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DETERMINING THERMAL CONDUCTIVITY FOR BIOLOGICAL MATERIALS IN A DRYING PROCESS

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Abstract. It is key issue for the Agricultural Engineering to identify thermo physical properties of biological materials. There is a special interest in the thermal conductivity and diffusivity coefficient estimation, which can be performed by comparing data obtained from a specific experimental procedure and theoretical data obtained from some mathematical model – capable for simulating the referred procedure. The experimental data of temperature were previously obtained from one thermal measuring system consisting of concentric cylinders to hold the biological material (a soybean sample in this study), with a heat source placed at the central axis and keeping the cylindrical, as well the circular cross sectional outer surfaces insulated . In such a procedure, only radial heat transfer is effective, minimizing the heat flux in the axial direction. Simulated data of temperature are obtained by using the mathematical model based on the Fourier's law with initial and boundary conditions according to the experimental procedure, which requires estimative of one the thermal conductivity value. The direct problem is solved by using an implicit forward time and centered space finite difference scheme, with Neumann boundary at the center and the outer surface. Results show a dependency of the thermal conductivity with radial component. The thermal property is estimated by computing the value having the best agreement.

1 INTRODUCTION

The knowledge of the thermal process can work as a support to the decision of choosing strategies to be implemented in order to improve the storage time of the agriculture products, task which also depends on the thermo physical properties of each product.

From this consideration, there are many research efforts developed in the Agriculture Engineering College (FEAGRI: Faculdade de Engenharia Agrícola) of the State University of Campinas (UNICAMP: Universidade Estadual de Campinas), most of them experimental ones. Part of this research is focused on the development of mathematical models.

The first research paper on this subject related to drying process is (Ito et al., 2002). The mathematical model is based on the heat conduction equation, in cylindrical coordinate with angular symmetry, and the algorithm based on an explicit scheme of finite difference method is used to simulate temperature data. This paper reinforced the research line on this subject, where the computer code for the numerical simulation is available for different but with the same nature products when submitted to the same particular drying process (Bossarino et al., 2005). The goal is to identify thermal parameters of biological material.

Basically, the approach used to this identification is based on considering acceptable interval values for this parameter, according to the literature, and then compute the value of the thermal conductivity by least square difference between the experimental data and simulated data.

Another algorithm including this approach but based on an implicit forward time and centered space finite difference scheme, as suggested (Amendola, 2005), is applied here, but we point out that the focused thermal parameter could have a dependency related to the space variable. Therefore, some identification strategies are suggested as an alternative approach to estimate the thermal parameters.

2 MATHEMATICAL FORMULATION FOR THE DIRECT MODEL

The mathematical model starts from the Fourier's law for the heat conduction transfer, using the cylindrical coordinates with angular symmetry, and considering concentric cylinders. The equation for the temperature field $T = T(r, t)$ under this assumption can be expressed as:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho Cp\ r}\frac{\partial T}{\partial r} + \frac{k}{\rho Cp}\frac{\partial^2 T}{\partial r^2}; \qquad R_1 < r < R_2 \tag{1}$$

where T is the biological material temperature [ºC], t is time [s], r is the space variable [m], k: the thermal conductivity [W / m*°C], ρ is the material density [kg / m^3], C_p is the heat capacity [J / kg*°C], and R_1, R_2 are the inner and outer cylinder radii [m], respectively. The initial condition is given by

$$\mathrm{T}(r, 0) = T_i; \qquad R_1 < r < R_2 \tag{2}$$

being T_i [ºC] the environmental temperature, and the boundary conditions are:

$$-k\left.\frac{\partial T}{\partial r}\right\|_{r=R1} = q_f$$

(3)

with q_f is the source heat flux in the experiment, for the internal cylinder, and for the outer surface the condition is expressed by

$$\left.\frac{\partial T}{\partial r}\right\|_{r=R2} = 0 \,. \tag{4}$$

Amendola (2006) has discussed about the application on the implicit versus explicit schemes for time integration for this problem, and she has concluded that implicit approach could be more efficient from computational point of view. An implicit scheme for equation (1) can be written as:

$$f_o\left(1-\frac{\Delta r}{r(j)}\right)T_{j-1}^{n}+\left[f_o\left(-2+\frac{\Delta r}{r(j)}\right)-1\right]T_j^n+f_oT_{j+1}^{n}=-T_j^{n-1} \tag{5}$$

where T_j^n = T($n*\Delta t$, $j*\Delta r$), with n = 1, ..., N_t; j = 1, ..., N_r, Δt is the time-step; Δr is the space discretization, $r(j) = j*\Delta r$; and f_o is the Fourier number

$$f_o=\left(\frac{k}{\rho C_p}\right)\left(\frac{\Delta t}{\Delta r^2}\right) \,. \tag{6}$$

The discretized initial and boundary conditions become

$$T_j^1 = Ti \ , \quad j = 1, \dots , N_r \tag{7}$$

$$T_1^n = T_2^n + b \, , \quad n = 2, \dots , N_t \tag{8}$$

being

$$b=\frac{q_f * \Delta r}{k} \tag{9}$$

$$T_{Nr}^{n} = T_{Nr-1}^{n} \quad , \quad n = 2, \dots , N_t \,. \tag{10}$$

To carry out the numerical simulation, the user should supply the following data set from the experimental procedure (1 up to 9) and literature (10):

1) the inner radius (R_1) [m];
2) the outer radius (R_2) [m];
3) heat flux (q_f) [W/m^2];
4) density of the biological material (ρ) [kg / m^3];
5) heat capacity of the biological material (C_p) [J / kg*K];
6) time period of the experiment (t_e) [s];

7) number of time-steps (N_t) (assuming that the time-step is known);
8) file with: N_t rows × 1 column (for the MatLab instruction: *load <file_name>.m*);
9) initial temperature for the biological material (T_i) [ºC];
10) two numerical values for thermal conductivity: $k_{\min}$ and $k_{\max}$ [W / m*C], and the size of the path Δk;

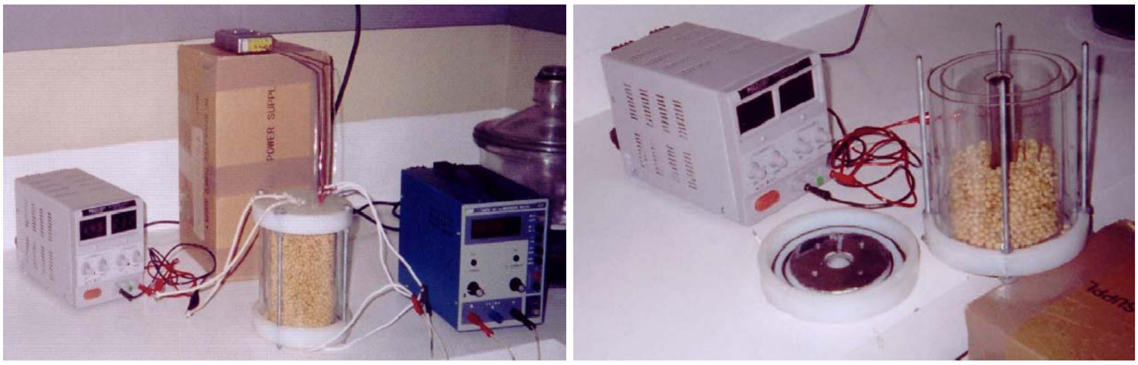
From this information, the computer code performs the numerical simulation, for several values of thermal conductivity k_m, where $k_m = k_1 + (m-1)\Delta k$ - where $k_1 = k_{\min}$. The effective thermal conductivity is determined from the value k_m with the smallest square difference between the simulated temperature and the measured temperature.

3 NUMERICAL RESULTS

Indeed, the estimation strategy for thermal properties for the biological material is a nice way to combine experimental data and theoretical data from a mathematical model. The worked example in this paper is focused on the soybean grains.

