Asociación Argentina
de Mecánica Computacional

Mecánica Computacional Vol XXXIII, págs. 1783-1791 (artículo completo)
Graciela Bertolino, Mariano Cantero, Mario Storti y Federico Teruel (Eds.)
San Carlos de Bariloche, 23-26 Setiembre 2014

UNCERTAINTIES FORMULATED AS A CLASSIFICATION PROBLEM APPLIED TO CHAOTIC SYSTEM

Pettras L. B. dos Santos, Sandra A. Sandri, and Haroldo F. de Campos Velho

Instituto Nacional de Pesquisas Espaciais (INPE), Av. dos Astronautas 1758, 12227-010, São José dos Campos (SP), Brazil, [pettrasleonardo@yahoo.com.br, sandra.sandri@inpe.br, haroldo@lac.inpe.br], http://www.inpe.br

Keywords: Neuro-fuzzy sistems, chaotic dynamics, uncertainties formulation, classification.

Abstract. Uncertainty is one main concern on operational prediction systems, because the initial conditions are not precisely determine, model parameters are estimated with few information, and the phenomena itself is not completely formulated – for instance: in the numerical weather prediction, turbulence is not fully understood and/or described. A quantitative evaluation how good is the prediction can be called *predictability*. The ”bred vector” methodology can be applied to characterized classes of dynamics. Two neuro-fuzzy systems are employed as class dynamics classifiers: (a) ANFIS (**A**daptive-**N**etwork-based **F**uzzy **I**nference **S**ystem) based on Takagi-Sugeno's approach, (b) GUAJE (**G**enerating **U**nderstandable and **A**ccurate fuzzy models in a **J**ava **E**nvironment) based on Mamdami's scheme. The technique is applied to a chaotic system: three coupled waves in solar physics. A better classification performance is obtained using the ANFIS, but the automatic rules generated by the GAUJE are more easily interpretable.

1 INTRODUCTION

A design for a prediction dynamical system can be expressed as: an observation system to provide systematic data from a given dynamical system, and a time-evolution mathematical model – with its computational implementation. An operational prediction system is not a simple task, it depends on appropriated facilities and specialized personal, dealing with advanced computer systems (hardware and software). In addition, the science itself is under development.

Here, the focus is to evaluate the goodness of the prediction, or the *predictability*.

One approach is to do a set of executions for the forward model, the *ensemble prediction* (Kalnay, 2003). The ensemble prediction is used for the operational centers, as employed by the CPTEC-INPE (Center for Weather Prediction and Climate Studies, National Institute for Space Research, Brazil) – see Mendonça and Bonatti (2004). From the ensemble, some statistics properties can be computed, such as the confidence interval, identifying zones of low and high predictability. The ensemble prediction can also be described as a uncertainty quantification of the forecast.

A different approach is presented, where the predictability is formulated as a classification problem. The bredding method is used to provide inputs for the dynamics classification. The bred vector is computed perturbing the reference dynamics (control) and calculating the difference between the perturbed and control dynamics, after some time-steps. This methodology has been employed to the American National Center for Environmental Prediction (NCEP) to evaluate the prediction (Toth and Kalnay, 1997), and it also be used in the several chaotic systems (Evans et al, 2004; Pasini and Pelino, 2005; Cintra and Campos Velho, 2008). The bred vector magnitude can be employed to establish different conditions to classify the dynamics. Evans et al. (2004) have derived some rules for the dynamics of the Lorenz system, and similar rules have been identified to the three coupling waves system to the solar physics (Cintra and Campos Velho, 2008). These two former experiments show that bred vectors provide a good tool for the prediction of behavior of chaotic systems. However, the derived rules, as in the cited studies, were obtained after tedious analysis by experts.

The present work uses neuro-fuzzy systems to achieve two goals: as a classification tool, and secondly for automatic rule generation for dynamics. The experiments were carried out on the chaotic non-linear coupled three-waves model (Chian et al., 1994). Two neuro-fuzzy systems are employed: ANFIS (Adaptive-Network-Based Fuzzy Inference System (Jang, 1993)) based on Takagi-Sugeno's approach, and GUAJE (Generating Understandable and Accurate fuzzy models in a Java Environment (Alonso et al., 2008, Alonso and Magdalena, 2011)) based on Mamdami's scheme. ANFIS presented a better accuracy, but the rules derived by GAUJE are more easily interpretable.

