \nabla^S \mathbf{u} = \mathbf{0} \tag{2}$$

where $\boldsymbol{\sigma}$ is the stress tensor, $\mathbf{u}$ the displacement, $\mathbf{C}$ the constitutive tensor and $\nabla^S \mathbf{u}$ the symmetrical gradient of the displacements. Some nonlinear models, such as damage, can also be written this way if $\mathbf{C}$ is allowed to depend on $\mathbf{u}$. Problem (1)-(2) has the structure of a Darcy-like problem. It is shown in Badia and Codina (2009) how to approximate it using *arbitrary* interpolations for $\mathbf{u}$ and $\boldsymbol{\sigma}$ and, likewise, *how to improve the accuracy of* $\boldsymbol{\sigma}$ depending on the variational formulation chosen. This is exploited in the context of problems involving localization and cracks in Cervera et al. (2010a,b, 2011). The idea is simple: if better accuracy is obtained for the stresses and the appearance of localization is driven by constitutive laws which depend on their prediction, failure mechanisms will be better predicted.

The incompressible counterpart of (1)-(2) is

$$-\nabla \cdot \boldsymbol{\sigma}' + \nabla p = \mathbf{f} \tag{3}$$

$$\nabla \cdot \mathbf{u} = 0 \tag{4}$$

$$\mathbf{C}_{\text{dev}}^{-1} : \boldsymbol{\sigma}' - \nabla^S \mathbf{u} = \mathbf{0} \tag{5}$$

where $\boldsymbol{\sigma}'$ is the deviatoric part of $\boldsymbol{\sigma}$, p is the pressure and $\mathbf{C}_{\text{dev}}$ the deviatoric component of the constitutive tensor. The analysis of finite element approximations of this problem accommodating arbitrary interpolations was presented in Codina (2009). Its extension to incompressible nonlinear solid mechanics problems is presented in Chiumenti et al. (to appear); Cervera et al. (submitted). It is shown there that this formulation allows one to solve plasticity problems with isochoric behavior, for example. The keys to this fact are also outlined in this work.

Problem (3)-(4)-(5) can also be considered a model problem for incompressible flows, now considering $\mathbf{u}$ the velocity field. In some cases it is either convenient or mandatory to interpolate the deviatoric part of the stresses $\boldsymbol{\sigma}'$. Quasi-Newtonian fluids are an example of the first situation, whereas viscoelastic flows involve a partial differential equation for the stresses that makes it necessary to interpolate them. The use of finite element formulations in both situations is respectively studied in Castillo and Codina (to appear a) and Castillo and Codina (to appear b).

The interest of approximating either problem (1)-(2) or (3)-(4)-(5) is thus clear. However, the main practical difficulty is that if the standard Galerkin formulation is used, there are compatibility conditions between stresses and the rest of unknowns that are difficult to meet in practice. The alternative is to use stabilized finite element formulations, which allow one to use any interpolation for all the unknowns, in particular equal interpolation. This is the approach followed in the references cited and that is summarized in this work.

REFERENCES

Badia S. and Codina R. Unified stabilized finite element formulations for the Stokes and the Darcy problems. *SIAM Journal on Numerical Analysis*, 47:1971–2000, 2009.

Castillo E. and Codina R. Stabilized stress-velocity-pressure finite element formulations of the Navier-Stokes problem for fluids with nonlinear viscosity. *Computer Methods in Applied Mechanics and Engineering*, to appear a.

Castillo E. and Codina R. Variational multi-scale stabilized formulations for the stationary three-field incompressible viscoelastic flow problem. *Computer Methods in Applied Mechanics and Engineering*, to appear b.

Cervera M., Chiumenti M., Benedetti L., and Codina R. Mixed stabilized finite element methods in nonlinear solid mechanics. Part III: compressible and incompressible plasticity. *Computer Methods in Applied Mechanics and Engineering*, submitted.

Cervera M., Chiumenti M., and Codina R. Mixed stabilized finite element methods in nonlinear solid mechanics. Part I: Formulation. *Computer Methods in Applied Mechanics and Engineering*, 199:2559–2570, 2010a.

Cervera M., Chiumenti M., and Codina R. Mixed stabilized finite element methods in nonlinear solid mechanics. Part II: Strain localization. *Computer Methods in Applied Mechanics and Engineering*, 199:2571–2589, 2010b.

Cervera M., Chiumenti M., and Codina R. Mesh objective modelling of cracks using continuous linear strain and displacement interpolations. *International Journal for Numerical Methods in Engineering*, 87:962–987, 2011.

Chiumenti M., Cervera M., and Codina R. A mixed three-field FE formulation for stress accurate analysis including the incompressible limit. *Computer Methods in Applied Mechanics and Engineering*, to appear .

Codina R. Finite element approximation of the three field formulation of the Stokes problem using arbitrary interpolations. *SIAM Journal on Numerical Analysis*, 47:699–718, 2009.