Asociación Argentina
de Mecánica Computacional

Mecánica Computacional Vol XXV, pp. 95-108
Alberto Cardona, Norberto Nigro, Victorio Sonzogni, Mario Storti. (Eds.)
Santa Fe, Argentina, Noviembre 2006

NUMERICAL ANALYSIS FOR THE FLOW STRUCTURES FOLLOWING A THREE-DIMENSIONAL HORIZONTAL FORWARD-FACING STEP CHANNEL

Juan G. Barbosa-Saldaña, Florencio Sánchez-Silva, Luis A. Moreno-Pacheco and Ignacio Carvajal-Mariscal

SEPI-ESIME-IPN
LABINTHAP
Unidad Profesional Adolfo Lopez Mateos
Mexico City, Mexico.
jgabriel_bs@yahoo.com.mx

***Keywords**: Numerical simulation, three dimensional, forward-facing step, laminar flow.*

Abstract. A numerical code based on the finite volume discretization technique is developed to simulate the flow structures following a three-dimensional horizontal forward-facing step. The link between the pressure distribution and velocity field are done by using the SIMPLE algorithm.

A rectangular channel encloses the forward-facing step such that the expansion ratio (ER) and the aspect ratio (AR) are equal to two and four respectively. The channel total length in the stream-wise direction is equal to 60 times the step height and the step edge is located 20 times the step height downstream the channel inlet. At the channel inlet the flow is considered as a three-dimensional fully developed flow.

Results for the reattachment line, the separation line, as well as for velocity profiles at different planes for different Reynolds are presented.

1. INTRODUCTION

Separation and reattachment flow is a phenomenon that is found in several industrial devices such as in pieces of electronic cooling equipment, cooling of nuclear reactors, cooling of turbine blades, flow in combustion chambers, flow in vertical plates with ribs, flow in wide angle diffusers, and valves. In other situations, the separation is induced in order to produce more favorable heat transfer conditions as in the case of compact heat exchangers and even more for understanding the onset of transition to turbulence in natural and mixed convection.

In the last decade several numerical studies have been conducted to achieve a better knowledge and understanding of the hydrodynamic of the separated flow. In this aspect the backward-facing step has been the central objective for several researches, and even more this problem is considered as a benchmark problem for validating numerical codes and procedures (Blackwell and Pepper, 1992; Williams and Baker, 1997).

On the other hand, the configuration of a forward-facing step has been investigated much less than the backward-facing step. Stuer *et. al* (1999) mentioned in their publication that very little has been published referring to the laminar separation over a forward-facing step, and neither its topology nor its re-circulation zones are know in a predictable form. Abu Mulaweh (2003) reports that the phenomena of convection over the forward facing step has not been studied due to its complexity. He concludes that depending on the magnitude of the flow Reynolds number, one or two flow-separation regions may develop adjacent to the step.

Some authors had conducted their researches to analyze the flow structures passing a forward facing step. Ratish and Naidu (1993) developed a stream-function-vorticity formulation for solving the two dimensional Navier-Stokes equations for laminar flow. In their publication they did not include the geometrical factors for the computational domain making their results difficult for being reproduced. In similar way, Houde et al. (1994) used a stream-function vorticity formulation for analyzing the steady two dimensional laminar flow problem following a forward-facing step. They implemented a second order difference scheme to numerically solve the problem. In their study, they found a re-circulation zone at the step corner and also they found that the flow separates from the bottom wall before the forward facing step. Even though their results presented an excellent agreement with previous results reported in literature, their approximation is for a two-dimension problem and cannot be useful for validating the results in this work. Others authors as Aseban et al. (2000) had conducted their studies for the forward facing step geometry to analyze the mixed convective flow in vertical plates or for studying the mixed convective flow in two dimensions channel for assisting and opposing flow as presented by Abu-Mulaweh et al. in several publications (1996, 1994, 1993). Although an important effort to analyze the flow passing a three-dimensional forward-facing step has been made, most of the studies are limited to the two dimensional case. In this research, the

analysis for the tri-dimensionality of the flow is the principal objective as it will be presented in later sections.
The importance of studying the flow passing a forward facing step is described by Stuer et. al (1999) and Ravindran (2002). They mentioned applications such that enhance heat transfer and flow mixing rates, flows over obstacles such buildings and cooling of electronic equipments as well as in the control of fluid flow for designing fluid dynamical systems. In this paper, some aspects of the fluid structures are presented for a numerical simulation for air flowing through a three-dimensional horizontal forward-facing step.

2. MODEL DESCRIPTION AND NUMERICAL PROCEDURE

The flow over a three-dimensional horizontal forward-facing step was numerically simulated via a finite volume discretization technique. The channel aspect ratio (AR=W/s) and expansion ratio (ER=H/s) were fixed in relation to the step height (s=0.01m) as AR=4 and ER=2, respectively. The step is located 20 times the step height downstream the channel inlet (*l*=20s) and the channel total length is equal to 60 times the step height (L=60s). The geometry is presented in fig. 1.