2 BREDDING METHOD AND CHAOTIC SYSTEM

Chaotic systems are deterministic dynamical systems with at least one positive (real part) Lyapunov exponent, and they are extremely sensitive to initial conditions. This feature imposes an important issue: are the chaotic systems predictable? Since observations always contain errors, the initial condition, for example, will never be precisely determined. For practical point of view, a prediction for a chaotic system only can be feasible for a short period of time, computing new initial condition from new observations. Therefore, the evaluation of quality of the prediction is an important issue.

The breeding method was designed as a scheme to generate perturbations on initial condition for ensemble forecasting in prediction of the atmospheric dynamics (Toth and

Kalnay, 1997). In this method, the model is executed two times. Firstly, it is run with the original data (control), and a second execution with a small perturbation on the initial fields. The difference between these two executions, after a certain number of time-steps, is the bred vector(s). A measure of the flow instability could be computed from the growth rate of the bred vectors. Figure 1 shows bred vectors growth. Alliggod et al. (1996) and Kalnay (2003) have reported the studies on stability of flows using Lyapunov vectors and bred vectors.

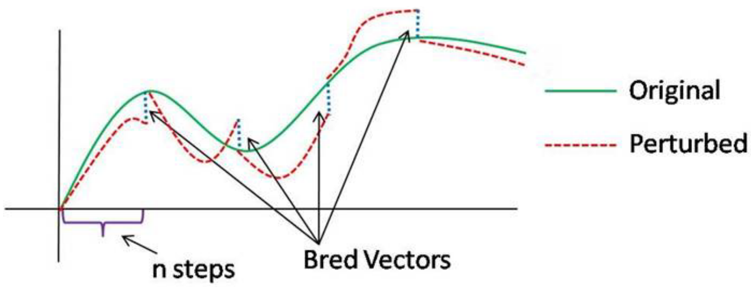

Figure 1: Bred vectors: systematic evaluations.

As mentioned above, the bred vector is obtained adding a small perturbation in the initial condition, computing the difference between perturbed and control runs, and normalization:

a) $\delta w_0 = w_0 + \Delta w$,

b) $\delta w_n = w(t_0 + n\Delta t) - \delta w(t_0 + n\Delta t)$,

c) $g_n = (1/n)\log\left[\|\delta w_n\| / \|\delta w_0\|\right]$.

2.1 Three coupled waves

Nonlinear three-waves coupling are of general interest in many branches of physics (Chian, 1994). Such coupling can explain the generation and modulation of plasma waves in the planetary magnetosphere and solar wind. A simple model for describing the temporal dynamics of resonant nonlinear coupling of three waves can be derived assuming monochromatic waves, where the time scale of the nonlinear interactions is much longer than the periods of the linear (uncoupled) waves. The electric fields are written in the form: $E_\alpha(x,t) = (1/2)A_\alpha(x,t)\exp\{i(k_\alpha x - \omega t)\}$, where $\alpha = 1,2,3$. However, for three-waves interactions to occur, the wave frequencies ω_α and wave vectors k_α must satisfy the resonant conditions

$$\omega_3 \approx \omega_1 - \omega_2 \ , \quad k_3 \approx k_1 - k_2 \ . \tag{4}$$

Under these circumstances, the nonlinear temporal dynamics of the system can be governed by the following set of three first-order differential equations expressed in terms of the complex slowly varying wave amplitude (Meunier et al., 1982)

$$dA_1(t)/d\tau = \nu_1 A_1 - A_2 A_3 \tag{5}$$

$$dA_2(t)/d\tau = i\delta A_2 - A_1 A_2 A_3^* \tag{6}$$

$$dA_3(t)/d\tau = \nu_1 A_1 A_3 A_2^* \tag{7}$$

where the variable $\tau = \chi t$, with χ is a characteristic frequency, $\delta = (\omega_1 - \omega_2 - \omega_3)/\chi$ is the normalized linear frequency mismatch, and the wave speed $\nu_\alpha = \hat{\nu}_\alpha/\chi$ gives the linear wave behaviors on the long time scale – parameter χ is a constant. In the experiments, wave A_1 is assumed to be linearly unstable ($\nu_1 > 0$) and the other two waves, A_2 and A_3, are linearly damped ($\nu_2 = \nu_3 = -\nu < 0$) (Chian et al., 1994, Lopes and Chian, 1996). The system admits both periodic and chaotic waves. For the chaotic dynamics, a strange attractor is found (see Figure 2). The coupled three-waves system has two *seasons* in the strange attractor. However, the seasons have no symmetry. One regime is identified as a line formed by a curve on XY plane followed by another curve on the YZ plane (Figure 2). Another regime is characterized by the straight line in the intersection between the XY and YZ planes.

