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# STUDIES ON AEROELASTIC OPTIMIZATION OF CONSTANT AND VARIABLE STIFFNESS LAMINATES WITH FAILURE AND STABILITY CONSTRAINTS

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**Abstract.** This paper presents studies on the aeroelastic optimization of three dimensional wings subject to strength and stability constraints. The design of aircraft wings usually demands that multiple requirements are met at once, among them, limiting stress and aeroelastic instabilities. They must be avoided inside the flight envelope. Nowadays, laminated composites are being widely employed as primary material for the wing skins. These materials offer multiple parameters to achieve all design goals, which are usually lightweight and compliance to safety requirements. The proposed optimization problem considers the minimization of total elastic energy, with a stress constraint expressed in terms of a failure criterion and minimum aeroelastic instability onset above certain prescribed speed. Multiple points of a flight envelope are considered for the static loads evaluation. The fiber orientations are the design variables, and both constant and variable stiffness are considered. In the first case, a single orientation is defined for each laminate ply, and in the latter, the fiber orientation varies constinuously on the wing surface. An heuristic method, Particle Swarm Optimization (PSO), is used as the search algorithm to find the best design. The aerodynamic loads are computed with the doublet-lattice method, and the structural normal modes and static response are obtained with a finite element code. The stability is verified with a classic PK method. A wing reinforced with internal ribs and spars is studied. It is observed that the requirements can be adequately achieved, specially using variable stiffness. The flutter speed might be found inside the flight envelope and thus it need to be evaluated during the optimization process. The methodology can be readily employed in the design of subsonic wings made of composite material.

## 1 INTRODUCTION

The development of automated manufacturing techniques applied to composite materials allow the increase in efficiency of structures made of such materials. One example is the use of fiber placement and tow-steering. Instead of using pre-manufactures fabric with specific orientations, usually cross-ply with fibers aligned perpendicularly, the fibers can be placed in a variable orientation, design to achieve optimal performance. Investigations of such techniques have been receiving attention for some time, and have matured (Setoodeh et al., 2006; Dillinger et al., 2013).

The term aeroelastic tailoring has been used to describe the optimization of wings that aim to improve the behavior in flight when considering the interaction among aerodynamics and structural elasticity and dynamics (Weisshaar, 1981; Hollowell and Dugundji, 1984). The application of variable stiffness (VS) laminates to aeroelastic tailoring has shown to be effective (Stodieck et al., 2013; Stodieck et al., 2015). Those works usually aim to improve the speed where instability occurs or response to gust loads.

Examples of important applications of VS laminates are the wings of remotely piloted and very light aircraft. This type of structure are designed not only to comply with strength requirements, by means of failure factors computation, but aeroelastic stability evaluation is also mandatory. The structural design of wings starts with the definition of a flight envelope by means of characteristic flight speeds and extreme load factors, defining a boundary that specifies a safety zone for operation in terms of true airspeeds on the aircraft (Raymer, 1992). Flutter and divergence onset speeds must be computed, and they should not occur below a prescribed value.

In the present work, a formulation for aeroelastic optimization is presented and discussed. The proposed goal is to minimize the compliance considering multiple load cases. The constraints are a limit on the wing composite material failure and flutter and divergence onset speed. The design variables used to find a design that prevents bothe problems are the fiber orientations of the laminates. To search for the best design, the Particle Swarm Optimization (PSO) algorithm is applied here.

In Section 2 it is described the optimization problem and how the constraints are applied. Section 3 presents the aeroelastic and structural models along with the material failure criterion. The numerical analysis and discussion are shown in chapter 4. The chapter 5 presents the conclusion remarks.

## 2 OPTIMIZATION PROBLEM

The optimization problem considers the compliance minimization under limiting fail factors and instability onset airspeed as constraints, while fiber orientations, $\theta$, are the design variables:

$$
\begin{aligned}
&\min_{\theta}\ \overline{W} = \max(W_j(\theta)) \\
&s.t.\ V_d \geq V_{lim} \\
&\qquad F \leq F_{max} \\
&\qquad -90 \ \leq\ \theta\ \leq\ +90
\end{aligned}
\tag{1}
$$

where $V_d$ is the onset speed of aeroelastic stability, which must be above a defined $V_{lim}$, $F$ is the failure factor and $F_{max}$ is the prescribed limit for the failure factor. The design variables are defined between -90$^o$ and +90$^o$. The objective function $\overline{W}$ is the maximum compliance among the considered load cases. Compliance is defined here as the product between the applied load f and the resulting displacement $\mathbf{q}$ for each load case $j$:

$$W_j = \mathbf{f}_j^T \mathbf{q}_j \tag{2}$$

and it gives a measure of the elastic energy for each load case. Its minimization, for constant loads, represents the minimization of the total displacement vector.

