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MODELING OF THE THERMO-ELASTOPLASTIC BEHAVIOR OF SPACE TRUSSES

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Abstract. Space trusses are structures composed of rods usually metalics (steel or aluminum) that are used mainly to sustain the roof of large spans present in sheds and pavilions. The present work has the aim of computationally modeling the coupled thermo-elastoplastic behavior in space trusses under large displacements and large strains. The present approach uses the heat transfer equation and an elastoplastic constitutive model, modified to include the thermal influence. Computer simulations of the resulting mathematical model can be used for the development of structural projects under large variations of temperature, as occurs in fire situations. The finite element method (FEM) was used to determine the temperature field in transversal section of rods. In order to find the displacements and strains in rods due to thermal variation and loadings, it was used the direct stiffness method. The Newton-Raphson method was used to solve the non-linear equilibrium equations of the thermomecanic constitutive model. The developed code was aplied to simulate a truss under a simplified standard fire situation, where the high temperature causes a decrease in the resistance and stiffness of the rods, which may plastificate and fail in a short time of exposure.

1 INTRODUCTION

Space trusses are a particular case of three-dimensional frame structures and are widely used in sheds and pavilions to sustain the roof of these places. Since the 1960s the use of steel and aluminum space trusses became very widespread in several countries (de Souza, 2003).

The increasing use of such structures encouraged more studies about their behavior under the effect of loadings (own weight and external forces) and under the effect of large variations of temperature. Both can cause changes in the conformation of the structures and if not previously planned can cause failure of the rods and even some physical damages. The number of fire situations in Brazil is increasing each day (Seito et al., 2008), which can be associated to the fast growing of the brazilian cities and to the urbanization occurred in the last century. Hence, building design should be more accurate, to avoid failures in security and maintenance.

Therefore, it is important to know the characteristics and the physical properties of the materials and how structures will behave in adverse situations to minimize the chance of a tragedy. Among the analysis which must be done is the elastoplastic mechanical behavior coupled to the thermal behavior, considering that the structure may be subjected to loadings and temperature variations in an exceptional combination of actions.

The elastic behavior of a material is observed when a structure under the effect of a loading is unloaded and the structure returns to its initial configuration with no permanent strain. On the other hand, the plastic behavior is described for a structure which acquires a permanent deformation even after the unloading.

In fire situations the structures undergo large variations of temperature due to heat transfer between the rods and the fire gases, leading to a change of some thermal and mechanical properties. In these cases a more elaborated analysis is needed to consider the reduction of the resistance and stiffness of the materials used.

Both the thermal and mechanical problems can be mathematically described by differential equations. This equations, most of the times, can not be solved analytically, requiring some numerical method for their resolution. The heat transfer in a fire situation involving trusses is a transient problem and is solved in this work by the FEM to determine the temperature field in the transversal section of the rods. Dependences of the thermal conductivity and of the specific heat on the temperature generate non-linear equations, which can be solved by a direct iterative method. The temperatures obtained in the thermal analysis are included in the structural analysis of trusses in the form of thermal strains to determine the field of total strains in the rods, also considering the decreasing of the yield stress, elasticity and plastic moduli. These considerations, together with the usual elastoplastic analysis (Bonet and Wood, 2008), generate a system of algebraic equations, which describes the equilibrium in terms of nodal displacements. Taking into account the possibility of large displacements and large strains and a elastoplastic behavior, this system of equations become non-linear and the Newton-Raphson method is used to solve it.

This work has the aim of develop and implement a simplified thermomechanical coupled model to computationally simulate the behavior of space trusses under large displacements and large strains from an elastoplastic constitutive model with thermal influence, to be capable of calculate the fire resistance time in a standard fire situation.

2 MODELING AND METHODS

The implementation and solution of the models studied in this work was done using the programmming language *Python* (Python, 2016), using the numerical library *numpy*. The thermal analysis was solved by the FEM, using the mesh generator program *Gmsh* (Geuzaine and Remacle, 2009) to create meshes with triangular elements in the cross-sectional area of the tubular rod analysed. The mechanical and coupled models were solved by the direct stiffness method using the nodes and connections of the rods in the truss adopting an increment control to establish the equilibrium at each increment step.

The next subsections treat the thermal model and the mechanical model coupled with the thermal influence.

2.1 Thermal Analysis

The first part of the structural analysis in fire situations is the determination of the temperature field of the elements exposed to fire. In this work it will be considered a mean temperature of the cross-sectional area to be the temperature of the rods in the truss, since it has a small variation due to the small thickness of the tubular rods.