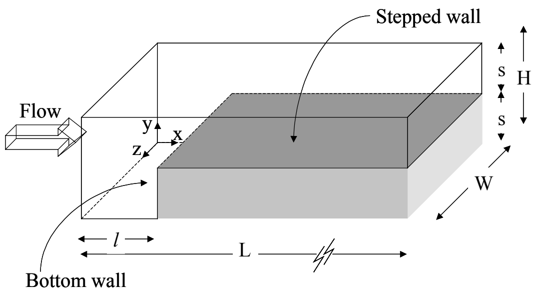

Figure 1: Computational domain for the forward-facing step

At the channel entrance the flow was treated as fully developed flow according to the correlation presented by Shah and London (1978). No-slip condition was applied at the duct walls, including the stepped wall and the thermo-physical properties inside the computational domain were assumed as constants. The fluid flow problem is considered to be steady state. Hence, the mass conservation and the momentum equations governing the phenomenon are reduced to the following forms according to Williams and Baker (1997):

Continuity Equation:

$$\frac{\partial u_i}{\partial x_i} = 0 \qquad (1)$$

Momentum Equation:

$$\frac{\partial}{\partial x_j}\left[u_i u_j - \frac{1}{\mathrm{Re}}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + p\delta_{ij}\right] = 0 \qquad (2)$$

The boundary conditions for the computational domain were established as follow:

$$0 \le x \le L, \quad 0 \le z \le W \begin{cases} y = 0 \\ y = H \end{cases} \{\phi = \phi^*$$

$$0 \le x \le L, \quad 0 \le y \le H \begin{cases} z = 0 \\ z = W \end{cases} \{\phi = \phi^*$$

$$0 \le y \le H, \quad 0 \le z \le W \begin{cases} x = 0 & \{\textit{Fully developed flow (Shah and London, 1978)} \\ x = L & \left\{ \left.\dfrac{\partial \phi}{\partial x}\right|_{x=L} = 0 \right. \end{cases}$$

Where, $\phi = u, v, w, p$

$*$ *guessed value or starred conditions*

The physical properties of air in the numerical procedure were treated as constant [μ=1.81x10^{-5} kg m^{-1} s^{-1} ρ=1.205 kg m^{-3}] and evaluated at the ambient temperature T_0 (T_0=293).

A FORTRAN code was developed to numerically study the stated problem. A finite volume discretization technique was implemented for discretizing the momentum equations inside the computational domain. The SIMPLE algorithm is utilized for linking the velocity and pressure distributions in the iteration procedure. At the final step of every iteration the velocity field and pressure distribution are corrected and updated until reach convergence as described by Patankar (1980). The power law scheme was utilized to represent the convection-diffusion term at the control volume interfaces. Velocity nodes were located at staggered locations in each coordinate direction while pressure and other scalar properties were evaluated at the main grid nodes. At the channel exit the natural boundary conditions $\left[(\partial\phi/\partial x)_{x=L} = 0\right]$ were imposed for all the variables. In addition, the overall mass flow in and out of the computational domain were computed and its ratio was used to correct the outlet velocity at the channel exit as described by Barbosa et al. (2005a).

To simulate the solid block inside the domain a very high diffusion coefficient for the momentum equations was chosen (μ=10^{50}). At the solid-fluid interface the diffusion coefficients where evaluated by a weighted harmonic mean of the properties in neighboring control volumes as described by Patankar (1980).

A combination of the line-by-line solver and the tri-diagonal matrix algorithm was used for each plane in x-, y-, and z-coordinate directions to compute the velocity and pressure inside the computational domain. Under-relaxation for the velocity components (α_u=α_v=α_w=0.4) and pressure (α_p=0.4) were imposed in order to guarantee convergence. Convergence for the solution was declared when the normalized residuals for the velocity components and pressure were less than $1x10^{-8}$.

A non-uniform grid size was considered for solving the numerical problem. In this sense, at the solid walls and at the edge of the forward-facing step the grid was composed by small-size control volumes (fine grid) and the control volume size increased far away from the solid walls. The grid size was deployed by means of a geometrical expansion factor, such that each control volume is a certain percentage larger than its predecessor. A detailed description for the grid generation can be found in Barbosa (2005).

The grid independence study was conducted by using several grid densities for a Reynolds number (Re=800) based on the step height. The location at the central plane in the span-wise direction (z/W=0.5) where the stream-wise component of the wall shear stress is zero was monitored to declare grid independence. A grid size of 150:40:40 does not represent important variation when compared with a 180:40:40 grid size. Hence, the former was proposed for the productive runs.