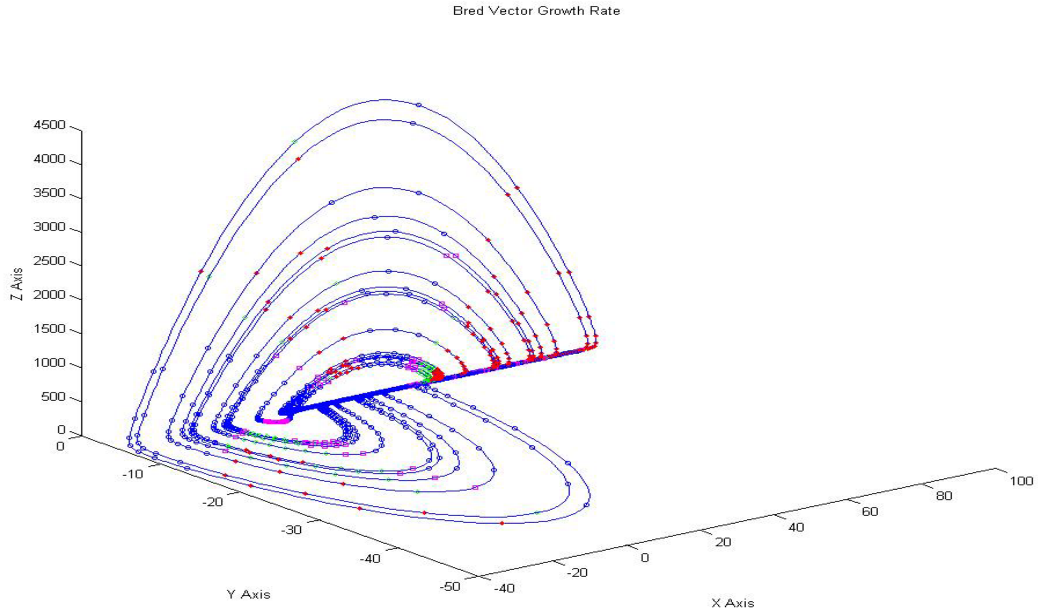

Figure 2: Three-waves model attractor colored with the bred vector magnitudes.

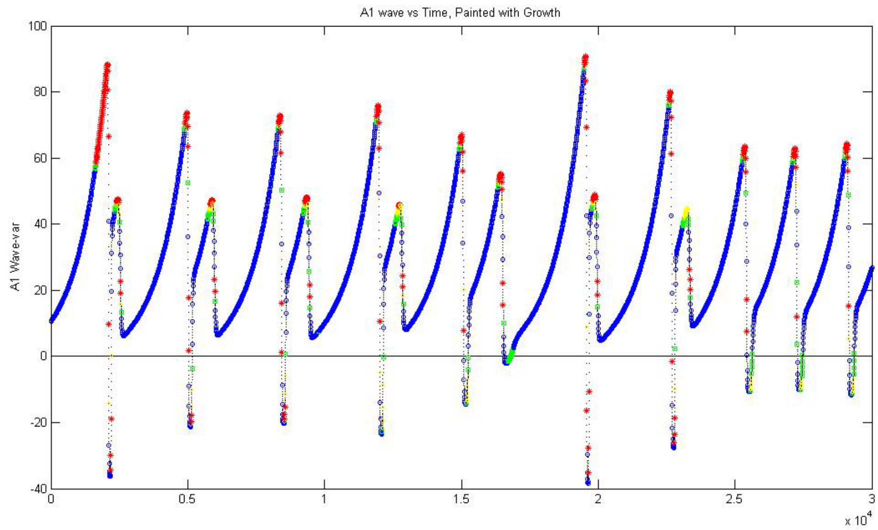

Figure 3: $A_1(t)$ for the three-waves model model.

Figure 3 shows the A_1(t) amplitude projection of the strange attractor. It is clearer to identify the regime changes after a red star appears. The red star means bred vector greater than 0.064, blue stars indicate negative growth rate, green and yellow stars the bred vector magnitude belongs to [0, 0.032] and [0.032, 0.064], respectively.

3 NETWORKS, FUZZY SYSTEMS AND NEURO-FUZZY SYSTEMS

Artificial Neural Networks (ANNs) (Haykin, 1999) were imagined to simulate the learning behavior of the human brain. The network is composed of processing units with weighted connection among them. The most cited method to compute the connection weights is the backpropagation method, where the adjustment is done to minimize the square difference between the target and the ANN output. Here, a neural network with a single hidden layer have been used, taken from the WEKA platform.