Global search optimization methods are capable of analyzing several points in the domain at the same time. Heuristic approaches have a higher computational cost compared to gradient-based methods, due to the large number of evaluations of the objective function. Nevertheless, one advantage is that it does not need sensitivities information and it is more likely to converge to the global maximum/minimum of the analyzed functional, since they present a stochastic variable selection.

Among several stochastic domain search optimization methods, the PSO is one of the most stable and generic algorithms (Kennedy and Eberhart, 1995; Bratton and Kennedy, 2007). The particle in PSO is a set of design variables defined as a $N$ dimensional vector, where $N$ is the number of design variables. The search is grouped into a swarm of an arbitrary number of particles, $n_p$.

### 2.1 Penalized problem

To account for the constraints, the objective function is now redefined, by multiplying the maximum compliance by a productory containing each constraint penalization

$$\overline{f}_o = \overline{W} \left[ \prod_{j=1}^{m} \left( h^{p g_j} \right) \right] \tag{3}$$

where $m$ is the number of inequality constraints, $h$ and $p$ are penalization factors, and $g$ are modified constraints. For the case of failure factors (F) and instability airspeed (V), the constraints are rewritten as

$$g_j^{(F)} = \max \left[ 0, \left( \frac{F}{F_{max}} - 1 \right) \right] \quad g_j^{(V)} = \max \left[ 0, \left( 1 - \frac{V_d}{V_{lim}} \right) \right] \tag{4}$$

The penalized optimization problem is now a minimization problem with side constraints on the design variables:

$$\begin{aligned} &\min_{\theta} \ \overline{f}_o \\ &s.t. \ -90 \ \leq \ \theta \ \leq \ +90 \end{aligned} \tag{5}$$

and the PSO is used to obtain the best set of design variables to satisfy the problem.

A flowchart with the optimization procedure is seen in Fig.1. The structural and dynamic routines are highlighted, since they are the most time consuming processes, specially the modes computation necessary to obtain the generalized matrices and to compute the aeroelastic evolution.

## 3 AEROELASTIC MODEL

Two types of aeroelastic problems are considered in this work: a static response, corresponding to a distributed load computed at a certain point from the flight envelope, and a stability problem, corresponding to a flutter analysis.

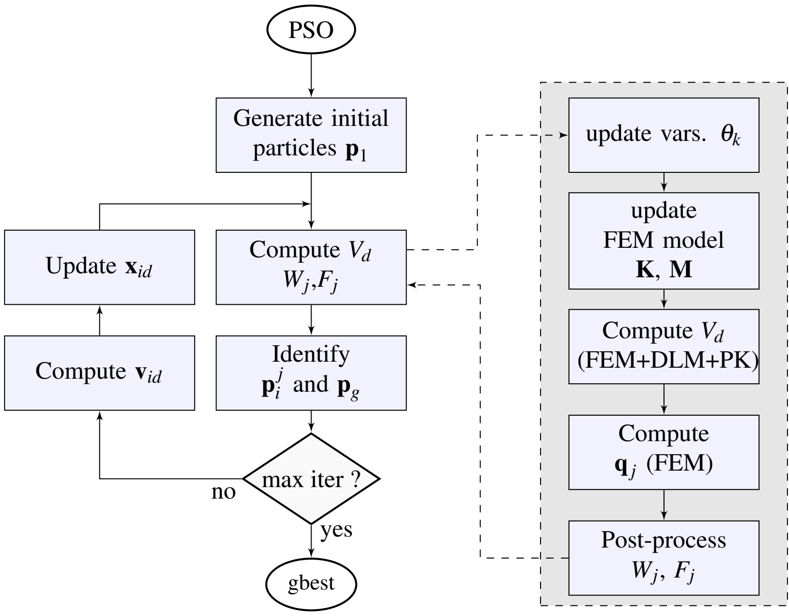



Figure 1: Flowchart of optimization and analysis process.

