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HEAT TRANSFER COEFFICIENT DETERMINATION OF QUENCHING PROCESS

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Abstract. Quenching is a widely-known industrial process where a piece at very high temperature is rapidly cooled. Internal modifications of the material and its properties occur during it. The analysis of quenching is relevant in order to characterize the process and, therefore, to tailor the final properties of the treated pieces. Thermal evolution will dictate the type of metallurgical transformations developed and the level of residual stresses and geometrical distortions that the piece will suffer.

Quenching tests are usually performed by instrumenting a probe and quenching it under controlled conditions. As a result, records of temperature evolution inside the probe are obtained. The determination of the total heat transfer coefficient during the quenching test is an ill-posed problem which is usually analyzed through inverse techniques.

In this work, heat transfer coefficient (HTC) is numerically determined for a set of quenching tests. This novel method comprises the use of analytical solutions of a similar problem, direct numerical resolution of the thermal evolution and an iterative algorithm of correction. The conditions of the tests were varied in order to assess the effect of different quenching conditions on the HTC. The physical modeling of different boiling mechanisms is briefly discussed.

1 INTRODUCTION

Quenching is an industrial process where a piece at high temperature is rapidly cooled. This process is mainly applied to heat treatment of metal components. The fast cooling produces the transformation of the material to metaestable phases or microstructures. The mechanical properties of the material are modified as a consequence of the metallurgical transformations. In addition to the modification of properties, residual stresses and geometrical distortions are induced into the piece. The modifications produced during the process depends on the rate of cooling at each point of the material. Therefore, the heat transfered during quenching will define the final properties of the piece. Due to this relationship between heat transfer and final properties, the prediction and modification of heat transfer during quenching is so relevant from the process engineering point of view [Totten (2007); Gür and Pan (2009)]

Accurate determination of total heat transfer coefficient (HTC) during quenching is even nowadays a challenging task for applied scientists and engineers. HTC determination is an inverse problem because the value of it is defined on the surface of the piece and has to be determined based on information taken inside of it. Different approaches have been applied to the resolution of this problem [Buczek and Telejko (2013); Felde and Réti (2010); Xiong (2010); Huiping et al. (2006)], from complex assumptions about the the HTC function up to the use of global optimization techniques to reproduce the temperature evolution. Disregarding the technique used to determine the HTC, it is important to note that the obtained results have to comply with at least the following requirements:

- The obtained HTC have to be accurate enough to reproduce the thermal history that was used to determine it when the direct problem is solved. A high level of detail in the HTC curve is needed in order to reproduce the different heat transfer mechanisms and their transitions.
- The HTC have to be free of numerical artifacts that can lead to unphysical behavior, such as constant phase oscillations or non-monotonicity on wall temperature.

In most of the cases, three different stages can be identified during quenching (from high to low wall temperature) (see Fig. 1):

1. Film Boiling: The piece is completely covered by a vapor blanket that isolates its surface from the quenching media. Heat transfer rates are moderate and a low net injection of vapor to liquid media occurs.
2. Nucleate Boiling: At this stage, the liquid is in contact with the wall of the piece at high temperature. An enhanced exchange of mass (vapor/liquid) and heat takes place during it. High heat transfer rates are observed an a high amount of vapor is injected into the quenching bath. As wall temperature drops, boiling intensity decreases.
3. Single Phase Convection: Once the wall temperature is within the range of saturation temperature of the liquid, boiling ceases and heat transfer is performed by the well-known single phase convection mechanism.

These mechanisms were separately studied in detail in [Kocamustafaogullari and Ishii (1983, 1995); Tu and Yeoh (2002); Yeoh and Tu (2004, 2006); Bromley et al. (1953)]. Due to the variety on behavior of each ones, it is fundamental to identify their combined contribution to heat transfer in order to accurately describe the quenching process and its effects.