The heat transfer can be understood as the propagation of energy between two regions at different temperatures. There are three types of heat transfer: conduction, when there is a gradient of temperature in a medium, solid or fluid; convection, which occurs between a surface and a fluid in motion when they are at different temperatures; and radiation, which is the heat transfer between two surfaces in different temperatures and can occur even in the absence of a medium (Bergman et al., 2011).

2.1.1 The heat conduction equation

Assuming energy conservation, which establishes that the energy rate within an element is equal to the net energy transport inside this element plus the energy rate generated in this medium, it follows that:

$$\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g \, . \tag{1}$$

The heat conduction is governed by the Fourier's Law, which establishes that the heat flux throughout a material is proportional to the negative gradient temperature, i.e.:

$$q_x = -k\frac{\partial T}{\partial x}, \tag{2}$$

where q_x is the heat flux in the x direction and k represents the thermal conductivity of the material.

From the energy conservation and the Fourier's Law, after some substitutions it comes to the heat difusion equation, considering a two-dimensional analysis:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + Q, \tag{3}$$

where ρ and c_p are the specific mass and the specific heat of the medium, respectively, and Q is the rate in which energy is generated by volume unit (W/m^3).

Considering an isotropic homegeneous medium and rearranging the Eq. (3), it comes to:

$$\frac{\partial T}{\partial t} - \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \frac{Q}{\rho c_p}. \tag{4}$$

2.1.2 The finite element method

The FEM used to solve the Eq. (4) depends on boundary conditions preestablished and an initial condition to obtain the temperature distribution, $T(x, y, t)$, in the cross-sectional area of the rods.

The three types of boundary conditions are: Dirichlet, in which a temperature is prescribed in a part of the boundary; Neumann, in which there is a constant heat flux in a part of the boundary; and Robin, in which is assumed a conditon of convection and radiation on the boundary. These three conditions can be summarized as:

$$-\mathbf{n} \cdot (k\nabla T) = h(T_s - T_\infty) - q_0, \tag{5}$$

where $\mathbf{n}$ is the normal direction out of the boundary, h is the combined coefficient of heat transmission by convection and radiation (see Ribeiro (2009)), T_s is the temperature on the surface of the structure, T_∞ is the temperature of the fluid which the structure is exchanging heat with (in this case the gases of the fire), and q_0 is the heat flux prescribed.

The FEM is based on a discretization of the domain considerated (cross-sectional area of the tubular rod) and from this obtain approximate numerical solutions of the differential equations. This solutions can be obtained through the method of weighted residuals together with the Galerkin method using a variational formulation to FEM application in these problems (Hughes, 1987; Larson and Bengzon, 2013). From this, the semi-discrete form of the heat conduction transient problem is given in the matrix form by:

$$\mathbf{M}\dot{\mathbf{T}}(t) + (\mathbf{A} + \mathbf{R})\mathbf{T}(t) = (\mathbf{b} + \mathbf{r})(t), \tag{6}$$

where each matrix is given by:

$$\begin{aligned} M_{ij} &= \int_\Omega \varphi_j \varphi_i d\Omega, \\ A_{ij} &= \int_\Omega \frac{k}{\rho c_p} \nabla\varphi_i \cdot \nabla\varphi_j d\Omega, \\ R_{ij} &= \int_\Gamma \frac{h}{\rho c_p} \varphi_i \varphi_j d\Gamma, \\ b_i &= \int_\Omega \frac{Q}{\rho c_p} \varphi_i d\Omega, \\ r_i &= \int_\Gamma (\frac{h}{\rho c_p} T_\infty + q_0) \varphi_i d\Gamma, \end{aligned} \tag{7}$$

where φ_i and φ_j are the shape functions associated to the FEM (Larson and Bengzon, 2013) and $i, j = 1, 2, \ldots, n_i$, being n_i the number of nodes of the computational mesh.

2.1.3 The transient problem solution

To solve the Eq. (6) it is adopted a numerical estrategy of time integration based on the Finite Difference Method (FDM). This method is based on the assumption that the Eq. (6) is satisfied only in discrete points $t_{n+\beta}$ of each time interval Δt in which the time domain was discretized and that the temperatures vary linearly along the time interval Δt, since the instant t_n until $t_{n+1} = t_n + \Delta t$. Fig. 1 shows these considerations and more details can be found in Ribeiro (2009).