Table 1 summarized the results for the grid independence study. By increasing the number of nodal points in the transverse (y-coordinate) and span-wise (z-coordinate) directions does not impact the numerical results.

Grid size x-y-z	Position at the central plane z=0 where $\tau_{xz}=0$	% Difference
180:40:40	0.1820	
150:40:40	0.1812	0.45
120:40:40	0.1870	2.75

Table 1: Grid independency study.

Once grid independence was established, the second step was to find a procedure to validate the numerical code. A direct validation was not possible because there is not published information dealing with the three-dimensional fluid flow problem through a forward-facing step. Then, it was observed that the difference in the numerical implementation between the backward- and forward-facing step is the location of the step. The former refers to a step at the channel's inlet, while the last one refers to a step and the channel's exit. Hence if the numerical procedure is validated for the backward facing step, it can be useful for solving the forward-facing step problem. In this sense, the forced convective flow trough a three-dimensional horizontal backward-facing step was studied and simulated with the same numerical technique and the results were presented by Barbosa et al. (2005b). It was found that the numerical predictions using the code presented errors less than 2% when compared with the experimental published data, thus validating the code for the backward-facing step case and then its application for a forward-facing step. Figure 2 presents a comparison for the x_u-line obtained with the numerical data and the experimental data obtained by Armaly et al. (2003). More information about the validation problem could be found in previous work published by the authors (Barbosa, 2005; Barbosa et al., 2005a; Barbosa et al., 2005b)

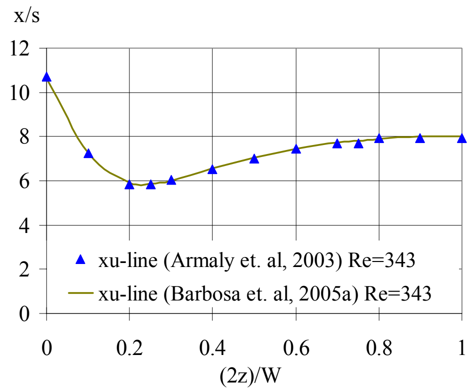

Figure 2: x_u-line numerical validation (Barbosa et al., 2005a)

3. NUMERICAL RESULTS AND DISCUSSION

The numerical study presented in this work considers the flow through a forward-facing step channel for three different Reynolds number (Re=200, 400 and 800). The Reynolds number ($Re = 2\rho U_b s/\mu$) is based on the bulk velocity at the duct entrance (U_b) and twice the channel's step height. The coordinate origin for the geometry was placed according to fig. 1.

A common concept to characterize the separated and reattached flow phenomenon is the end of the re-circulation zone or the point where the wall shear stress is equal to zero. As mentioned by Nie and Armaly (2003), for a three-dimensional backward-facing step there is a series of points along the span-wise direction (z-coordinate direction or span-wise direction/coordinate) where the wall shear stress is equal to zero. The collection of these points is called the x_u-line and is used to delimit the re-circulation zone along the span-wise direction. Numerically the x_u-line is defined as the point in the mainstream flow direction where the u-velocity component changes its value from positive to negative or vice versa. In a similar way, for the case of the forward-facing step a re-circulation zone should be developed adjacent to the bottom wall and before the step. Hence, there is also a line, which delimits the starting point for the re-circulation zone before the step. This line will be referred as the x_u-line or separation line, and its distribution along the span-wise direction is presented in fig. 3 for the three different study cases. A symmetry behavior for the x_u-line respect to the span-wise direction is observed for the three cases. The flow separation occurs in an earlier position as the Reynolds number is increased, as can be observed in fig. 3. According to Schlichting and Gersten (2000) the separation is governed by the pressure gradient and the friction along the wall. In this sense, it can be considerer that the pressure drop and the friction along the wall are larger for higher Reynolds. Figure 3 also reveals that near to the sidewalls the lowest x/s values for the x_u-line are found. This behavior could be explained due to the presence of the sidewalls and the no slip condition imposed for the numerical simulation.

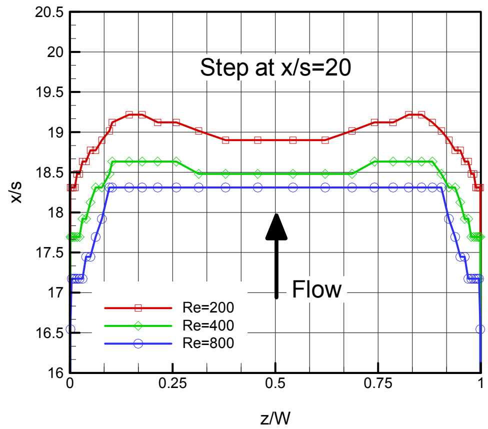

Figure 3: Separation line before the step and adjacent to the bottom wall (x_u-line)