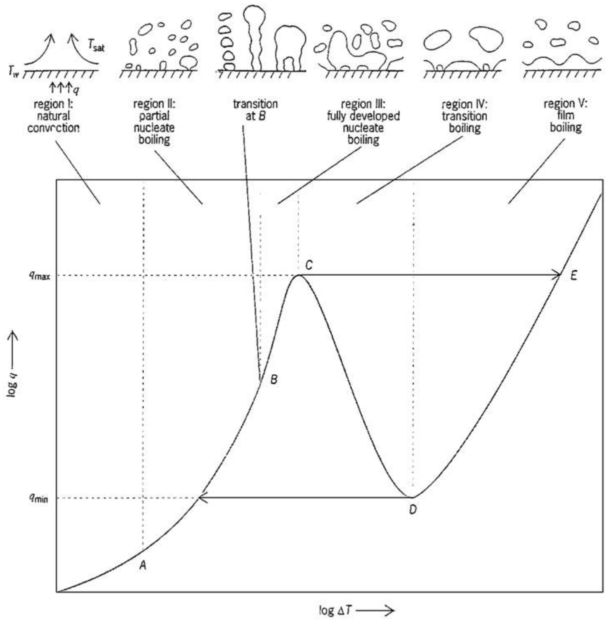

Figure 1: Sketch of wall heat flux during boiling. Extracted from Dhir (1998)

2 HTC DETERMINATION METHOD

The method to determine the HTC presented here is based on the iterative resolution of the direct problem of cooling of a probe subjected to different $h(t)$ functions. After each iteration, a correction function is applied to $h(t)$ in order to improve the reproduction of the experimental values. The method has two main blocks: the experimental data conditioning and generation of initial iterator for $h(t)$, and the iterative corrector method to reach HTC within an specified tolerance. Along this work we will refer to HTC as the (T_w, h) curve, while $h(t)$ is the time-dependent function applied in the boundary condition of the thermal problem.

The test probe and experimental conditions used in this work are based on the ISO 9950, ASTM D 6200-01 and ASTM D 6482-06 standards. That is to say, the analysis of a slim cylinder oriented parallel to the fluid flow direction is considered. The cylinder is slender enough to assume an infinite long geometry for the events that occur at half length of it. This assumption implies that the $h(t)$ is the same along the whole surface of the probe. Once the heat transfer coefficient is correlated to wall temperature, this simplification is swept.

The case presented in this work can be analyzed as a 1-D axisymmetric problem (see Eq. 1).

$$\begin{aligned} \rho_s c_{p,s}\frac{\partial T}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left(r k_s \frac{\partial T}{\partial r}\right) &= 0, \qquad T(r,t) \in (0,R)\times(0,t_f) \\ T(r,0) &= T_s, \text{ at } t=0 \qquad \forall\, r \\ \frac{\partial T}{\partial r} &= 0, \text{ at } r=0 \qquad \forall\, t \\ -k_s\frac{\partial T}{\partial r} &= h(t)\left(T(R,t)-T_l\right), \text{ at } r=R \qquad \forall\, t \end{aligned} \tag{1}$$

The inverse problem arises when based on records of temperature $T(0,t)$ in the center of the probe, an $h(t)$ function has to be determined at the surface of it. In this work, the adequateness of the $h(t)$ function is defined on the basis of its ability to reproduce the experimental cooling velocity curve $(t, dT/dt)$ when applied into problem in Eq. 1. In addition to the ability to reproduce the cooling curve, a high level of smoothness is required, so solutions with oscillations of high amplitude are *a posteriori* discarded.

2.1 Initial approximation for $h(t)$

The problem presented in Eq. 1 has analytical solution for the particular case of $h(t) = h$ constant. This solution is presented in Eq. 2 for a constant temperature, T_s, as initial condition. Based on this case and the experimental results, the initial iterator, h_0, used to determine $h(t)$ is obtained.

$$T(r,t) = T_l + \frac{2}{R^2}\sum_{n=1}^{\infty}\exp\left(-\alpha\beta_n^2 t\right)\frac{R\beta_n^2 J_0\left(\beta_n r\right) J_1\left(\beta_n R\right)}{\left(\beta_n^2 + \left(\frac{h}{k_s}\right)^2\right) J_0\left(\beta_n R\right)\beta_n}\left(T_s - T_l\right) \tag{2}$$

The β_n values are the solutions of the transcendental equation (see Eq. 3) and α is the solid thermal diffusivity.

$$\beta R J_1(\beta R) - \frac{hR}{k} J_0(\beta R) = 0 \tag{3}$$

To obtain the initial approximation of $h(t)$ only the mode in Eq. 2 is considered to describe $T(r,t)$. The time derivative of this expression takes the form presented in Eq. 4. From this equation, the value of β_1 can be obtained from experimental records of T and dT/dt. Once, $\beta_1(t)$ is determined, this value is plugged into Eq. 3 in order to obtain the initial iterator, h_0. This approximation for h is calculated for each time step, giving the corresponding curve $h_0(t)$ of the test (see blue line in figure 2). Even though this first approximation is a very good one and can capture the different boiling mechanisms, it is not accurate enough for our purposes. If a detailed characterization of heat transfer mechanisms, and correlation to physical properties of quench bath and process parameters is pursued, a much more accurate determination of $h(t)$ should be obtained, i.e. the $h(t)$ function should be able to reproduce the experimental data with a small total error. In this method, the desired $h(t)$ is obtained through successive correction of it at each iteration.

