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PARAMETRIC PROBABILISTIC APPROACH IN THE DYNAMICS OF POROUS FGM CURVED BEAMS

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Abstract. In this article we evaluate the uncertain dynamic response of inhomogeneous curved beams constructed with ceramic and metallic materials that vary in a given functional forms. The construction process of this type of structures conducts to the presence of porosity in its domain. The porosity can be source of uncertainties in the dynamic behavior. In order to study the dynamics of these structures, we employed the Principle of Virtual Work to derive a curved beam model. The model incorporates shear flexibility, variable curvature and variable porosity. It serves as a mean deterministic reference to the studies on stochastic dynamics and uncertainty quantification, which are objectives of this article. The uncertainty quantification procedure considers the introduction of random variables to characterize the uncertainty in material or geometric properties such as elasticity modulus and/or density of the material constituents, curvature radius of the beam, porosity parameters, among others. The probability density functions (PDF) of the random variables are derived appealing to the Maximum Entropy Principle. Then, the probabilistic model is constructed with the basis of the deterministic model and both calculated within finite element approaches. Once the probabilistic model is constructed, the Monte Carlo Method is employed to calculate random realizations. In order to identify the sensitivity of the random parameters, a number of scenarios are evaluated in which the random variables can have different distributions/variations according to the level of information known or at least assumed.

1 INTRODUCTION

Functionally Graded Materials (FGM) are increasedly employed in high tech goods, p.e. aeronautics, astronautics, instrumental for medical care, artifacts and tools for energy harvesting and so on. FGM are traditionally constructed with two or more components which can be customized such that material properties in the structure can vary in selected directions according to given functions. Constructive patterns can also be designed in order to optimize the structural response. Depending on the constructive procedure the deposition of ceramic and metallic constituents together with the effect of pressure and/or localized heat leads eventually to the formation of porosities or little cavities in the structural domain. This context shaped what nowadays is known as Porous Functionally Graded Materials (PFGM). Porosity can be observed as the unexpected result of the manufacturing process or as a part of regularly designed microstructure with little cavities (Miao and Sun, 2010). In both cases there is a background of constructive procedures or modeling approaches that involve uncertainty (Kou and Tan, 2010).

In very recent years the interest of studying the response of structures constructed with PFGM has been unleashed. The research effort in this topic included mainly the dynamics and elastic stability of plates (Rezaei et al., 2017), shells (Jouneghani et al., 2018; Ramezani and Talebitooti, 2015), straight beams (Fazzolari, 2018; Chen et al., 2015), and micro/nano structures (Ebrahimi et al., 2017; Ebrahimi and Daman, 2017), among others. The study of naturally curved beams is of interest in a wide field of industrial applications as a support structure or a simple part of complex technological system. However there is a lack of studies related to curved beams constructed with PFGM, employing 1D formulations.

As it was previously mentioned the manufacturing of this type of structures has a number of uncertainty sources that can substantially alter the response of the structure. Possible sources of uncertainty can be found in material properties, boundary conditions, loads (Sampaio and Cataldo, 2011), the hypotheses of model or the model itself (Soize, 2005), etc. In order to characterize the uncertain response in dynamics of structures there are approaches that can be collected into the following groups: parametric probabilistic approach (PPA) and non-parametric probabilistic approach (NPPA) (Soize, 2005). In the first case the source of uncertainty are the parameters of the model in the second case the model as a whole. In the PPA each uncertain parameter is associated to a random variable whose probability density function (PDF) is defined according to given information about them (mean values, standard deviation, bounds, etc.).

The objective of this article is directed toward offering some contributions in the mechanics/dynamics of curved PFGM beams and especially to quantify the propagation of the uncertain in the dynamic response of the structure. In this context, the present article is arranged according to the following scheme: As first step the hypotheses of the constitutive model are enunciated and the deterministic structural model is introduced and conceived in the context of first order shear theories. A finite element formulation is proposed and then employed to carry out calculations of the deterministic model. Subsequently, the probabilistic model is constructed employing the previous finite element formulation in which the random variables are incorporated. The PDF of the random variables (some elastic, electric properties, elastic foundations, etc) are deduced by employing the Maximum Entropy Principle (Jaynes, 1957) subjected to given known information such as expected values and/or coefficients of variation (CoV) of the parameters. Then the Monte Carlo method is employed to simulate realizations. A statistical analysis is done and the results presented in the form of frequency response functions or other graphics of statistical interest.