According to White (1999) the flow passing the step edge is separated and somewhere downstream it will be reattached. This phenomenon was observed in the numerical simulation and the results for Re=200, Re=400 and Re=800 are presented in figures 4a), 4b) and 4c) respectively. As can be appreciated in these figures, the amplitude of this zone in the main flow direction (x-coordinate direction or stream-wise direction/coordinate) is on the order of few centimeters and the trend for the re-circulation zone is similar for the three study cases.
In the three cases, two re-circulation zones can be clearly identified. One before the step (x<0.2) and the other over the stepped wall (x>0.2). The largest re-circulation zone corresponds to the higher Reynolds (fig. 4c) while the smallest re-circulation zone belongs to Re=200 (fig. 4a). Figure 4 shows that the starting of the re-circulation zone over the step is almost a straight line. The reason of this particular behavior is associated with the fact that the abrupt change in the geometry that produces the separation is located at the same position for all the span-wise direction (the step edge). However, at the end of this re-circulation zone the line along the span-wise direction delimiting the re-circulation zone presents a no-regular line. This could be associated with the development of zones of high tri-dimensional flow after the step edge and mainly inside the re-circulation zone that is developing in this zone. As can be appreciated, the delimiting re-circulation zone for Re=800 (fig. 4c) has more irregularities than the other two cases as results of higher tree-dimensional behavior of the flow in this zone. For Re=800 and Re=400 it can be appreciated a small zone of positive values for the u-velocity component before the step. This particular behavior is not presented at Re=200. Hence it can be said that for Re=800 and Re=400 the primary re-circulation zone before the step does not finish at the step edge but it finish earlier.

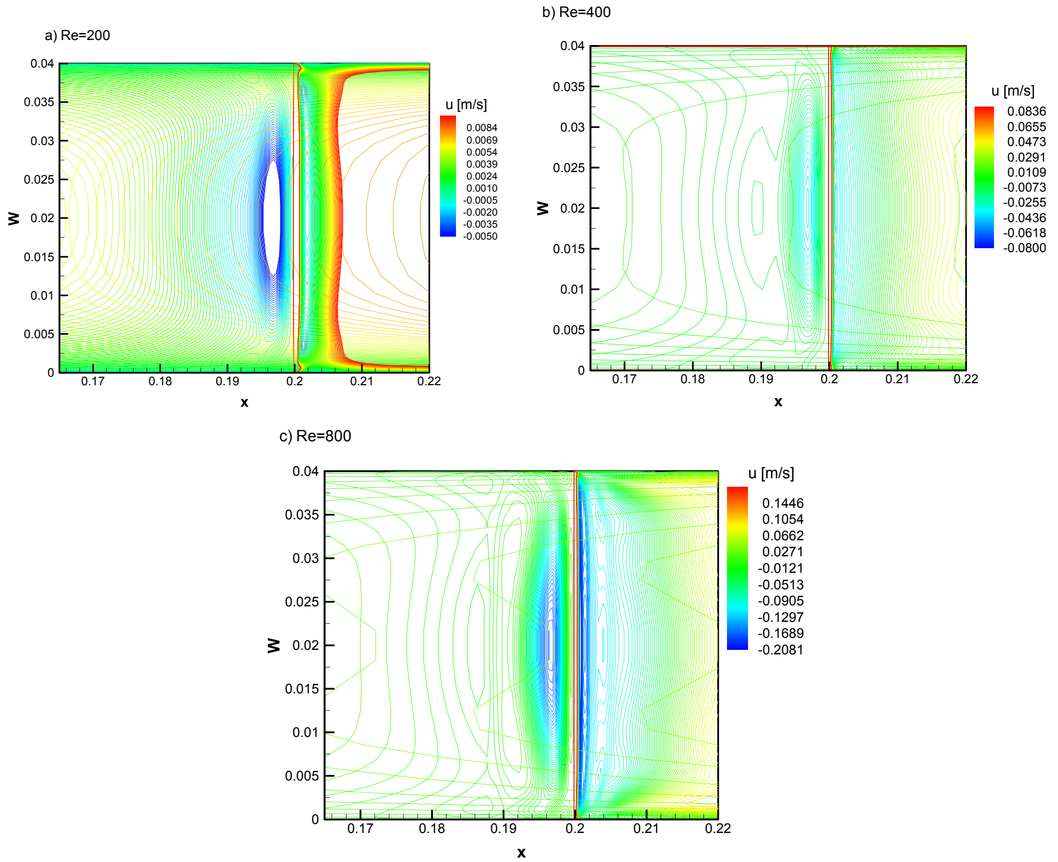

Figure 4: Re-circulation in a horizontal plane adjacent to the stepped wall a) Re=200, b) Re=400 c) Re=800