$$\frac{dT}{dt} = -\alpha\beta_1^2\left(T(r,t) - T_\infty\right) \tag{4}$$

$$h_0 = \frac{\beta_1 J_1\left(\beta_1 R\right) k}{J_0\left(\beta_1 R\right)} \tag{5}$$

The initial $h(t)$ iterator (h_0) is obtained from the combination of (t, T) and $(t, dT/dt)$ curves, and, as it will be presented in next section, the $(t, dT/dt)$ curve will also be used as target on the HTC determination. This curve plays an important role along the whole method, therefore, an adequate computation of it is fundamental. The experimental (t, T) is usually recorded at a frequency sampling in the order of 100 Hz. This velocity of sampling, in combination with the inherent error of measurement produces a signal with noise. The level of noise in this type of experimental records prevents the use of simple numerical derivations schemes to compute dT/dt. Signal conditioning and filtering are most usual techniques applied to obtain smooth $(t, dT/dt)$ curves [Felde and Totten (2012); Felde and Réti (2010)]. But, the misuse of filters or poor signal conditioning has the inconvenient of information loss or the addition of spurious numerical artifacts, such as the results presented in [Ferguson and MacKenzie (2012); Maniruzzaman et al. (2012)]. In this work, we use the unfiltered (t, T) signal. Local low order polynomials were fitted to the experimental data (usually using a 51 to 101 points stencil) and the derivative was obtained from the fitted curve. If necessary, a second fit of similar nature was performed on the derivative curve. The selection of the order and stencil of the interpolating polynomial are selected in a way to maximize the noise reduction and reduce the information loss from the experimental records.

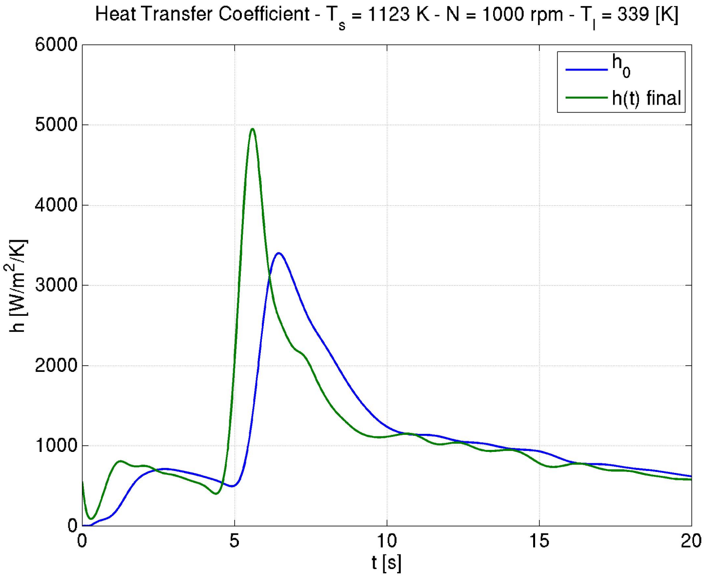

Figure 2: Example of initial and optimal h(t) functions

2.2 Iterative correction method

The iterative correction process of $h(t)$ is based on the direct resolution of the thermal problem (see Eq. 1) until a desired error on the $(t, dT/dt)$ curve is attained. $h(t)$ is used in the flux boundary condition on the resolution of the cooling of an infinite long cylinder of radius R at initial constant temperature T_s. This problem is solved using a commercial Finite Element Method software (Comsol Multiphysics V3.5a). The numerical $(t, dT/dt)$ curve is obtained and compared against the experimental one. At each iteration, the relative error between numerical and experimental results is obtained according to Eq. 6. This relative error function contains the information when the applied $h(t)$ is higher or lower than the experimental one, therefore, it can be used to correct it. After each direct numerical solution of the problem, the corrected $h(t)$ will be the one presented in Eq. 7, where $t*$ is a displaced timescale. The displacement on $t*$ has to be taken into account due to the inherent thermal resistance of the probe. An error of cooling velocity in the center of the probe at time t is due to an erroneous h applied at the surface at time $t - \delta t$. The shift in timescale can be obtained from the definition of Fourier Number *Fo* (see Eq. 8).