The experimental device to carry out the experimental procedure is shown in Figure 1 (from Ito (2003)).

The soybean grains are released in a cylindrical container – two concentric cylinders, with a thermal source coming from the inner cylinder, and keeping the cylindrical, as well the circular cross sectional outer surfaces insulated, with four thermo-couple placed in different positions along the radii (see figure 1b for details)-. This equipment produces a dataset of time-series for temperature for several points.



(a) (b)

Figure 1: Experimental devices for thermal process for soybean grains: (a) assembled equipment, (b) experimental device with more details (from Ito (2003)).

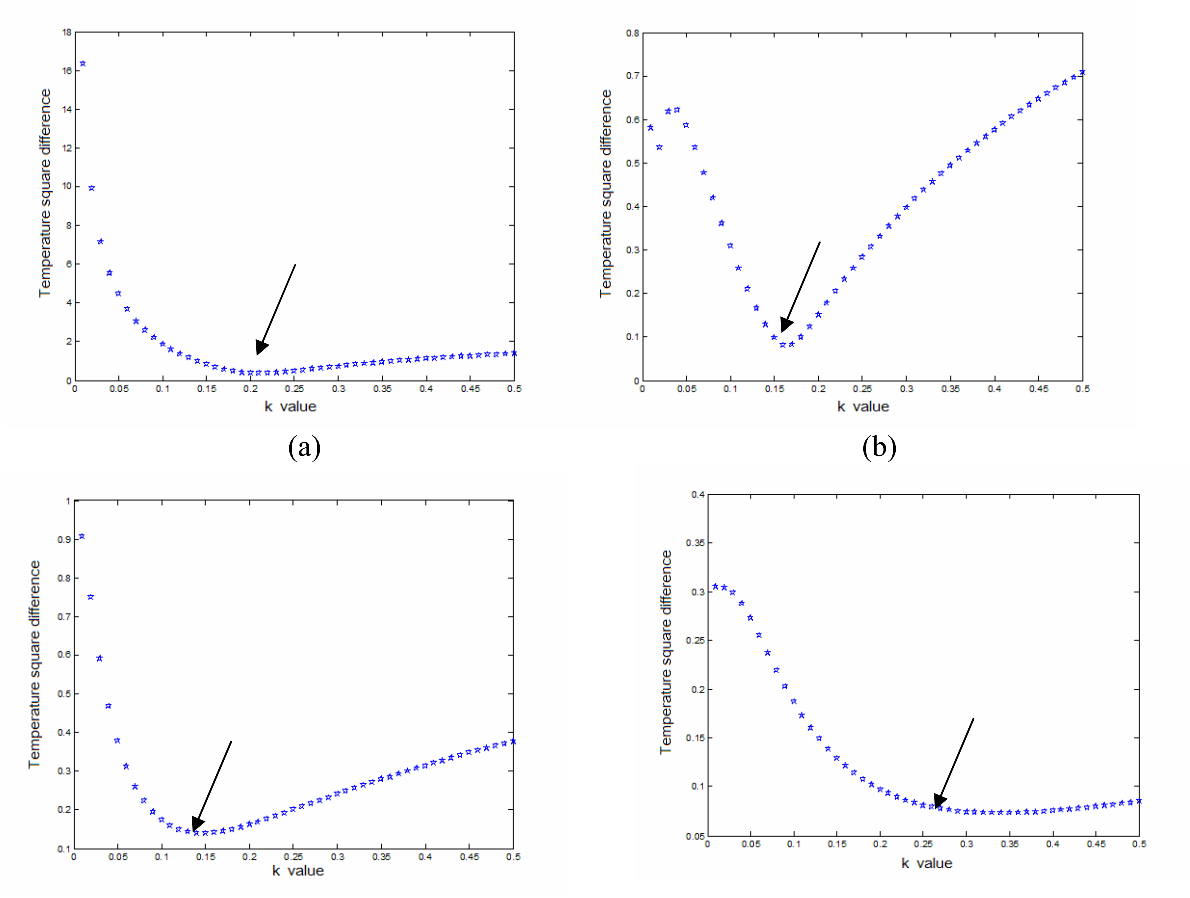
Here, we are considering part of the same temperature dataset for the soybean used by Ito (2003).