In order to have a detailed understanding of this phenomenon the wall shear stress averaged (τ) over the span-wise direction is plotted along the main flow direction (x) in fig. 5. As can be appreciated the wall shear stress presents a similar behavior for the three studies cases. At the channel inlet the flow was considerer as a fully developed flow and therefore the horizontal line in the plot. However at the proximity of the step (x/L=0.33) the lines present negative values for (τ) associated with the presence of the primary re-circulation zone. At the step edge the τ-lines present a discontinuity due to the abrupt change in the geometry. After the step edge, the values for the shear stress present high values (redeveloping zone) and then the values have a tendency towards an asymptotically value at the channel's exit.
A zoom for the zone at the vicinity of the step is also presented in fig. 5. Here it can be appreciated fluctuations from positive to negative values for Re=800. This behavior is not too apparent for Re=400 and definitely it does not appear for Re=200. The fluctuations just mentioned for Re=800 are associated with the presence of zones with positive values for the u-velocity component before the step as discussed earlier.

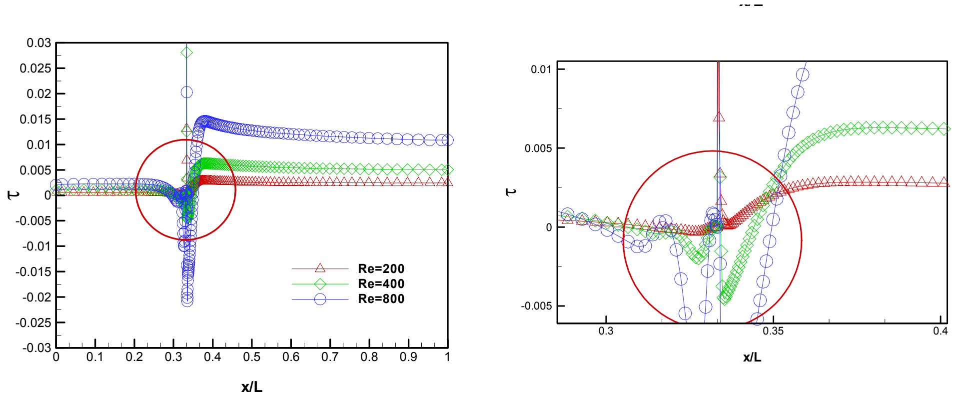

Figure 5: Average wall shear stress averaged over the span-wise direction

Figure 6 presents the stream traces along the central plane in the span-wise direction (z/W=0.5) for Re=200. Figure 6a) presents a zoom augmentation to detail the flow structures at the edge of the forward-facing step, while fig. 6b) details the corner at the bottom wall and the step. In both figures the re-circulation zone on the stepped wall and the re-circulation zone on the bottom wall are perfectly defined respectively, while fig 6c) is used to represent the flow structures at the upper wall and at the step corner and is also used to present the pressure contours in this zone.

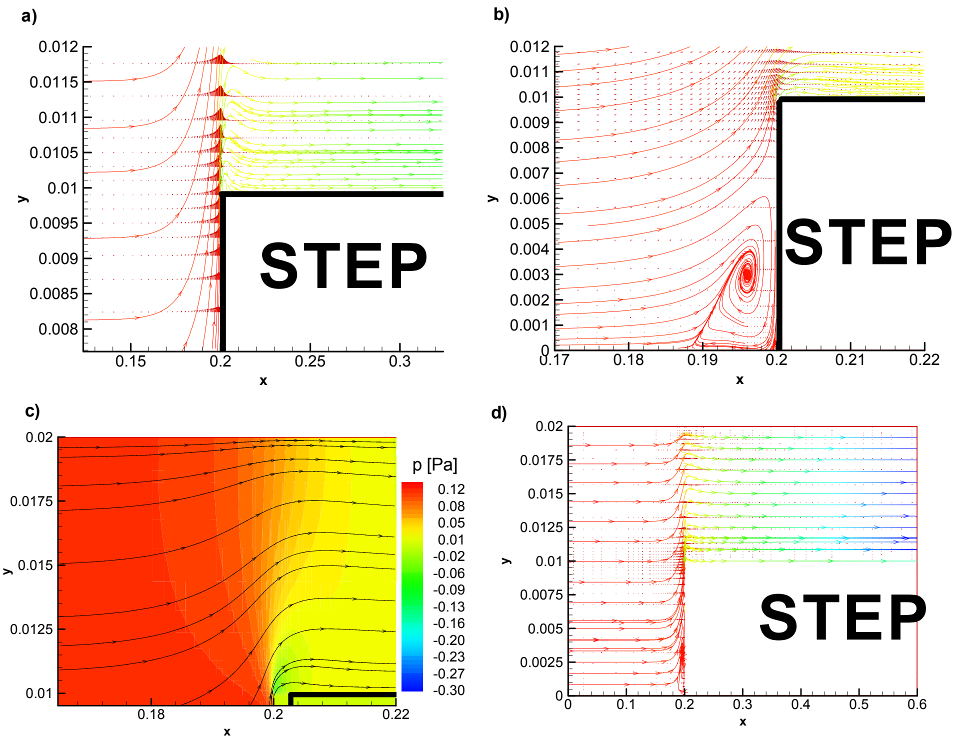

Figure 6: Stream traces and pressure contours for Re=200 at the central plane in the span-wise direction (z/W=0.5) a) step edge, b) step corner at the bottom wall, c) step edge and top wall, d) stream traces