2 BRIEF DESCRIPTION OF THE DETERMINISTIC MODEL

2.1 Main hypotheses and Kinematics

Fig.1 represents a curved beam with variable curvature radius and opening angle α. It has rectangular cross-section and it is contained in the plane π. The reference system is located in the geometric center of the cross-section c. The present technical theory of curved beam is based in the following assumptions:

- The cross-sectional shape is rigid in its own plane.
- Warping function is defined according to the reference point c.
- The beam is constructed with metallic and ceramic constituents varying in z-direction.
- Shear flexibility due to bending and twisting is incorporated.
- The model is derived in the context of linear elasticity.

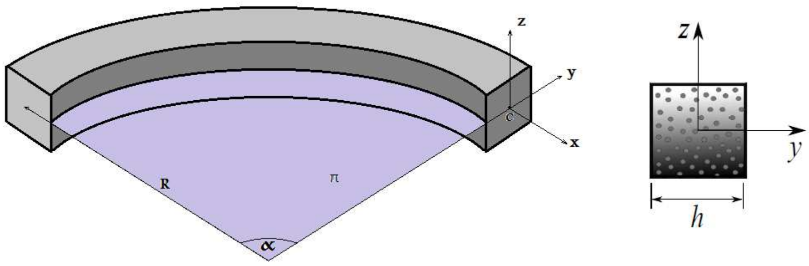


Figure 1: Curved beam .

$$\left\{ \begin{array}{c} u_x \\ u_y \\ u_z \end{array} \right\} = \left\{ \begin{array}{c} u_{xc} - \omega\phi_w \\ u_{yc} \\ u_{zc} \end{array} \right\} + \left[\begin{array}{ccc} 0 & -\phi_z & \phi_y \\ \phi_z & 0 & -\phi_x \\ -\phi_y & \phi_x & 0 \end{array} \right] \left\{ \begin{array}{c} 0 \\ y \\ z \end{array} \right\} \tag{1}$$

where ϕ_x, ϕ_y, ϕ_z y ϕ_w are defined according to the following forms:

$$\phi_x = \theta_x,\ \phi_y = \theta_y,\ \phi_z = \theta_z - \frac{u_{xc}}{R},\ \phi_w = \theta_w + \frac{\theta_y}{R} \tag{2}$$

In the previous equations u_{xc}, u_{yc}, u_{zc} are the displacements of the reference point, θ_y, θ_z are bending rotational parameters, θ_x is the twisting angle and θ_w is a measure of the warping intensity.

The warping function is approximated according to the following expression:

$$\omega = \bar{\omega}\mathcal{F} \text{ with } \mathcal{F} = \frac{R}{R+y} \text{ and } \bar{\omega} = -xy \tag{3}$$

2.2 Virtual work equations

The expression of the linearized virtual works can be written as:

$$\mathcal{W}_T = \int_L \left(\delta\widetilde{\mathbf{D}}^T\widetilde{\mathbf{Q}}\right) dx + \int_L \delta\widetilde{\mathbf{U}}^T\mathbf{M}_m\ddot{\widetilde{\mathbf{U}}}dx - \int_L \delta\widetilde{\mathbf{U}}^T\widetilde{\mathbf{P}}_X dx + \delta\widetilde{\mathbf{U}}^T\widetilde{\mathbf{B}}_X\Big|_{x=0}^{x=L} = 0, \quad (4)$$

where:

$$\begin{gathered} \widetilde{\mathbf{U}}^T = \{u_{xc}, u_{yc}, \theta_z, u_{zc}, \theta_y, \theta_x, \theta_w\}, \\ \widetilde{\mathbf{D}} = \mathbf{G}_{DU}\widetilde{\mathbf{U}}, \\ \widetilde{\mathbf{Q}}^T = \{Q_x, M_y, M_z, B, Q_y, Q_z, T_w, T_{sv}\}, \end{gathered} \quad (5)$$

In previous expressions, $\mathbf{G}_{DU}$ is a differential operator matrix, $\mathbf{M}_m$ is a matrix of mass coefficients. $\widetilde{\mathbf{P}}_X$ and $\widetilde{\mathbf{B}}_X$ are vectors of applied forces and applied boundary conditions, respectively. $\widetilde{\mathbf{Q}}^T$ is the vector of internal forces.