The linear static problem is defined as

$$\mathbf{K}\mathbf{q} = \mathbf{G}\mathbf{f}_a \tag{6}$$

where $\mathbf{K}$ is the structural stiffness matrix, $\mathbf{q}$ is the structural displacement, $\mathbf{G}$ is the interpolation matrix between the aerodynamic and structural models, and $\mathbf{f}_a$ is the aerodynamic force vectors computed at the aerodynamic panels.

The usual practice for flutter problem solution is to rewrite the structural equation of motion into the Laplace domain as a set of linear systems, which requires the assumption that the aerodynamic response is linear with respect to the structural deformation amplitude, for a sufficiently small amplitude. The equation of motion can be written in the generalized form in the Laplace space as

$$\left[\tilde{\mathbf{M}}s^2 + \tilde{\mathbf{B}}s + \tilde{\mathbf{K}} - q_\infty \tilde{\mathbf{Q}}(\mathrm{i}\ k)\right] \mathbf{X}_s(s) = 0 \tag{7}$$

where $\tilde{\mathbf{M}}$, $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{K}}$ are, respectively, the generalized mass, damping and stiffness matrices, while $q_\infty$ is the dynamic pressure, given by $\rho V^2/2$, where $V$ is the flow velocity. The aerodynamic force matrix $\tilde{\mathbf{Q}}(ik)$ is defined through the PK method, being decomposed into real and imaginary parts Rodden and Johnson (1994). The aerodynamic influence coefficient matrices are obtained using the non-planar version of the Doublet Lattice Method (DLM)(Albano and Rodden, 1969; Giesing et al., 1971) for a list of reduced frequencies.

The flutter solution considers the evolution of frequencies and damping as the airspeed increases, searching for the condition where the real part changes its sign. Results are usually displayed in the form of VGF plots, where graphs of $v \times f$ and $v \times g$ are shown together, allowing evaluation of the type of instability.

## 3.1 The structural model

The finite element method (FEM) is used to compute the structural matrices. The generalized matrices in Eq. (7) are obtained by the modal shape matrix $\mathbf{\Phi}$:

$$\tilde{\mathbf{M}} = \mathbf{\Phi}^{\mathsf{T}}\mathbf{M}\mathbf{\Phi}, \qquad \tilde{\mathbf{K}} = \mathbf{\Phi}^{\mathsf{T}}\mathbf{K}\mathbf{\Phi}, \tag{8}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the physical structural mass and stiffness matrices.The stiffness matrix is function of the fibers orientation, the optimization design variables.

The laminated wing is modeled using a 3-node triangular shell element, which is a combination of an optimal membrane element (OPT) (Felippa, 2003) and a discrete Kirchhoff triangle (DKT) bending element (Batoz et al., 1980). The element assembly proposed by Khosravi et al. (2007) is used, keeping only the linear form, with 6 degrees of freedom on each node. The formulation of the structural matrices are summarized by Khosravi et al. (2007). They are obtained according to the classical lamination theory (CLT), where the contribution of each lamina is obtained from integration in the normal direction. The lamina constitutive matrix is given in the laminate frame, which is obtained from the material coordinate system through a plane rotation that depends on the angle $\theta_k$. The reader is referred to classical FEM texts for the application of coordinate transformations in shell elements (Bathe, 1996; Hughes, 1987) .

The Hoffmann failure model is considered in the present work,as given in Jones (1999). It considers different strength on each direction, such that the factor that indicates failure, $F$, is computed as

$$F = F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{12}\sigma_1\sigma_2 + F_{66}\tau_{12}^2 \leq 1 \tag{9}$$

The parameters in the above equation are given by:

$$F_1 = \frac{1}{X_{1T}} + \frac{1}{X_{1C}} \quad F_2 = \frac{1}{X_{2T}} + \frac{1}{X_{2C}} \quad F_{11} = -\frac{1}{X_{1T}X_{1C}} \quad F_{22} = -\frac{1}{X_{2T}X_{2C}} \quad F_{66} = \frac{1}{S_6} \tag{10}$$

where $X_{iT}$ and $X_{iC}$ are the tensile and compressive strength of the lamina on the directions 1 and 2, and $S_6$ is the in-plane shear strength. For optimization purposes, the maximum failure factor for each load case on the whole model is considered.