The pressure contours (p) in the figure before mentioned manifest the adverse pressure gradient at the vicinity of the step-edge and then the flow separation in this zone. In fig. 6d) is observed that once the flow is reattached to the stepped wall it continues developing towards the channel exit.
Figure 7 presents the same flow structures as fig. 6 but the Reynolds parameter is Re=800. The flow structures for Re=800 not only present a more complicated vortexes inside of the re-circulation zones, but also reveals a larger size of these zones in the x as well as in the y coordinate direction (transverse direction/coordinate) . In fig. 7b) the existence of two vortexes inside the re-circulation zone adjacent to the bottom wall and step is found (x<0.2m). As mentioned earlier, the presence of adverse pressure gradients in the vicinity of the step becomes more evident here for Re=800 than for Re=200. The zone of low pressure at the edge of the step is associated with the flow separation at the stepped wall as presented in Fig. 7c).
Figure 7b) shows very clear the formation of the re-circulation zone adjacent to the step edge. A difference from fig. 6b), it is observed that for Re=800 the separation flow occurs closer to the step edge than for Re=200. In similar way, for Re=800 the re-circulation zone over the step is perfectly defined in fig 7c) and this effect is less well-defined for Re=200 in fig 6c).

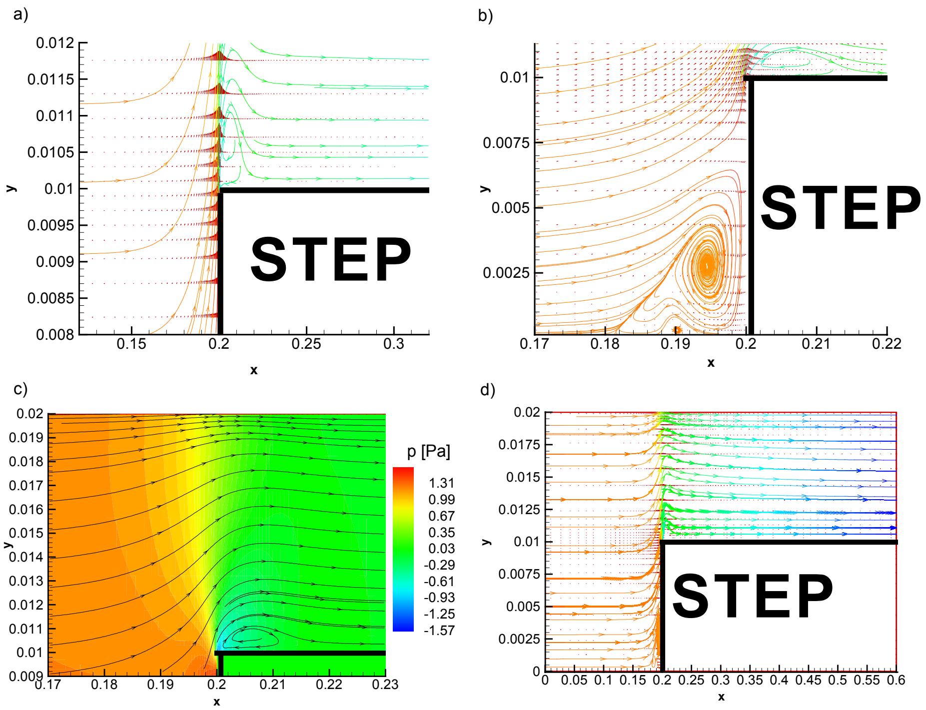

Figure 7: Stream traces and pressure contours for Re=800 at the central plane in the span-wise direction (z/W=0.5) a) step edge, b) step corner at the bottom wall, c) step edge and top wall, d) stream traces

The stream traces presented in figs. 6d) and 7d) for Re=200 and Re=800 respectively, shows that in both cases the flow structures experienced a kind of hydraulic jump after the step. This behavior is a particular feature for the flow passing an obstacle (forward-facing step). After this point, the stream traces show that the flow continues developing towards the channel exit to its three-dimensional fully developed conditions. The case for Re=400 is not presented in this work for question of space.
Finally to give more information of the flow structures some plots for the u-velocity at constant z- and x-planes are discussed.
In fig. 8, the u-velocity profile at the central plane in the span-wise for two different x-constant planes before the step are presented. The negative values for the u-velocity component is evidence about all discussed before referring to the re-circulation zone adjacent to the bottom wall before the step (Fig. 8a). In order to have more information a different plane is presented in fig 8b. Here the vertical axis of the figure is shortened and only the values near to the bottom wall are plotted. Close to the bottom wall for Re=800 a zone of positive values for the u-component is found. This particularity indicates that the re-circulation zone for Re=800 does not finish at this point but some point before, as mentioned earlier. Another implication is that the v-velocity component at this zone must have positive values in order to satisfy continuity. On the other hand, for Re=200 and Re=400 the trend for the u-velocity component is similar to that for Re=800.