These internal forces are defined as:

$$\begin{aligned} &\{Q_x, M_y, M_z, B\} = \int_A \sigma_{xx}\{1, z, -y, w\}dydz, \\ &\{Q_y, Q_z\} = \int_A \{\sigma_{xy}, \sigma_{xz}\}dydz, \\ &T_w = \int_A \left(\sigma_{xy}\frac{\partial\bar{\omega}}{\partial y} + \sigma_{xz}\frac{\partial\bar{\omega}}{\partial z}\right) dydz, \\ &T_{sv} = \int_A \left[-\sigma_{xy}\left(z + \frac{\partial\bar{\omega}}{\partial y}\right) + \sigma_{xz}\left(y + \frac{\partial\bar{\omega}}{\partial z}\right)\right] dydz \end{aligned} \quad (6)$$

with:

$$\begin{aligned} \sigma_{xx} &= E_{xx}(y, z)\,\varepsilon_{xx} \\ \sigma_{xy} &= 2G_{xy}(y, z)\,\varepsilon_{xy} \\ \sigma_{xz} &= 2G_{xz}(y, z)\,\varepsilon_{xz} \end{aligned} \quad (7)$$

$$\begin{aligned} \varepsilon_{xx} &= (u_{x,x} + u_y/R)\,\mathcal{F} \\ 2\varepsilon_{xy} &= (u_{y,x} - u_x/R)\,\mathcal{F} + u_{x,y} \\ 2\varepsilon_{xz} &= u_{z,x}\mathcal{F} + u_{x,z} \end{aligned} \quad (8)$$

To model the porosity, assumed uniformly distributed (Fazzolari, 2018):

$$\begin{aligned} E_{xx}(z) &= (E_{xxc} - E_{xxm})\,V_c + E_{xxm} - \tfrac{\beta}{2}(E_{xxc} + E_{xxm}) \\ G_{xy}(z) &= (G_{xyc} - G_{xym})\,V_c + G_{xym} - \tfrac{\beta}{2}(G_{xyc} + E_{xym}) \\ G_{xz}(z) &= (G_{xzc} - G_{xzm})\,V_c + G_{xzm} - \tfrac{\beta}{2}(G_{xzc} + E_{xzm}) \\ \rho(z) &= (\rho_c - \rho_m)\,V_c + \rho_m - \tfrac{\beta}{2}(\rho_c + \rho_m) \end{aligned} \quad (9)$$

where:

$$V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^p \quad for \quad z \in \left\{-\frac{h}{2}, \frac{h}{2}\right\} \quad (10)$$

In Eq. (9) and Eq. (10), sub-indexes c and m identify Ceramic and Metallic counterparts, parameter $\beta \in [0,1], \Rightarrow \beta << 1$ identifies the level of porosity and exponent $p \in \mathbb{R}$ identifies the type of graded mixing.

2.3 Finite Element Discretization

A Finite Element formulation can be derived through discretization of Eq. (4). The discretization is carried out using isoparametric elements with five nodes and shape functions of quartic order (Piovan and Cortinez, 2007). The vector of kinematic variables can be written as:

$$\bar{\mathbf{U}}_e = \left\{\bar{\mathbf{U}}_e^{(1)}, ..., \bar{\mathbf{U}}_e^{(5)}\right\}, \quad \bar{\mathbf{U}}_e^{(j)} = \left\{u_{xc_j}, u_{yc_j}, \theta_{z_j}, u_{zc_j}, \theta_{y_j}, \phi_{x_j}, \theta_{x_j}\right\}, \quad j = 1, ..., 5 \tag{11}$$

Then, the following finite element equation is derived:

$$\mathbf{K}\bar{\mathbf{W}} + \mathbf{C}_{RD}\dot{\bar{\mathbf{W}}} + \mathbf{M}\ddot{\bar{\mathbf{W}}} = \bar{\mathbf{F}}. \tag{12}$$

where, $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices, $\mathbf{C}_{RD} = \eta_1\mathbf{M}+\eta_2\mathbf{K}$ is the Rayleigh damping matrix introduced as a part of an eigenvalue calculation in order to extract normal modes to reduce de discretized model. η_1 and η_2 are computed employing given damping coefficients (Piovan et al., 2013).