$$e_r(t) = \frac{(dT/dt)_{exp} - (dT/dt)_{num}}{(dT/dt)_{exp}} \tag{6}$$

$$h_k(t) = h_{k-1}(t)\,(1 + e_r(t*)) \tag{7}$$

$$\delta t(t) = \frac{Fo\,R^2}{\alpha(t)} \tag{8}$$

The iterative correction process is concluded when the norm of the $e_r(t)$ vector reaches a given tolerance. An adequate selection of the $h_0(t)$ initial guess and *Fo* number can lead to the determination of $h(t)$ with a total relative error lower than 10^{-4} to the experimental cooling rate curve (see figure 3) in few iterations (usually 15 or less). Once the $h(t)$ produces a cooling history that complies with the given the tolerance, the HTC curve is constructed from the numerical results of wall temperature, T_w (see figure 4).

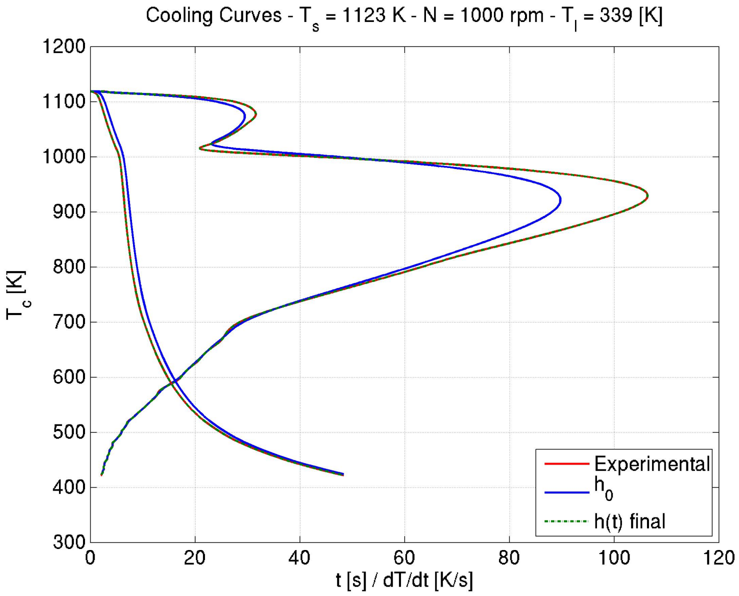

Figure 3: Example of experimental and numerical cooling curve for h_0 and optimal $h(t)$

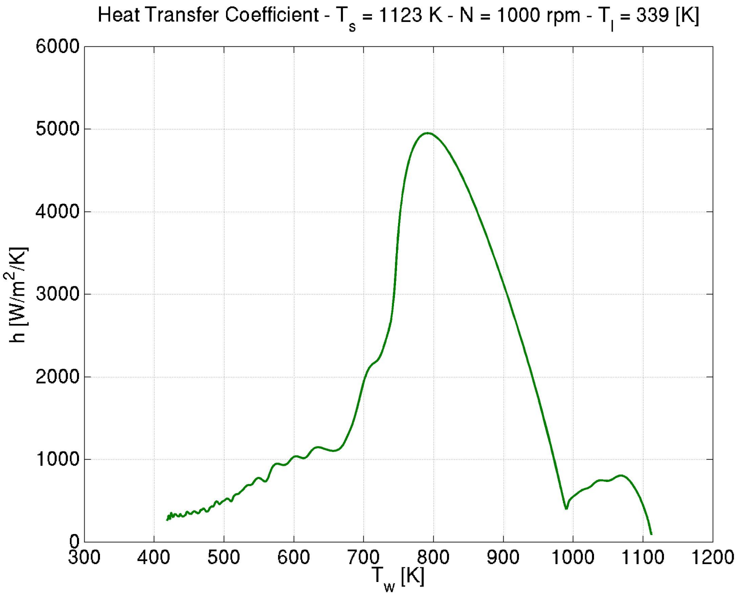

Figure 4: Example of numerical HTC obtained

The results of heat transfer coefficients obtained using this method can be used to characterize in detail boiling mechanisms during quenching process. Transition between boiling mechanisms and the effect of flow conditions on HTC can be assessed with higher accuracy compared to other available methods [Felde and Réti (2010); Xiong (2010); Huiping et al. (2006)].