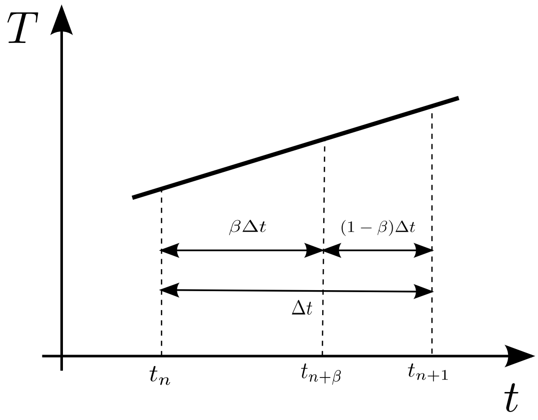

Figure 1: Temperature variation in the time interval Δt

After the manipulation, the equation adopted to the calculation of the temperatures at each time step is:

$$[\mathbf{M}_{n+\beta} + \beta\Delta t(\mathbf{A} + \mathbf{R})]\mathbf{T}_{n+\beta} = \mathbf{M}_{n+\beta}\mathbf{T}_n + \beta\Delta t(\mathbf{b}_{n+\beta} + \mathbf{r}_{n+\beta}), \tag{8}$$

where $\beta \in [0, 1]$ is a time integration parameter. After the system of Eqs. (8) is solved for $\mathbf{T}_{n+\beta}$, the temperatures at the end of the time interval t_{n+1} are given by:

$$\mathbf{T}_{n+1} = \frac{1}{\beta}\mathbf{T}_{n+\beta} + \left(1 - \frac{1}{\beta}\right)\mathbf{T}_n. \tag{9}$$

For fire situations Ribeiro (2009) and Vila Real (1988) suggested $\beta = \frac{2}{3}$, which is the Galerkin time integration scheme, and will be the one used in this work. If the thermal conductivity and specific heat vary with the temperature the problem becomes non-linear. Both linear and non-linear problems involving the Eq. (8) can be solved by various iterative processes, being used in this work the simple iterative method, which uses as convergence criterion the euclidian norm. This and other iterative processes can be found with more detais in Vila Real (1988).

2.2 Mechanical analysis

The second part of the structure analysis in fire situations is to evaluate how the temperatures obtained in the thermal analysis together with permanent load (own weight) and accidental load can influence the behavior of trusses under large displacements and large strains, using for this a coupled thermo-elastoplastic model, which in this work will be detailed in one-dimensional cases.

2.2.1 The constitutive model

This model can be called a solid constitutive model of stress-strain and it is composed of a set of equations which describes the stress at any instant in function of the strain history until that instant. When an external force F is applied in the axial direction of a rod, it generates an internal axial force T, that divided by the deformed cross-sectional area, a, results in the Cauchy stress, σ (see Fig. 2). Then, the internal force is given by $\mathbf{T} = \sigma a \mathbf{n}$, where $\mathbf{n}$ is the unit vector, pointing out of the rod.

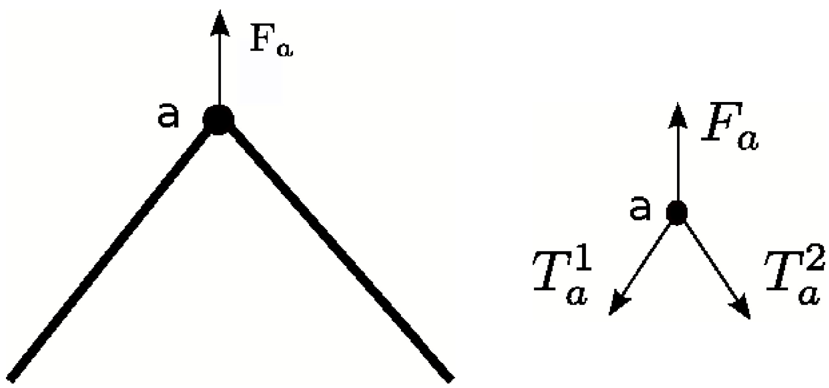

Figure 2: Internal and external forces in a node of a rod

However, for large strains the elastoplastic behavior is better described by the Kirchhoff stress, τ, given by:

$$\tau = \sigma \frac{v}{V}, \tag{10}$$

where v is the current volume of the rod and V is the initial volume of the rod. The strain is expressed by:

$$\varepsilon = \ln \lambda = \ln \left(\frac{l}{L} \right), \tag{11}$$

where λ is called the stretch, l is the current length and L is the initial length. The relation between the current and initial length and the current and initial volume is given by the Poisson coefficient, ν, in isotropic materials as:

$$\frac{v}{V} = \left(\frac{l}{L} \right)^{(1-2\nu)}. \tag{12}$$

The parameter, characteristic of each material, that relates the stress τ with the strain ε in elastic conditions is the elasticity modulus, given by:

$$E = \frac{d\tau}{d\varepsilon}, \tag{13}$$

where it is initially assumed that this value is constant for the same material without temperature change. Then, the Kirchhoff stress τ can also be given by:

$$\tau = E \ln \lambda, \tag{14}$$

and the internal forces can be rewriten as:

$$\mathbf{T} = \frac{VE}{l} \ln(\frac{l}{L})\mathbf{n} = \tau \frac{V}{l}\mathbf{n}. \tag{15}$$

2.2.2 The coupled thermo-elastoplastic behavior

To analyse the thermo-elastoplastic behavior it is necessary first to know about the elasto-plastic behavior. More details can be found in Bonet and Wood (2008). In this work a coupled model will be considered so all the formulation will take into account the elastic and plastic equations.