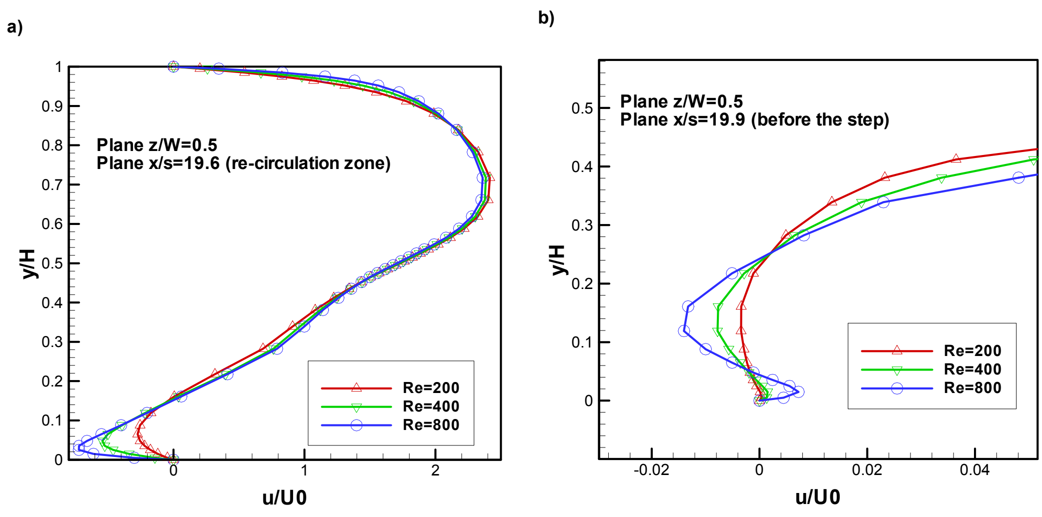

Figure 8: u-velocity profile at the central span-wise z/W=0.5 plane a) x/s=19.6 re-circulation zone b) x/s=19.9 before the step

Figure 9 shows the u-velocity profile for the central plane in the span-wise direction for a constant x-plane just passing the step edge. The u-component negative values for Re=800 and Re=400 indicate that the flow separation along the stepped wall start earlier than for Re=200. This effect was discussed before.

The u-velocity profile at the channel exit for the middle plane in the span-wise direction is presented in fig. 10. Here is evident that the channel is long enough to accommodate fully develop flow for Re=200 and Re=400. However for Re=800 there are slight differences from the fully developed flow conditions.

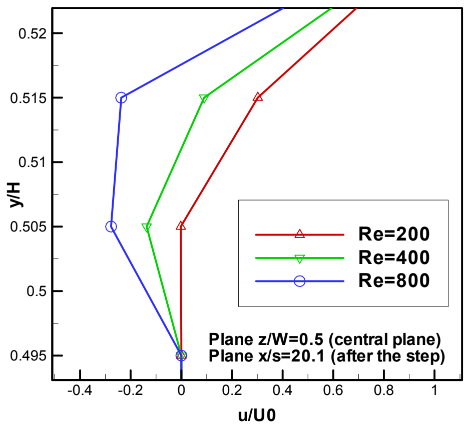

Figure 9: u-velocity profile at the central span-wise plane and x/s=20.1

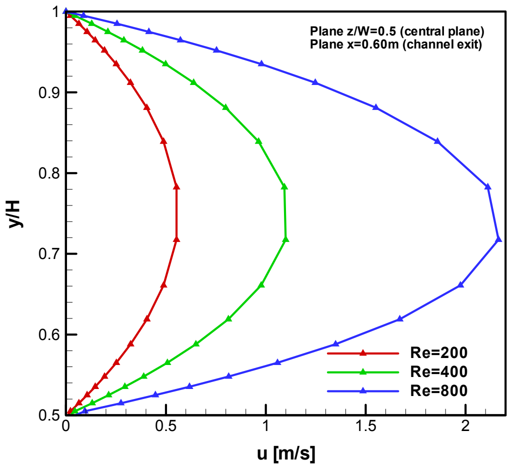

Figure 10: u-velocity profile at the central span-wise plane (z/W=0.5) and channel exit (x/L=1)

4. CONCLUSIONS

The numerical results for simulating airflow through a horizontal channel with a forward-facing step were presented for three different Reynolds parameters.