The response in the frequency domain of the linear dynamic system can be written as:

$$\widehat{\mathbf{W}}(\omega) = \left[-\omega^2\mathbf{M} + i\omega\mathbf{C}_{RD} + \mathbf{K}\right]^{-1} \widehat{\mathbf{F}}(\omega), \tag{13}$$

where $\widehat{\mathbf{W}}$ and $\widehat{\mathbf{F}}$ are the Fourier transform of displacement and force vectors, respectively.

3 CONSTRUCTION OF THE PROBABILISTIC MODEL

The Finite Element formulation of the previously developed deterministic model is employed as a reference expected response to construct the Probabilistic model. The Maximum Entropy Principle (MEP) is used to derive the probability density functions (PDF) of the random variables associated with the uncertain parameters (Jaynes, 1957). This particular is quite sensitive in stochastic analysis and PDF's should be deduced according to the given information (normally and sensitively scarce) about the uncertain parameters. The deterministic model developed in the previous sections has many parameters that can be uncertain, however the most relevant could be the exponent of the functionally graded variability, the level of porosity and curvature parameter that contemplates variation in the curvature radius.

In the present problem random variables V_i, $i = 1, 2, 3$ are introduced such that they represent the aforementioned parameters. The random variables have bounded supports whose upper and lower limits can be defined in terms of available information. The bounds, applying common definitions, are employed to calculate mean and standard deviation or the coefficient of variation, and viceversa. It is assumed that the mean value (calculated with the given bounds) of the random variable may coincide with the deterministic value in order to check convergence. Provided that there is no information about the correlation or dependency of material properties, random variables V_i, $i = 1, 2, 3$, according to MEP, are assumed independent and non correlated. Consequently, taking into account the previous context, the PDF's of the random variables can be written as:

$$p_{V_i}(v_i) = \mathfrak{S}_{\left[\mathcal{L}_{V_i}, \mathcal{U}_{V_i}\right]}(v_i) \frac{1}{2\sqrt{3}\underline{V}_i\delta_{V_i}}, i = 1, ..., 13 \tag{14}$$

where $\mathfrak{S}_{[\mathcal{L}_{V_i},\mathcal{U}_{V_i}]}(v_i)$ is the support, whereas $\mathcal{L}_{V_i}$ and $\mathcal{U}_{V_i}$ are the lower and upper bounds of the random variable V_i. $\underline{V}_i$ is the expect value of the i^{th} random variable, whereas δ_{V_i} is its coefficient of variation.

The Matlab function unifrnd$\left(\underline{V}_i\left(1-\delta_{V_i}\sqrt{3}\right), \underline{V}_i\left(1+\delta_{V_i}\sqrt{3}\right)\right)$ can be used to generate realizations of random variables V_i, $i=1,2,3$. Then, using Eq. (14) in the construction of the matrices of finite element formulation given in Eq. (13), the stochastic finite element model can be written as:

$$\widehat{\mathbb{W}}(\omega) = \left[-\omega^2\mathbb{M} + i\omega\mathbb{C}_{RD} + \mathbb{K}\right]^{-1}\widehat{\mathbf{F}}(\omega). \tag{15}$$

Notice that in Eq. (15) the math-blackboard typeface is employed to indicate stochastic entities.

The Monte Carlo method is used the simulate the stochastic dynamics, which implies the calculation of a deterministic system for each independent realization of random variables V_i, $i=1,2,3$. The convergence of the stochastic response $\widehat{\mathbb{W}}$ can be calculated with the following expression:

$$conv(N_S) = \sqrt{\frac{1}{N_{MS}}\sum_{j=1}^{N_{MS}}\int_{\Omega}\left\|\widehat{\mathbb{W}}_j(\omega) - \widehat{\mathbf{W}}(\omega)\right\|^2 d\omega}, \tag{16}$$

where N_S is the number of Monte Carlo samplings and Ω is the frequency band of analysis. Clearly, $\widehat{\mathbb{W}}$ is the response of the stochastic model and $\widehat{\mathbf{W}}$ the response of the mean model or deterministic model.