Fig. 3 shows the theoretical decomposition of a rod in the coupled model.

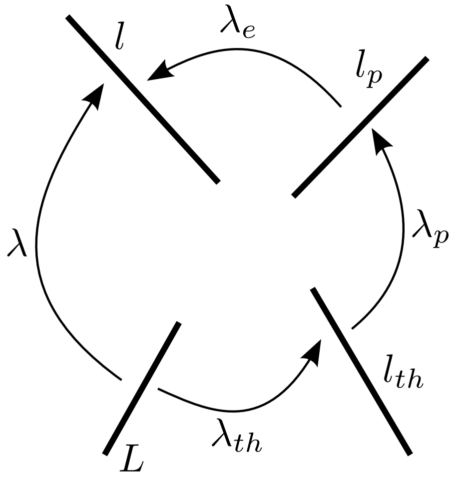

Figure 3: Stretch configurations of the rod

The equations which describe the thermo-elastoplastic behavior are given by:

$$\frac{l}{L} = \frac{l}{l_p}\frac{l_p}{l_{th}}\frac{l_{th}}{L}, \tag{16}$$

$$\lambda = \lambda_e \lambda_p \lambda_{th}, \tag{17}$$

$$\ln \lambda = \ln \lambda_e + \ln \lambda_p + \ln \lambda_{th}, \tag{18}$$

$$\varepsilon = \varepsilon_e + \varepsilon_p + \varepsilon_{th}, \tag{19}$$

where the subscribed letters e, p and th represent the elastic, plastic and thermal terms of the model, respectively. The total strain contains now the contribuitions of the three types of strain.

However, the normal stress along the rod depends only on the elastic component of the strain:

$$\tau = E(T)\varepsilon_e = E(T)(\varepsilon - \varepsilon_p - \varepsilon_{th}) \ , \tag{20}$$

where now the elasticity modulus varies with the temperature.

The function $f(\tau, \bar{\varepsilon}_p)$ is used to determine the nature of the behavior, working as a yield condition:

$$f(\tau, \bar{\varepsilon}_p) = |\tau| - (\tau_y^0(T) + H(T)\bar{\varepsilon}_p) \leq 0 \ , \tag{21}$$

where $\bar{\varepsilon}_p$ is the hardening parameter, which is defined as the absolute plastic strain accumulated in time, and H is a property of the material called plastic modulus. Eq. (21) brings also the dependence of the yield stress τ_y^0 with the temperature. If $f(\tau, \bar{\varepsilon}_p) < 0$ the strain is elastic. Alternatively, if $f(\tau, \bar{\varepsilon}_p) = 0$, a further strain can be plastic or elastic, depending on the next load.

Although the analysis consists of a quasi-static behavior, it is convenient to associate the development of the strain to a parameter analogous to time (pseudo time), which will be referred from now on as time. To evaluate the material behavior studied in time and space it is needed to consider discrete time steps Δt when the rod moves from a position n in a time t to a position $n+1$ in a time $t+\Delta t$. These time steps are associated to increments of load or displacements and the strategy adopted to know the correct behavior is to assume, as a trial for an initial evaluation, that the strain in the time interval Δt is purely elastic, calculating the value of the function f. Given the result, the variables are updated according to the Table 1. This process is known as *Retturn-Mapping* (Bonet and Wood, 2008).

Table 1: Variables update in the evaluation of the material behavior.

Elastic: $f(\tau_{n+1}^{trial}, \bar{\varepsilon}_{p,n}) \leq 0$	**Plastic:** $f(\tau_{n+1}^{trial}, \bar{\varepsilon}_{p,n}) > 0$
$\Delta\gamma = 0$	$\Delta\gamma = \frac{f_{n+1}^{trial}}{E(T_{n+1})+H(T_{n+1})}$
$\tau_{n+1} = \tau_{n+1}^{trial}$	$\Delta\varepsilon_p = \Delta\gamma sign(\tau_{n+1})$
$\varepsilon_{p,n+1} = \varepsilon_{p,n}$	$\tau_{n+1} = \tau_{n+1}^{trial} - E(T_{n+1})\Delta\varepsilon_p$
$\bar{\varepsilon}_{p,n+1} = \bar{\varepsilon}_{p,n}$	$\varepsilon_{p,n+1} = \varepsilon_{p,n} + \Delta\varepsilon_p$
	$\bar{\varepsilon}_{p,n+1} = \bar{\varepsilon}_{p,n} + \Delta\gamma$