Although the selection of the order of polynomial and stencil length in the calculation of dT/dt was made in order to produce a smooth curve that maintains all the details of the process, a small amount of low amplitude ripples can still be found in our results (see Fig. 2 & 4). These ripples are related to the polynomial fit and up to this level are deemed acceptable for the purpose of this work.

3 EXPERIMENTAL CONDITIONS

A factorial set of tests was performed using the data acquisition device Ivf SmartQuench® designed by Swerea IVF AB[1]. The experimental setup involves the complete immersion of the selected test probe in the quenching bath. The probe was previously heated to a initial standard temperature in an electrical resistance furnace. Time-temperature records during the test are measured at the center of probe body using a type K thermocouple (NiCr/NiAl, 1.5mm diameter). The stored data can be then evaluated and displayed as cooling curves, diagrams and tables. The quenching media was oil Q8 Bellini FNT® [2], a fast quenching oil with high thermal stability and low volatility. Its limited sludge formation during heat treatments ensures insignificant deviations in the results from normal performance, as well as a long service life of test probe. The cylinder-shape probe was 400mm overall length, 12.5mm diameter, with a mass of 240 g and made of Inconel 600, a nickel-chromium alloy with good physical properties (high corrosion and oxidation resistance at high temperatures) and no phase change in the analyzed interval. The test apparatus is the commercially available Tensi agitation assembly designed according to the standard ASTM D 6482-06 and ISO 9950. The quenching chamber has a tubular opening where the heated probe must be inserted. An electric motor connected to a standardized impeller (50mm dia., 3 blades, 42mm pitch setting) allows the agitation of the quenching medium. The original transparent PVC test chamber was replaced by a stainless steel device. This change was made in order to perform tests in oil at high temperature. The dimensions were preserved: 125 x 60 mm wide, 205 mm height, volume of 1.5 liters and with a mass of 7.6 kg (including motor). (see Fig. 5).

The variables under control were: initial temperature of the probe (T_s), rpm of the impeller (N) and oil temperature (T_l). The states of each variable are summarized in Table 1. A total of 48 different conditions were tested based on the variables and states considered. It is important to note that mean oil velocity ($v_{l,\infty}$) is not controlled with this setup. Each combination of T_l and N will produce a different $v_{l,\infty}$, and it is supposed that $v_{l,\infty}$ doesn't depend on T_s. Therefore it is assumed that each combination (N,T_l) corresponds to one $v_{l,\infty}$ that has to be indirectly determined.

T_s [K]	N [rpm]	T_l [K]
1023	0	306
1073	1000	339
1123	2000	373
1623	3000	–

Table 1: Variables and states analyzed

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[1] http://swerea.se/en/Start2/

[2] http://www.q8oils.com/

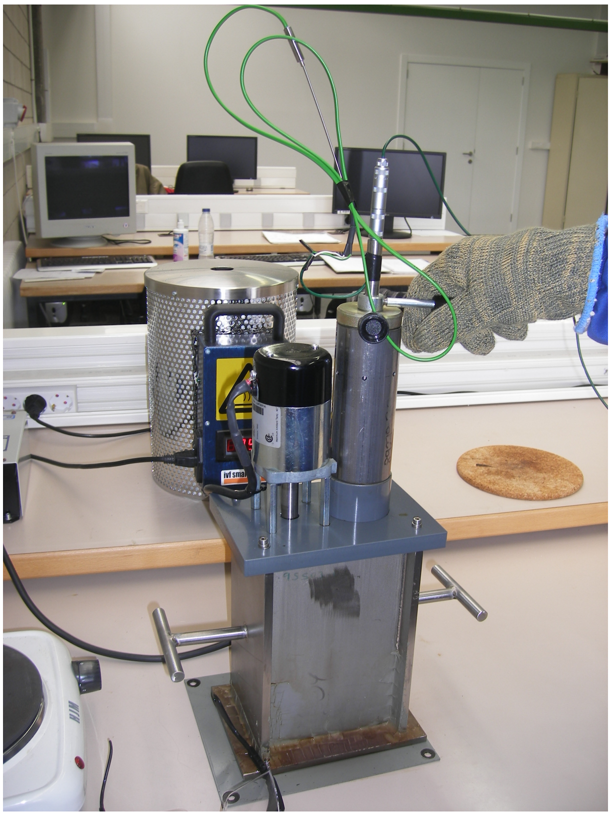

Figure 5: Test apparatus

Tests were performed according the the following procedure:

- Set the oven temperature at +5K of T_s. Set T_l and agitation velocity.
- Place the probe into the oven. Record the temperature including the last stage of heating and verify that the probe and oven had reached thermal balance.
- while still recording the temperature, place rapidly the probe into the quenching chamber. Record the temperature up to maximum acquisition time is attained.