4 COMPUTATIONAL STUDIES

4.1 Preliminary validations and comparative studies

In this section a comparison and validation of the deterministic PFGM curved beam model with respect to other approaches is performed. The first example corresponds to a comparison of the present PFGM curved beam model reduced to the case of straight beam (i.e. $R \to \infty$), with respect to the beam model of Fazzolari (2018). The structure is constructed with Aluminium ($E_m = 70\ GPa, \rho_m = 2702\ Kg/m^3, \nu_m = 0.3$) and Alumina ($E_c = 380\ GPa, \rho_c = 3960\ Kg/m^3, \nu_c = 0.3$) configuring a PFGM beam with porosity $\beta = 0.2$ and material distribution given by exponent $P = 1.0$. The beam is clamped in both extremes. In Table 1 a comparison of Fazzolari's model and the present model (reduced to the case of straight beam) is shown. The frequency parameter evaluated is defined as $\widetilde{\omega} = \omega(L^2/h)\sqrt{\rho_m/E_m}$. It is possible to see the matching between both models. The percentage error is no greater than 0.32%.

	l/h				
	5	10	15	20	50
Present Red. Model	7.7609	8.6615	8.8696	8.9465	9.0325
Fazzolari Model	7.7388	8.6806	8.8962	8.9746	9.0594

Table 1: Comparison of frequency parameters of different approaches

Fig. 2 shows a comparison of frequency response functions of the present 1D model with a Finite Element 3D model for a PFGM beam with variable radius of curvature. The material constituents and properties were the same of the previous example (i.e. Aluminium, Alumina, with $P = 1.0$ and $\beta = 0.2$), but curved with the following law of variation: $R(x) = R_0 + \Gamma_c\,\theta\,x$ (i.e. an Archimedes spiral), where: $R_0 = 2.0\,m$, $\Gamma_c = 0.2$, $\alpha_0 = 2.0\,rads$. The damping parameters η_1 and η_2 for stiffness and mass matrices were calculated assuming damping coefficients of 10 % and 5 % in 1^{st} and 2^{nd} natural frequencies.

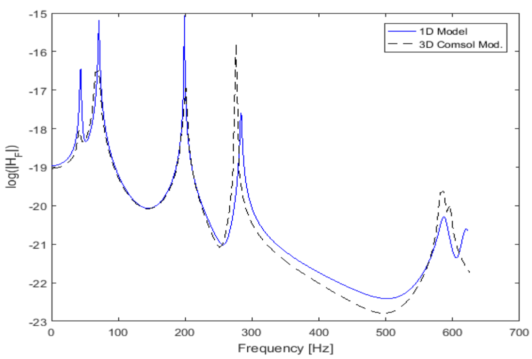


Figure 2: FRF of the present 1D curved beam and COMSOL 3D.

It is possible to see an acceptable matching between both approaches, despite the Rayleigh damping matrix was approximated only with two frequency terms. It certainly will match better if more terms were included in the calculation of Damping matrix as suggested by Bathe (1982) and Meirovith (1997). However, it should be soundly said that there is an important time saving in the calculation of the FRF. Actually, the 1D procedure was 5 times faster than the 3D approach, for nearly the same results.

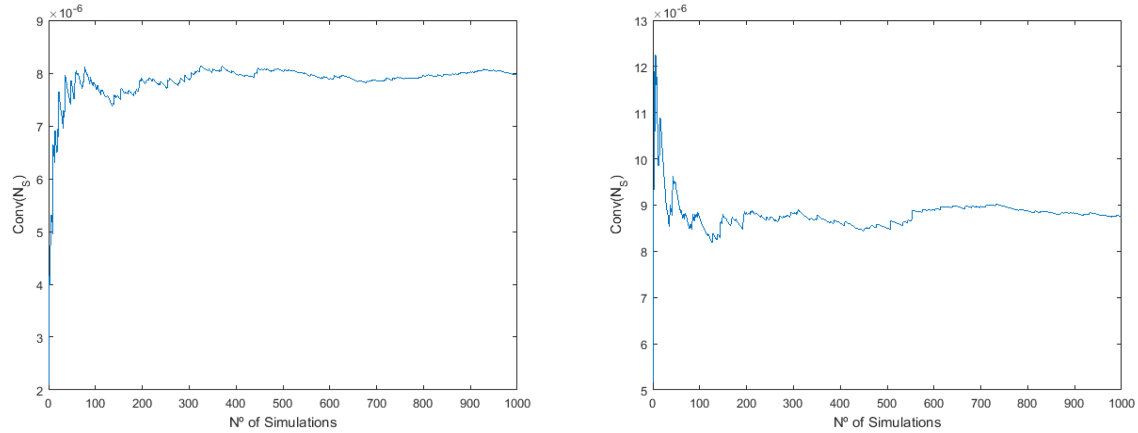
4.2 Uncertainty quantification of PFGM beams with variable curvature