2.2.3 Non-linear equilibrium equations and the Newton-Raphson method

The equilibrium equations of a node in a rod are established in relation to the current position by assembling the typical internal forces $\mathbf{T}$ and the external forces $\mathbf{F}$ at all nodes in the truss. This assembly is done by the contribuition of each element associated to a typical node and is expressed by the residual nodal force $\mathbf{R}$ as the balance between the internal and external forces:

$$\mathbf{R} = \sum_{el=1}^{n_{el}} \mathbf{T}_{el} - \mathbf{F} = \mathbf{0} \ , \tag{22}$$

where n_{el} is the total number of elements that meet at each node. The equilibrium equations above described are non-linear functions of the nodal positions, since the nodal internal force is a function of the length l and the unit vector $\mathbf{n}$, and can be solved by the Newton-Raphson method. A more compact form of the Eq. (22) can be writen for all the nodes of the structure:

$$\mathbf{R}(\mathbf{x}) = \mathbf{T}(\mathbf{x}) - \mathbf{F}. \tag{23}$$

The Newton-Raphson procedure involves the stiffness matrix $\mathbf{K}$, the incremental displacements u and the unbalanced residual forces $\mathbf{R}$ and are writen to an iterative step i as:

$$\mathbf{K}(\mathbf{x}_i)\mathbf{u} = -\mathbf{R}(\mathbf{x}_i). \tag{24}$$

The tangent stiffness matrix $\mathbf{K}$ is assembled in the usual manner from the contribuition of the stiffness matrix of each individual element $\mathbf{K}_{el}$:

$$\mathbf{K}^{(el)}(\mathbf{x}_i^{(el)}) = \begin{bmatrix} \mathbf{K}_{aa}^{(el)} & \mathbf{K}_{ab}^{(el)} \\ \mathbf{K}_{ba}^{(el)} & \mathbf{K}_{bb}^{(el)} \end{bmatrix} \ , \tag{25}$$

where, in function of a stiffness term k_s and the axial force $T = \sigma a$, the components of the consistent tangent stiffness matrix are:

$$\mathbf{K}_{aa}^{(el)} = \mathbf{K}_{bb}^{(el)} = k_s \mathbf{n} \otimes \mathbf{n} + \frac{T}{l}\mathbf{I}_{3\times 3} = -\mathbf{K}_{ab}^{(el)} = -\mathbf{K}_{ba}^{(el)} \ , \tag{26}$$

where the stiffness term k_s is given by:

$$k_s = \left(\frac{V}{v} \frac{d\tau}{d\varepsilon} \frac{a}{l} - \frac{2T}{l} \right) = \frac{V}{l^2} \left(\frac{d\tau}{d\varepsilon} - 2\tau \right). \tag{27}$$

In the elastic case $\frac{d\tau}{d\varepsilon} = E$ as already mentioned before and in plastic case $\frac{d\tau}{d\varepsilon} = \frac{EH}{E+H}$.

3 RESULTS AND DISCUSSION

In this section the behavior of a truss submitted to a fire situation will be analysed using the coupled thermo-elastoplastic model described. But first it will be shown two examples (a thermal and a mechanical) used to validate the development and the implementation of the model.

3.1 Thermal validation

To validate the thermal implementation it was considered a situation where a rod exchange heat with gases in a standard fire situation. The equation of the temperature of these gases is given by EN 1991-1-2:2002 (2002):

$$T_g = T_o + 345 \log_{10}(8t + 1), \tag{28}$$

where T_g is the temperature of the gases (in ^{o}C), T_o is the initial temperature, adopted as 20^oC and t is the time spend since the beginning of the fire (in minutes).

The rod studied has $60mm$ of diameter and $2mm$ of thickness, and the analysis was done in a 10^o section of its transversal section. Due to the symmetry and small thickness of the rod the medium of the temperatures calculated was assumed the temperature of the rod at each instant of time. The mesh built and the boundary and initial conditions adopted are shown in Fig. 4.