Fuzzy Systems (Gomide and Pedrycz, 1998) is focused to emulate some of the human capacity of reasoning with diffuse information. Different from the Boolean logic, the fuzzy logic is a many-valued logic, supporting the fuzzy set theory. Membership to a fuzzy set is a number in the real interval [0, 1] – leaking from the binary values (true or false). In general, systems based on fuzzy sets theory have rules of the type "if condition then conclusion" (IF-THEN rules), where the variables in the contexts of the condition and conclusion are elements of fuzzy sets. Probably, Sugeno and Mamdani models are the most well-known kinds of fuzzy systems. The Sugeno fuzzy systems have been applied to design control systems (Takagi, T., Sugueno, 1985). The Mamdami fuzzy systems has also been applied to control, but due to the their characteristics easier interpretation rules are obtained from these system, by comparing with Sugeno systems.

Neuro-fuzzy systems are models combining artificial neural networks features with fuzzy rule-based systems. A neuro-fuzzy system (Lin and Lee, 1996) is used to derive a fuzzy rule based system, whose defining parameters (fuzzy terms and rules) are identified in a learing phase, in a similar process used by neural networks, where a set of input-output pairs is employed. Neuro-fuzzy systems are models combining artificial neural networks features with fuzzy rule-based systems. A neuro-fuzzy system (Lin and Lee, 1996) is used to derive a fuzzy rule based system, whose defining parameters (fuzzy terms and rules) are identified in a learing phase, in a similar process used by neural networks, where a set of input-output pairs is employed.

Artificial neural networks are very effective to pattern recognition, but it is hard to explain how works the "black box" of the ANN. On the other hand, fuzzy rule based systems, after to create the inference (IF-THEN) rule, is much easier to understand how results are obtained. Neuro-fuzzy systems, using neural networks learning engine for creating the inference rules, entail to join both approaches.

Neuro-fuzzy systems are linked to their fuzzy system framework. Therefore, there are at least two types of such systems: derived from Sugeno and Mamdami approaches. The initial layers of Sugeno and Mamdami neuro-fuzzy systems are the same, but considering other layers the implementations are very different. The similar layers are commented bellow.

In the first layer, the current value of each input fuzzy variable is compared with the fuzzy terms associated with that variable, resulting in a compatibility degree for each term. In the second layer, the compatibility degrees from the different input variables are combined, resulting in the overall compatibility degree of the potential rules. In the last layers, a fuzzy set (respect. a set of constants) is learned for each rule in a Mamdami (respec. Sugeno) systems. The numerical experiments are performed with ANFIS (Shing and Jang, 1993), and GUAJE (Alonso at al., 2008) neuro-fuzzy computer codes. The configuration of the classifier designed by the ANFIS uses the backpropagation algorithm.

4 RESULTS

The ANN and the ANFIS experiments were produced using platforms WEKA and MATLAB, respectively. Training with ANFIS was performed considering 3 triangular fuzzy terms for each input variable and constant output, with 300 epochs. The Multilayer Perceptron (MLP) with 3 layers (input layer, one hidden layer, and output layer) was chosen as the ANN architecture. The ANNs used the following configuration: learning rate = .3, momentum = .2, epochs = 500. The number of neurons in the hidden layer was determined by the formula:

$$n_{HL} = \frac{1}{2}(n_{IN} + n_C) \tag{8}$$

where n_{HL} is the neurons in the hidden layer, and n_{IC}, n_C are the number of inputs (= 4) and classes, respectively.

The main focus for the GUAJE system is to design a easier interpretable Fuzzy Rule-Based Classifiers (FRBS) from the Mamdami's approach. The package is able to combine knowledge from the expert information and/or from the data. For many applications, the expert information is hard to obtain or difficult to express in a formal way. Considering N_C classes $C = \{C^1, C^2, C^3, \ldots, C^{Nc}\}$, a FRBC is a fuzzy system with capacity to select one class from a pre-defined NC classes. For an input vector ($x^p \in R^n$), it is possible to determine an activation degree for the vector x^p associated to each class C^i by using a fuzzy inference. Clearly, the input vector can have a membership degree different of zero for more than one class, but the output class C^i is derived from the highest $\mu_{C^i}(x^p)$: membership degree of x^p to the class C^i. The GAUJE system uses a Highly Interpretable Linguistic Knowledge (HILK) engine, where the fuzzy classification is based on the Max-Min inference scheme with the winner rule fuzzy reasoning tool. The whole process is made up of four main steps:

1. **Selection**: Identify the most discriminative variables.
2. **Partition**: includes partition learning (automatic generation of fuzzy partitions from data) and partition selection.
3. **Rule base learning**: linguistic rules are automatically extracted from the data.
4. **Knowledge base improvement** : iterative refinement process for partitions and rules.