In this section the propagation of uncertainty associated with three modeling parameters is carried out. The random variables to construct the probabilistic model are the variability in the curvature through parameter Γ_c, the variability of porosity, through parameter β and the variability of material distribution by means of the exponent P. The propagation of uncertainty is evaluated in the frequency response functions of a clamped-free curved beam with variable curvature of an Archimedes spiral (such that $R(x) = R_0 + \Gamma_c\,\theta\,x$, $R_0 = 2.0\,m$), with a sudden Impact Force $\|F(L)\| = 1.0\,N$ applied in the free end. The expected values of the random variables are: $\bar{\Gamma}_c = 0.2$, $\bar{\beta} = 0.1$, $\bar{P} = 1$.

The propagation of the uncertainty related to the aforementioned parameters is evaluated by three ways: (a) by taking each random variable alone and assuming the remaining with their nominal expected values, (b) by taking two by two random variables and the remaining deterministic and (c) taking into account all random variables in the realizations. Thus in Fig. 3 one can see the convergence of two cases of realizations with the coefficient of variation in all random variables ($CoV = 0.1$). In the most of realizations performed a stable convergence was reached from $N_S = 500$ Monte Carlo realizations.

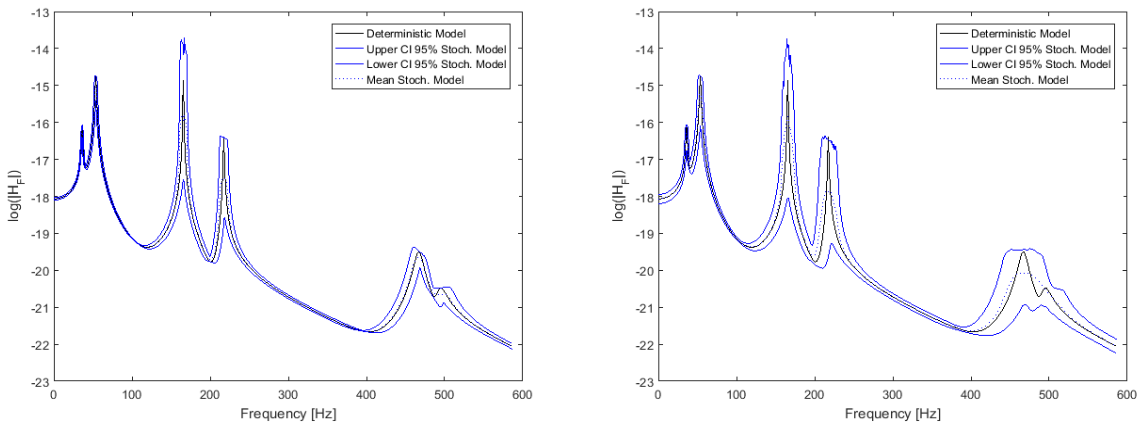
In Fig. 4 one can see the 95% Confidence Internal of of the Frequency Response Functions in the case of random variables with the same input Coefficient of variation: $CoV = 0.1$. Fig. 5 shows the Coefficient of variation of the frequency response outcome for different levels of input uncertainty. As it can be seen in Fig. 5 the uncertainty of the response is quite relevant in the frame of the third and fourth natural frequencies.

Fig. 6 shows the effect of different levels of input uncertainty and how they propagate in the response measured through an output coefficient of variation δ_{out}.