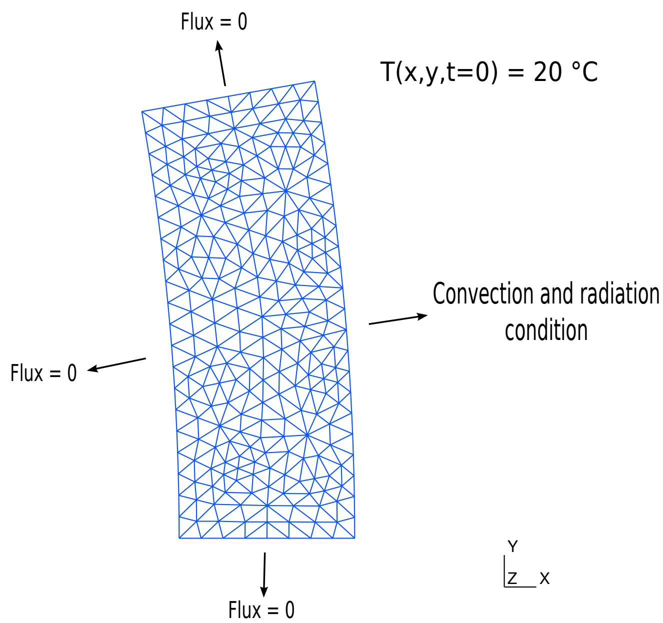

Figure 4: Mesh and conditions of the problem

The material of the rod used was carbon steel, whose specific mass is $7850kg/m^3$ (William D. Callister, 2008). The emissivity adopted in this work was $0,7$ and for the convection coefficient the EN 1991-1-2:2002 (2002) recommends the value of $25W/m^2\ ^oC$. The conductivity coefficient and the specific heat of the steel were considered dependents of the temperature and this dependence can be found on EN 1994-1-2:2005 (2005).

Fig. 5 shows the temperature versus time of the standard fire situation and the curves obtained with the algorithm used to implement the heat problem in this work and the one obtained with the Thersys 2.0 software, implemented and validated by Ribeiro (2009).

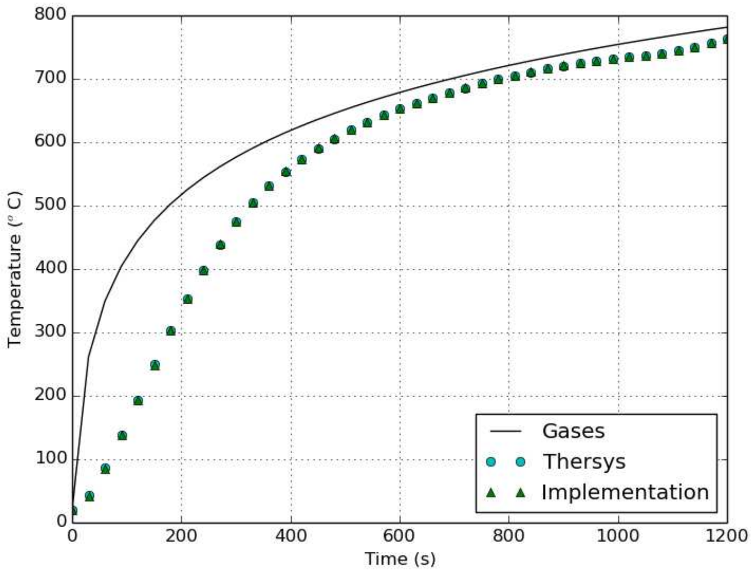

Figure 5: Temperature evolution in the fire situation

The results presented in Fig. 5 show good agreement with other works and thus confirm the validation of the algorithm used to the thermal problem, allowing its use to the coupled problem that will be treated in this work.

3.2 Mechanical validation

To validate the mechanical problem it was used the following example, in which an external force **F** is applied to a rod (Bonet and Wood, 2008). Fig. 6 shows the configuration of the problem:

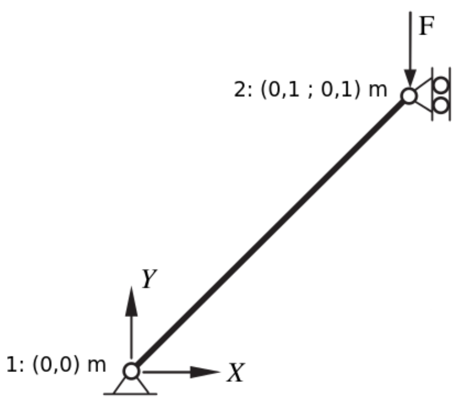

Figure 6: Initial configuration of the problem

The value of the parameters used in this problem are summarized in Table 2.

Table 2: Parameters of the material.

Parameter	**Simbol**	**Unit**	**Value**
Elasticity modulus	E	(GPa)	$210,00$
Yield stress	τ_y^0	(GPa)	$25,00$
Plastic modulus	H	(MPa)	$1,00$
Poisson's ratio	ν	Dimensionless	$0,30$
Initial Area	A	(m^2)	$1,00 \times 10^{-6}$

Making the displacement control of the node 2 under the effect of the force **F** it is possible to reproduce the results of the elastoplastic behavior of the rod. Fig. 7 shows the variation of the ratio F/EA versus the ratio $(Y-y)/L$, being $(Y-y)$ the displacement and L the initial length, in comparison with the result presented in Bonet and Wood (2008).