The built-in filter provided by IVF was disabled during the test. The data analysis is performed using the *.ivf file were the raw values of temperature acquired during the test are recorded. The data analysis had to be done in this way in order to avoid numerical artifacts introduced by the IVF filter based on Fourier Analysis [Felde and Réti (2010)].

4 RESULTS

In order to analyze the different heat transfer mechanisms developed during quenching, the study of the effect of flow conditions is performed based on the total heat flux released by the piece ($q_w = h(T_w - T_l)$). This parameter is more convenient to assess the combined contribution of different stages during quenching. Variation of HTC depending on test conditions is presented in next section. Finally, a general discussion of heat transfer mechanisms is developed based on the obtained results.

4.1 Effect of flow conditions on HTC

HTC curves for different agitation and T_s values at each T_l are presented in Fig. 6. For all the T_l temperatures considered, three mechanisms of heat transfer are clearly identified: film boiling up to $1000K$ approximately, nucleate boiling from $1000K$ up to $650K$ approx. and finally single convection heat transfer. From the results presented in Figs. 6 & 7 it can be extracted the following remarks:

- HTC curves obtained from different T_s tend to collapse into a single one.
- HTC curves present less scattering at low T_l.
- The film boiling stage is independent to $v_{l,\infty}$, except for high T_l and high $v_{l,\infty}$. It also presents a very slight dependence on T_l.
- The nucleate boiling mechanism is independent to the variables analyzed.
- The single convection heat transfer stage is rather independent to N at low to medium T_l. At high T_l, the effect of flow velocity is clearly identified.

Based on these results, it can be observed that given a flow velocity ($v_{l,\infty}$, as a result of combination of T_l and N), HTC is only dependent on T_w for the studied geometrical configuration. The total heat transfered is the result of three clearly defined heat exchange mechanisms: film boiling, nucleate boiling and single phase convection. Due to the physical nature of each mechanism, the transition from film boiling to nucleate is presented as an abrupt change in HTC curve. This transition is smoothed at high T_l and flow velocity, this effect is probably due to a change on film boiling mechanism at that condition. The transition from nucleate to single phase convection is smooth, therefore the identification of the transition point, or region, between mechanisms is a non trivial task.

The film boiling mechanism (see Fig. 7, right column) is totally independent of $v_{l,\infty}$ for T_l = 306 & 339 K, showing a slight increase of heat transfered as T_l increases. The behavior at maximum flow velocity (maximum T_l & N) is different. This change may be attributed to change of mechanism dependent on Re number of the flow. Further analysis on this subject is needed.

The nucleate boiling mechanism (see Fig. 6) appears totally independent on the analyzed test variables. Some scattering on their results may be due to inherited effects of the transition from film boiling. This mechanism seems to only depend on wall temperature, T_w and thermo-physical properties of quenching media.

Single phase convection (see Fig. 7, left column) shows a moderate dependence on $v_{l,\infty}$ for T_l = 306 & 339 K, while for 373 K, the effect of flow velocity can be clearly distinguished. For this mechanism, as it is widely known in elementary heat transfer, the higher the stream velocity, the higher the heat transfer. For oils, the effect of variation of Pr number has to be taken into account when enhancement of heat transfer occurs at high T_l. The small amplitude ripples observed in this work are attributed to the polynomial approximation of dT/dt. Up to this moment are deemed acceptable for the evaluation of the optimization method and further work is being carried out in order to reduce, or totally eradicate them.

4.2 Heat flux partition model

After the study of dependences of heat transfer on flow conditions, a general view about the distribution of mechanisms is presented. Further analysis on the functionalities of each