The goal here is to identify with class of dynamics the chaotic system is going to drop. This is an alternative to compute the confidence interval, where several executions should be performed for calculating some statistical properties.

In the three-waves experiment, we have used 170 samples, divided into training (100), validation (30) and test (40). The input is the number of bred vectors in each class (color) found in the preceding straight line regime.

In the two-classes experiment, the classes description are given as:

A: the straight line trajectory will last up to 1200 time steps;

B: the straight line trajectory will last more than 1200 time steps.

In the multi-classes experiment, we have used 6 classes, numbered from 0 to 9. Each class is associated to the interval bounding the number of time Each class is associated to the interval bounding the number of time steps in which the straight line trajectory is predicted to last: Class-0: [0, 1200], Class-1: [1201, 1600], Class-2: [1601, 2000], Class-3: [2001, 2400], Class-4: [2401, 2800], Class-5: > 2800.

Confusion matrices were computed by the use of ANN, ANFIS, and GAUJE. The classification performance is shown in Table 1.

Table 1: Classification performance for 2 and 5 classes.

Precision	2 classes	5 classes
ANN	92,5%	75.0%
ANFIS	87,0%	77.5%
GUAJE	87,5%	70.0%

The main focus for the GUAJE system is to design a easier interpretable Fuzzy Rule-Based Classifiers (FRBS) from the Mandami's approach. The package is able to combine knowledge from the expert information and/or from the data. For many applications, the expert information is hard to obtain or difficult to express in

Rules

Rule	Type	Active	If Vermelhas	AND Amarelas	AND Verdes	AND Azuis	THEN Saida
1	I	yes	average	average	low	low	Classe 3
2	I	yes	low	low	high	low	Classe 12
3	I	yes	average	average	average	low	Classe 7
4	I	yes	low	average	average	low	Classe 8
5	I	yes	low	average	low	low	Classe 5
6	I	yes	average	high	low	low	Classe 0
7	I	yes	average	average	low	average	Classe 3
8	I	yes	low	low	average	low	Classe 10
9	I	yes	low	high	low	low	Classe 1
10	I	yes	low	average	high	low	Classe 13
11	I	yes	average	average	average	average	Classe 5
12	I	yes	low	average	low	average	Classe 3
13	I	yes	average	high	low	average	Classe 2
14	I	yes	average	low	high	low	Classe 12
15	I	yes	average	high	average	low	Classe 4
16	I	yes	low	high	low	average	Classe 3
17	I	yes	average	average	high	low	Classe 12
18	I	yes	low	high	average	low	Classe 5
19	I	yes	high	high	low	low	Classe 0
20	I	yes	high	average	low	low	Classe 0

Figure 4: Automatic rules generated by GUAJE/

Figur3 4 shows the automatic rules (25) generated by the GUAJE system. Several rules can be grouped to become even easier interpretable system.

5 CONCLUSIONS

We have investigated the ability of ANFIS and GAUJE neuro-fuzzy systems to evaluate the predictability of chaotic system. This task is formulated as a classification problem, where classes of dynamics are identified. Experiments were performed on the coupled three-waves model from solar physics. Our results were compared with a standard artificial neural network (multilayer perceptron). The results obtained so far show that neuro-fuzzy systems of the Sugeno type (ANFIS) are useful for the prediction of chaotic systems. The use of other systems (neuro-fuzzy and otherwise) is under investigation, with the potential for producing fuzzy systems by learning, in particular those based in the Mamdami paradigm. The ultimate goal is to use the derived fuzzy systems as a basis for the automatic production of interpretable rules, such as those created by observation for the three-waves system.

As already mentioned, the dynamics identification formulates as a classification problem is an alternative to evaluate the predictability – another approach for predictability quantification is to use statistical analysis for computing the confidence interval. Applying the neuro-fuzzy formulation as classification tool, we can also derive automatic rules. Such rules can be employed for forecasters in the practical operations as an auxiliary decision tool.

ACKNOWLEDGMENT

The authors thank FAPESP and CNPq, Brazilians agencies for research support.

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