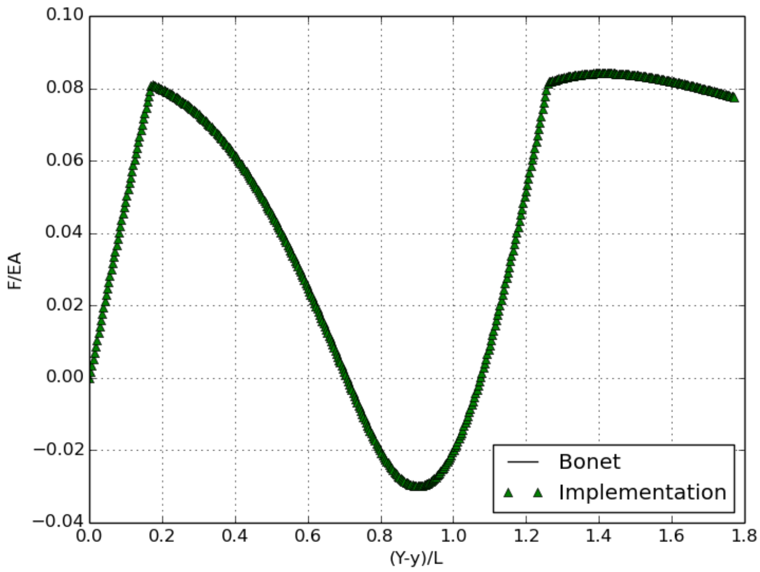

Figure 7: Behavior of the elastoplastic deflection force

The analysis of the curves shows that the algorithm used in the mechanical problem is valid, allowing it to be used in the main coupled problem.

3.3 Coupled problem

The coupled thermo-elastoplastic behavior will be analysed in a space truss with square-square arrangement, 30 x $30m$ of vain, $1,5m$ of height, and $2m$ of module as recommended by Marsh (2000) to a cost optimization in the structure. To avoid buckling it will be used rods with $76mm$ of diameter and $2mm$ of thickness. The sketch of this truss can be visualized in Fig. 8, where the white dots (on the right) are the support and the red dot is the node that will be analysed.

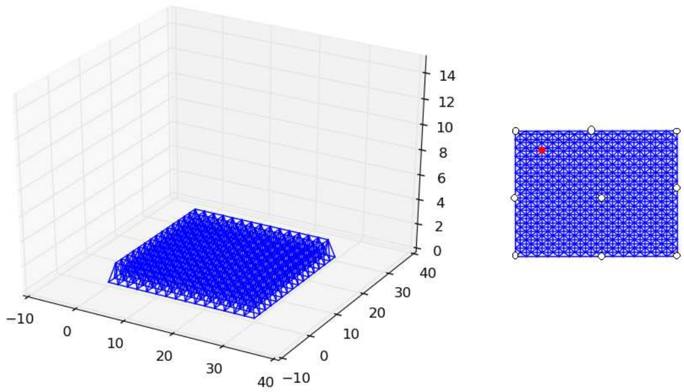

Figure 8: Side and top view of the truss

The material used in the rods of the truss was carbon steel 1040, whose properties are shown in Table 3.

Table 3: Carbon steel 1040 properties (at 20^oC).

Parameter	**Simbol**	**Unit**	**Value**
Specific mass	ρ	(kg/m^3)	$7850,00$
Elasticity modulus	E	(GPa)	$207,00$
Yield stress	τ_y^0	(MPa)	$290,00$
Poisson's ratio	ν	Dimensionless	$0,30$

The plastic modulus H is often determined experimentally and for this work the value adopted was $1,0GPa$ for a linear kinematic hardening model (de Souza Neto et al., 2011). To analyse the variations in E, τ_y^0 and H with the temperature increasing it was used the reduction factors $k_{E,T}$, $k_{\tau,T}$ and $k_{H,T}$, respectively, related to rolled structural steels. This values are shown in Table 4 (EN 1993-1-2:2005, 2005), considering $k_{H,T} = k_{E,T}$.

Table 4: Reduction factors for rolled steels.