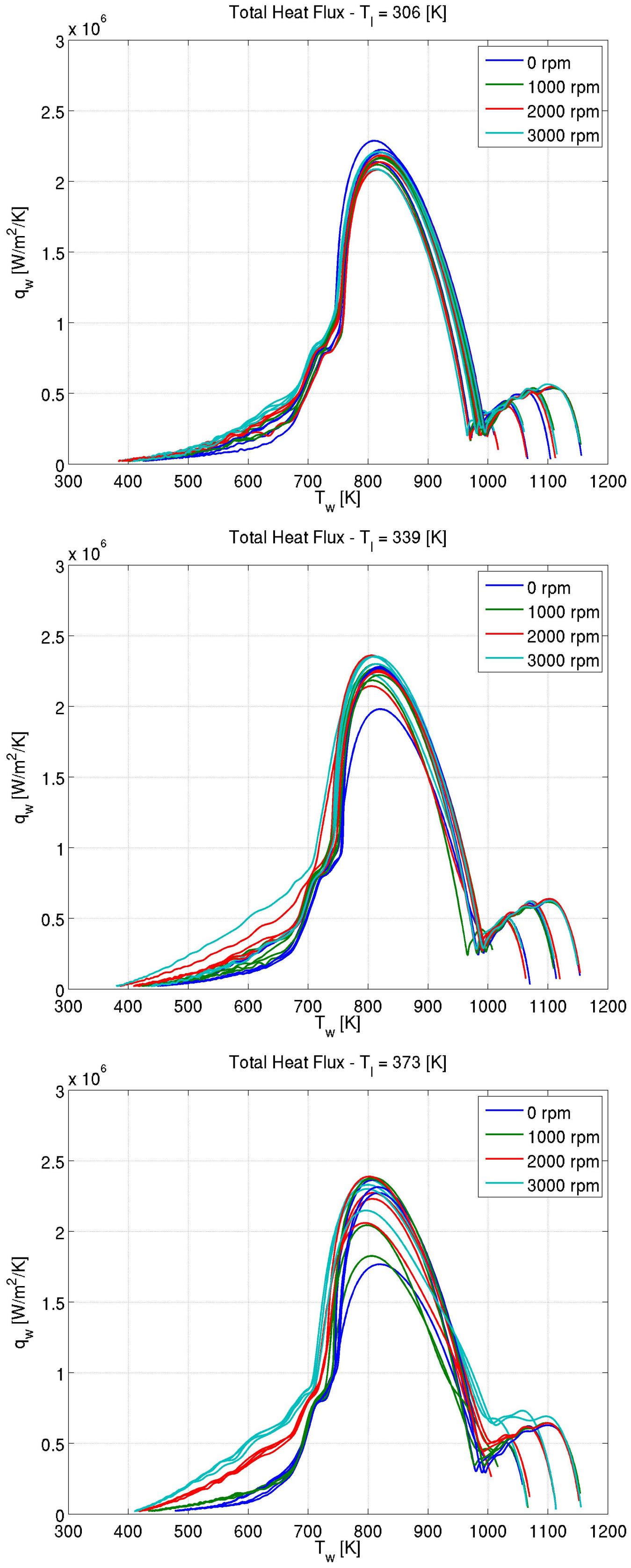

Figure 6: Total Wall Heat Flux at different T_l and rpm

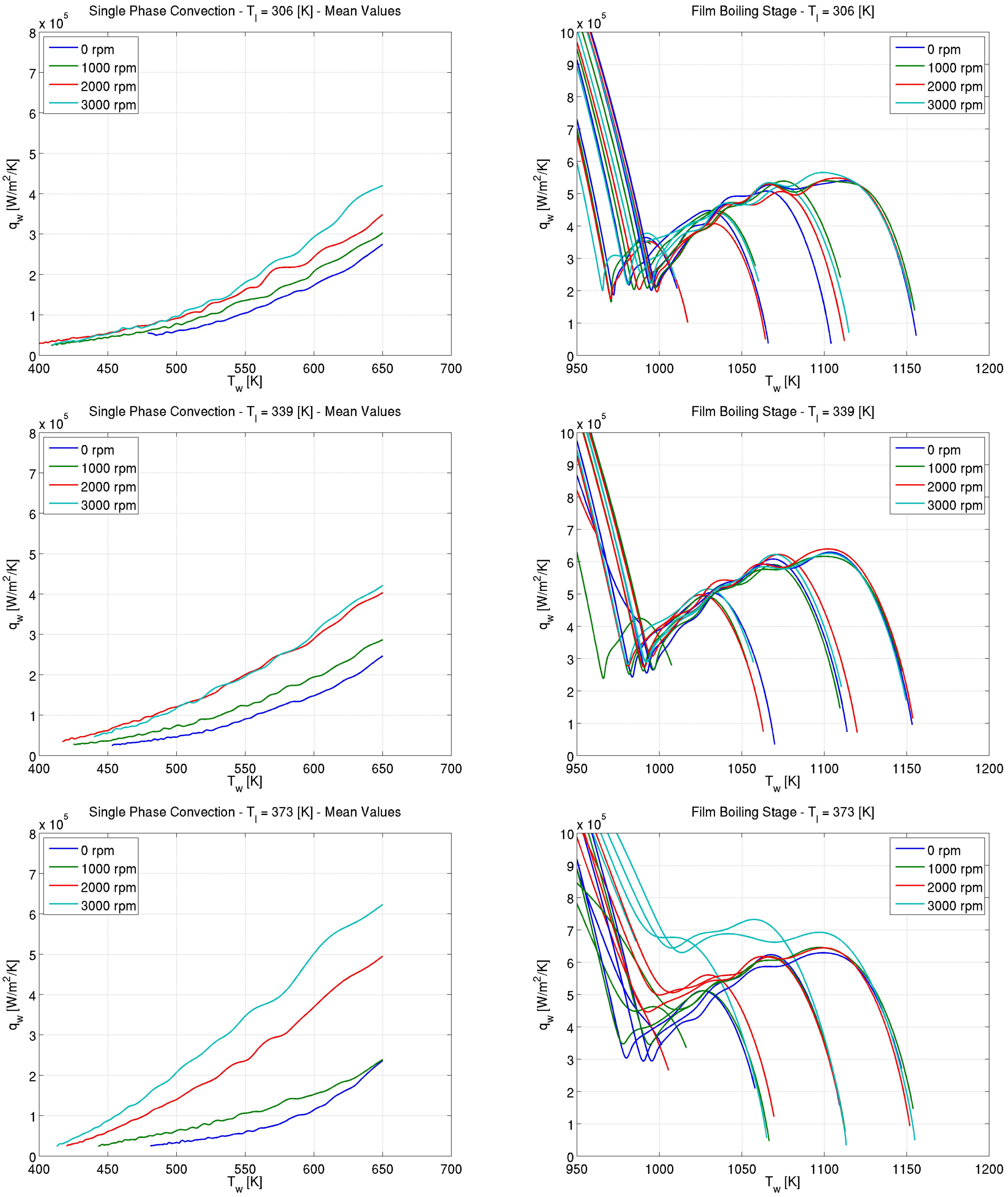

Figure 7: Single Phase convective and Film Boiling Heat Flux stages at different T_l and rpm

mechanism with each of its variables requires a more extensive analysis and is not covered by this work.

In general terms, the total heat transfered during quenching can be partitioned in the following way:

$$q_w = q_{cond} + q_{nucl} f + q_{fb}(1-f) + q_{rad} \tag{9}$$

where each component can be proposed as:

- $q_{cond} = q_{cond}(T_w, T_l, v_{l,\infty}, oil)$
- $q_{nucl} = q_{nucl}(T_w, oil)$
- $q_{fb} = q_{fb}(T_w, T_l, ...?)$

In Eq. 9 a blending function, f, to operate the transition between film and nucleate boiling is proposed as necessary. Also, the typical contribution of radiation heat transfer to a gray medium is included (Bromley).

Based on the HTC determination performed in this work and previous studies of heat transfer during boiling [Kocamustafaogullari and Ishii (1983, 1995); Tu and Yeoh (2002); Yeoh and Tu (2004, 2006); Bromley et al. (1953)] a complete characterization of each mechanism in Eq. 9 could be performed. A high level of detail and accurateness on the determination of HTC is necessary in order to capture the details and dependences of a complex phenomenon such as quenching.

5 CONCLUSIONS

The heat transfer coefficient was determined for several quenching tests performed under different conditions. The conditions were selected in order to assess the effect of parameters such as initial piece temperature, agitation and oil temperature on heat transfer.

The inverse problem of the HTC determination was solved using a novel method that it is based on the iterative resolution of the heat equation. At each iteration, the time dependent heat transfer function is corrected in order to reproduce the experimental cooling history. The evolution of the experimental time derivative is used as objective function to be reproduced by the numerical result. This parameter was selected in spite of the temperature because is much more sensitive than it at high values.

It can be observed that, for the analyzed conditions, HTC is independent of the initial solid temperature. For a given combination of flow temperature and impeller velocity, HTC is only dependent on wall temperature, having all the curves collapsed into a single one. The different heat transfer mechanisms (film boiling, nucleate boiling and single phase convection) can be easily identified from the HTC curves. The developed method to obtain the HTC allows the analysis of this process, the mechanisms that produces it and the transitions between mechanisms with a level of accurateness higher than the current techniques.

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