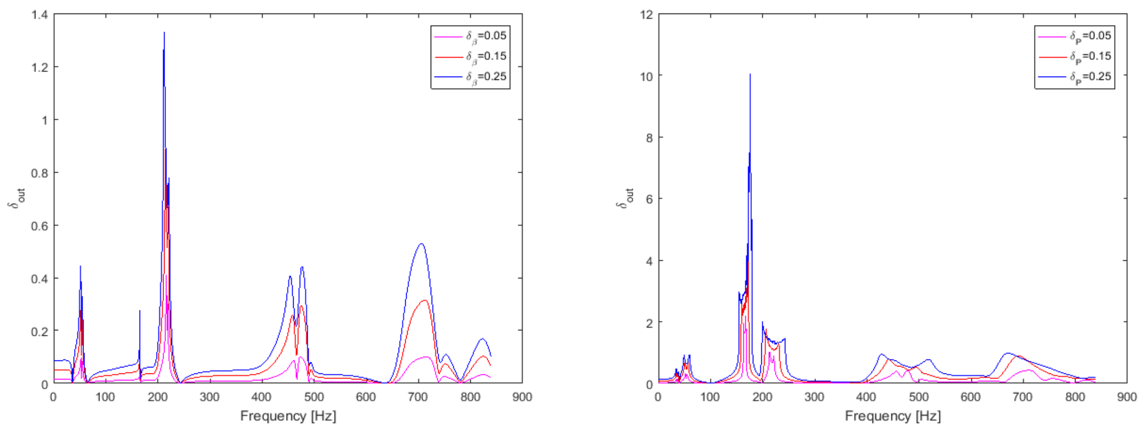
(a) (b)

Figure 3: Convergence for a case with $CoV = 0.1$. (a) Random curvature only, (b) All random variables.



(a) (b)

Figure 4: Confidence intervals for $CoV = 0.1$. (a) For the random variable Γ_c, (b) All random variables.



(a) (b)

Figure 5: Output δ_i for different inputs (a) For β only, (b) For P only.

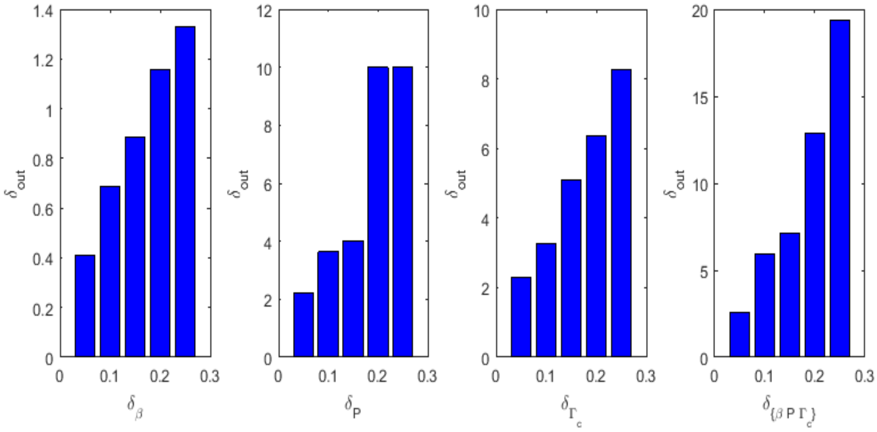


Figure 6: Sensibility of the parameters.

5 CONCLUSIONS

In the present paper a new model for porous functionally graded curved beams has been presented. The model can reproduce results of porous functionally graded straight beams of other authors. Also the curved beam model with variable curvature compares well with 3D approaches of the FEM implemented in commercial software. The following points are remarked:

- The variability of material properties propagates the uncertainty of the dynamic response.
- The uncertainty for graded properties is quite important.
- The uncertainty related to variable curvature radius is average.
- The uncertainty related to porosity is not so sensitive than the previous two.
- The variability of the response is intensely propagated in the vicinity of some natural frequencies.

Future extensions to this research will include: Incorporation of piezo and magnetic layers in a 1D formulation; the variation porosity and material properties as stochastic fields instead of random variables; incorporation of other types of damping models and the variability of properties with temperature, among others; Employment of reduced order modeling in order to accelerate computing time; use of this simplified 1D model to construct basic meta-structures to evaluate their sensitivity in stochastic dynamics.

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