The flow structures showed that the flow is separated and reattached in two different regions. One before the step adjacent to the bottom wall and the other is developed adjacent to the stepped wall after the step edge. The size and location of these re-circulation zones depend on the Reynolds parameter. As Reynolds is increased, the re-circulation zones before and after the step increases their size. It is also observed that as Reynolds is increased the separation flow occurs at earlier positions in the main flow direction.

It was found that as the Reynolds is increased, more complex flow structures are found and then the flow is a strongly three-dimensional.

Even some results in this geometry were presented; it is necessary to continue a methodic study in order to characterize this important phenomenon.

REFERENCES

Abu-Mulaweh H.I., A review of research on laminar mixed convection flow over a backward- and forward-facing steps. *Int. J. of Thermal Sciences*, 42: 897-909, 2003.

Abu-Mulaweh, H.I., Armaly, B.F., and Chen, T.S., Measurements in buoyancy-opposing laminar flow over a vertical forward-facing step. *Int. J. Heat Mass Transfer*, 39: 1805–1813, 1996.

Abu-Mulaweh, H.I., Armaly, B.F., Chen, T.S., and Hong, B., Mixed convection adjacent to a vertical forward-facing step. *Proceedings of the 10th International Heat Transfer Conference*, 5: 423–428, 1994.

Abu-Mulaweh, H.I., Armaly, B.F., and Chen, T.S., Measurements of laminar mixed convection flow over a horizontal forward-facing step. *J. Thermophys Heat Transfer*, 7: 569–573, 1993.

Armaly, B. F., Li, A., & Nie, J. H., Measurements in three-dimensional laminar separated flow. *Int. J. Heat and Mass Transfer*, 46: 3573-3582, 2003.

Asseban, A., Lallemand, M., Saulnier, J.B., Fomin, N., Lavinskaja, E., Merzkirch, W., and Vitkin, D., Digital Speckle Photography and Speckle Tomography in Heat Transfer Studies. *Optics & Laser Technology*, 32: 583-592, 2000.

Barbosa Saldana J.G., *Numerical Simulation of Mixed Convection over a Three-Dimensional Horizontal Backward-Facing Step*, Texas A&M University Doctoral Dissertation, College Station, ISBN:05-4208-8487, 2005.

Barbosa-Saldana, J. G., Anand, N. K. & Sarin, V., Numerical Simulation of Mixed Convective Flow over a Three-Dimensional Horizontal Backward Facing Step. *Int. J. Heat Transfer*, 127:1027-1036, 2005a.

Barbosa-Saldana, J. G., Anand, N. K. & Sarin, V., Numerical simulation of forced convection over a three-dimensional horizontal backward-facing step. *Int. J. of Computational Methods in Engineering Science and Mechanics,* 6: 225-234, 2005b.

Blackwell, B.F., & Pepper, D.W., (eds). *Benchmark problems for heat transfer codes*, ASME-HTD-222: Anaheim, 1992.

Houde, H., Lu, J., and Bao, W., A Discrete Artificial Boundary Condition for Steady Incompressible Viscous Flows in a No-Slip Channel Using a Fast Iterative Method. *J. Computational Physics*, 114: 201-208, 1994.

Nie, J.H., and Armaly, B.F., Reattachment of three-dimensional flow adjacent to backward-facing step. *Int. J. of Heat Transfer*, 125: 422-428, 2003.

Patankar, S.V., *Numerical heat transfer and fluid flow*, Taylor and Francis, Philadelphia, 1980.

Ratish-Kumar, B.V., and Naidu, K.B., A streamline upwinding streamfunction-vorticity element analysis of Navier-Stokes equations. *Applied Numerical Mathematics*, 13: 335-344, 1993.

Ravindran, S. S., Control of flow separation over a forward-facing step by model reduction. *Computer methods in applied mechanics and engineering*, 191: 4599-4617, 2002.

Schlichting, H. & Gersten, K., *Boundary Layer Theory*, Springer, Berlin, 2000.

Shah, R. K., & London, A. L., *Laminar flow forced convection in ducts*, Academic Press, New York, 1978.

Stuer, H., Gyr, A., & Kinzelbach, W., Laminar separation on a forward facing step. *Eur. J. Mech. B/Fluids*, 18: 675-692, 1999.

White, F. M., *Mecanica de Fluidos*, McGraw Hill, Ciudad de Mexico, 1999

Williams, P.T., & Baker, A.J., Numerical simulations of laminar flow over a 3D Backward-facing step. *Int. J. Numerical Methods in Fluids*, 24: 1159-1183, 1997.