Temperature	**Rolled steels**	
T(°C)	$k_{\tau,T} = \frac{\tau^0_{y,T}}{\tau^0_y}$	$k_{E,T} = \frac{E_T}{E}$
20	$1,000$	$1,000$
100	$1,000$	$1,000$
200	$1,000$	$0,900$
300	$1,000$	$0,800$
400	$1,000$	$0,700$
500	$0,780$	$0,600$
600	$0,470$	$0,310$
700	$0,230$	$0,130$
800	$0,110$	$0,090$
900	$0,060$	$0,0675$
1000	$0,040$	$0,045$
1100	$0,020$	$0,0225$
1200	$0,000$	$0,000$

With respect to the relative thermal elongation of steel ($\Delta l/L$), the EN 1993-1-2:2005 (2005) recommends the following values:

$$\frac{\Delta l}{L} = \begin{cases} -2,416 \times 10^{-4} + 1,2 \times 10^{-5} T_s + 0,4 \times 10^{-8} T_s^2 & \text{for} \quad T_s \leq 750^o C \\ 0,011 & \text{for} \quad 750 < T_s \leq 860^o C \\ -0,0062 + 2 \times 10^{-5} T_s & \text{for} \quad T_s > 860^o C \end{cases} \tag{29}$$

Considering the simulation, an accidental load of $0,5kN/m^2$ was applied in the nodes of the top stringer, simulating a roof sustained by the truss, in addition to the permanent load of its own weight. In sequence it was simulated a standard fire situation with 20 minutes of duration with the same conditions of the ones used in the thermal validation. The vertical displacement of the node analysed due to the temperature variation is shown in Fig. 9.

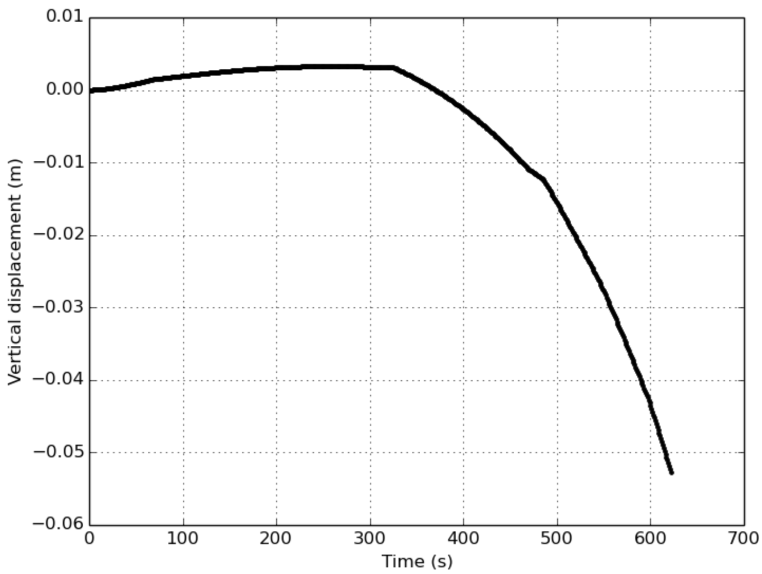

Figure 9: Vertical displacement of the node analysed during the fire situation

The simulation could not run until the end of the fire situation, stopping at 10 minutes and 23 seconds, which leads to believe that some parts of the structure could not resist and failed. The configuration of the truss in the moment before the simulation stopped can be seen in Fig. 10, where the rods in black are already plastified, while the others have a certain amount of stress but are still in the elastic range.

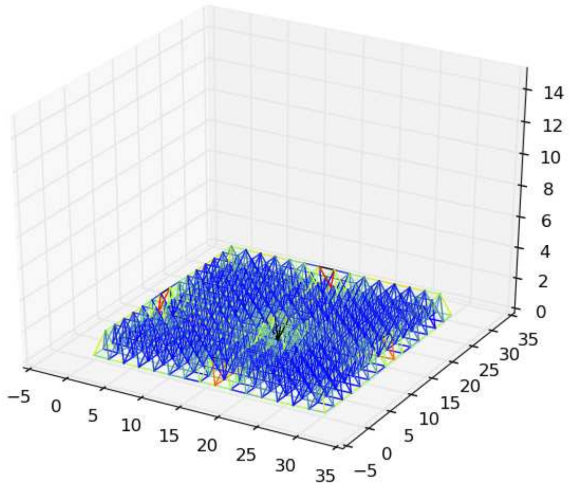

Figure 10: Final configuration of the truss

At this final configuration 10 rods were already plastified and it was possible to confirm that the most stressed rods were the ones connected to the supports.

4 CONCLUSIONS

The proposal to develop and implement a coupled thermo-elastoplastic model was accomplished. It was possible to validate the thermal and mechanical models and make their connection to apply to a problem involving a space truss in a fire situation, where a large temperature variation influenced the strain and stress suffered by the rods and the material properties: elasticity and plastic modulus and yield stress. This work made possible to predict an approximate time in which a space structure can resist to a fire situation before it fails and avoid bigger damages, like an accident envolving people. With this prediction it was possible to determine an evacuation time in case there are people in danger.

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