The referred container is defined between two cylinders with radii R1 = 0.013 [m] and R2 = 0.0.049[m], subject to a heat source, qf = 393.7 [W/m^2], placed at its central axis, where, for a certain height of the equipment and at four positions along the radii of the spatial domain the values of soybean temperature, T [°C], were recorded along the time, t [s], approximately at each 100s during 6000s.

The following fixed parameters of the process or product were considered: initial temperature $T_0 = 23.7$ [°C], density $\rho = 1180$ [kg/m^3] and heat capacity $C_p = 1970$ [J/kg°C].

In this example, the thermal conductivity ranges from k_{min}=0.1 Wm^{-1}C^{-1} up to k_{max}=0.5 Wm^{-1}C^{-1}, and Δk=0.001 Wm^{-1}C^{-1}. The computational code associated to the established algorithm is executed for all of these k_m (m=1, …, 400) values, where k_1= k_{min} and k_{400}= k_{max}. Therefore, it is possible to compare the measured temperature against the simulated temperature using each of all these k values, at each time-step.

Figure 2 shows the square difference between the simulated and measured temperatures for each thermo-couple position. The arrow in the figure is pointing to the minimum square difference, associated to the effective value (minimum difference) for thermal conductivity.



(a) (b)

(c) (d)

Figure 2: Square difference between experimental and computed temperatures in the interval k = [0.1, 0.5] for four different positions of thermo-couples: (a) position 1 – close to the inner surface, (b) position 2 – between the position-1 and the central point of the cylinders, (c) position 3 – between position-2 and position-4, (c) position 4: close to the outer surface.

The minimum difference between simulated and measured values is used to identify the effective thermal conductivity for each of the referred four positions along the equipment. The computed values for thermal conductivity are: k_1 = 0.211 Wm^{-1}C^{-1} (square difference = 0.3479), k_2 = 0.1640 Wm^{-1}C^{-1} (square difference = 0.0800), k_3 = 0.21460 Wm^{-1}C^{-1} (square difference = 0.1391), and k_4 = 0.337 Wm^{-1}C^{-1} (square difference = 0.0732). This is semi-automatic method for determining the thermal conductivity, but it is simple and efficient.

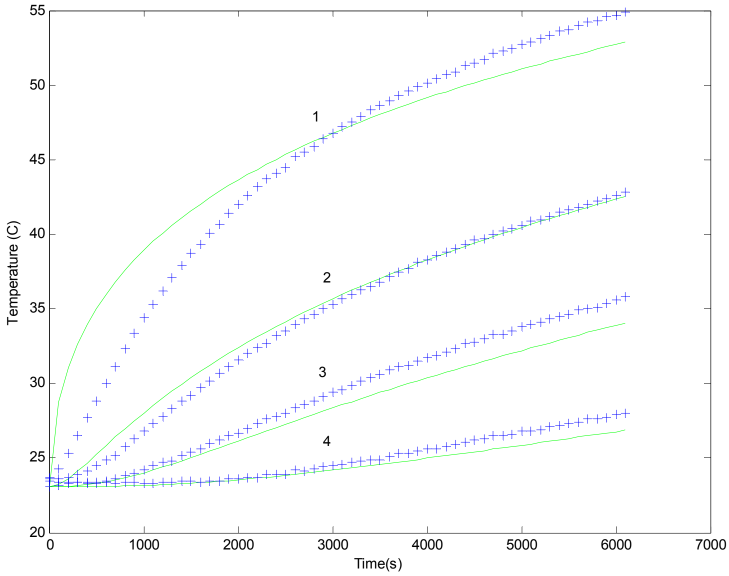


Figure 2: Numerical simulation (green) and experimental measurements (cross points).

Figure 3 displays the simulation of the temperature along the time according to the thermal conductivity mentioned before. The simulated curve (continuous line) is compared with the measured temperature (represented by the cross points). There is some disagreement between the simulated and the experimental values. As noticed in the previous and related works, the disagreement is more for the position-1. This behavior is expected, because the real situation is more complex than that described by the simple model used. However, the mathematical model's answer is close to the experimental results, indicating an effective modeling. One important issue to mention is the methodology employed here for the identification of the physical property.

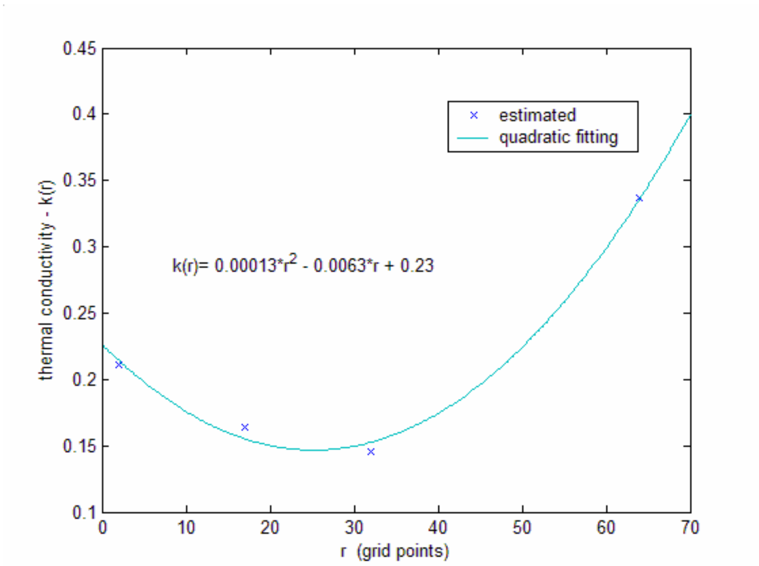


Figure 3: Thermal conductivity values (*k)* as a function of the grid points related to the four thermo-couples positions, and a quadratic fitting curve considering these four points.

3 FINAL REMARKS

This paper has shown a simple mathematical model to perform the numerical simulation of a drying thermal process specific carried out for biological material. The model is an important tool not only for the understanding of the process itself, but also because it can be used to enhance our understanding of similar processes such as drying, cooling, etc, for food processing. In addition, the model can be employed in the identification of some physical properties. The identification procedure was illustrated for estimating the thermal conductivity of the soybean. The thermal conductivity k was estimated from the least square difference between simulated data and measured data.

With four measured positions, one can compute the thermal conductivity for these temperature time-series. Figure 3 shows the estimated k=k(r) according to the four sensor positions. The continuous curve represents a fitting curve by a quadratic function from this four estimated thermal conductivities.

The results are indicating that the thermal conductivity k(r) could have a space dependency (radial variable). Under this condition, more sophisticated inverse problem technique can also be employed, for example inverse regularized solutions (Tikhonov and Arsenin, 1977). For this approach the inverse problem is formulated as a non-linear optimization problem, being the objective function given by:

$$J(k) = \left\| T^{Exp} - T^{Mod}\left[k(r)\right] \right\|_2^2 + \alpha\Omega\left[k\right] \tag{11}$$

where Ω[.] is the regularization operator, and α is the regularization parameter. In this formulation two strategies can be employed (Campos Velho et al., 2002): parameter estimation and function estimation.
(i) parameter estimation:

$$k(r) = \sum_{j=0}^{N_p} c_j r^j \tag{12}$$

where c_j are the unknowns and N_p is the number of expansion terms.

(ii) function estimation, considering a sampled function:

$$K = \left[k(r_1) \quad k(r_2) \quad \cdots \quad k(r_N)\right]^T = \left[k_1 \quad k_2 \quad \cdots \quad k_N\right]^T$$

The regularization process is more effective when function estimation is considered.

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