## 4 NUMERICAL STUDIES

To evaluate the methodology, a 3D wing with internal structural elements is analyzed. The wing dimensions are shown in Fig. 2a. Internally, it contains two spars and several ribs in the leading and trailing edges. This configuration allows for a manufacturing with fiber placement on the skins over curved molds and final assembly of separate parts. The aerodynamic profile is the NACA 0012, which has curved surfaces and is symmetric, with maximum thickness of 12% of the chord length at a position equivalent to 30% of the chord. The spars are then placed at 20% and 40% of the chord.

The material considered is carbon fiber with epoxy matrix, with properties detailed in Table 1. It has a ratio $E_1/E_2$ equal to 4.2, what helps improving directional stiffness capabilities to reduce compliance and to avoid flutter and divergence. The tested models all considered two plies at the top and bottom skin. The internal structure is modeled as two-plies laminates, with [$0^o$/$90^o$] sequence, and remain constant for all models.

Unidirectional (UD) and variable stiffness (VS) laminates are considered in the study. In the first case, each layer is defined by a single orientation with angle $\theta$. In the latter, the fiber orientation is described by two variables, the fiber orientation at the root and at the tip, $\theta_r$ and $\theta_t$. This procedure for establishing the variable stiffness is similar to what was proposed by Gürdal and Olmedo (1993), to comply with manufacturing limitations. Here, to describe the local fiber orientations at a certain element, a linear variation between the root and tip of the wing is considered, so that:

$$\theta(x_2) = \frac{\theta_t - \theta_r}{L} x_2 + \theta_r \tag{11}$$

where $x_2$ is the coordinate along the span and $L = x_{2t} - x_{2r}$, the distance between the tip and root.

Table 1: Carbon fiber-Epoxy mechanical properties and strength.

| $E_1$ [GPa] | $E_2$ [GPa] | $\nu_{12}$ | $G_{12}$[GPa] | $\rho$ $[kg/m^3]$ |
|---|---|---|---|---|
| 290 | 69 | 0.25 | 4.962 | 1824 |
| $X_1^T$ [MPa] | $X_1^C$ [MPa] | $X_2^T$ [MPa] | $X_2^C$ [MPa] | $S_6$ [MPa] |
| 621 | 621 | 13.8 | 193 | 27.6 |

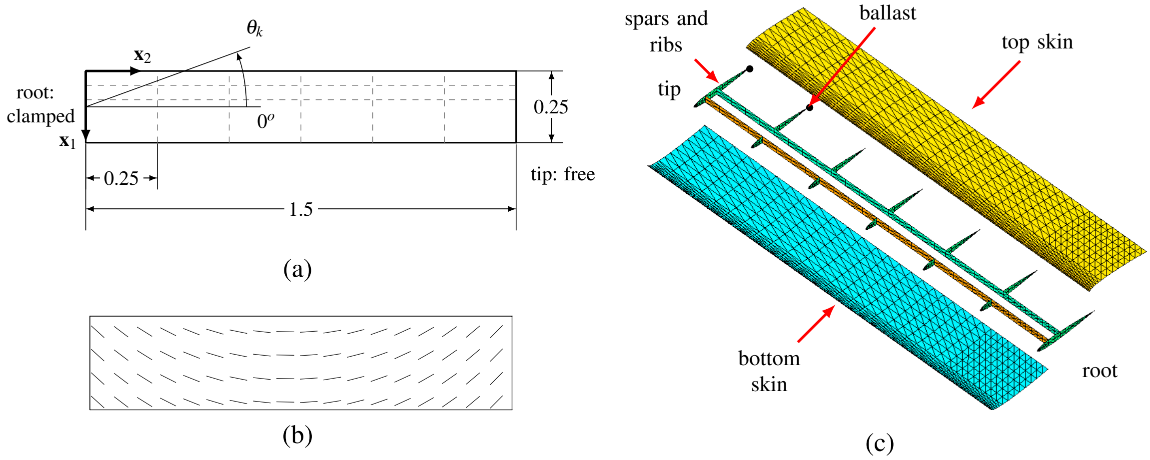


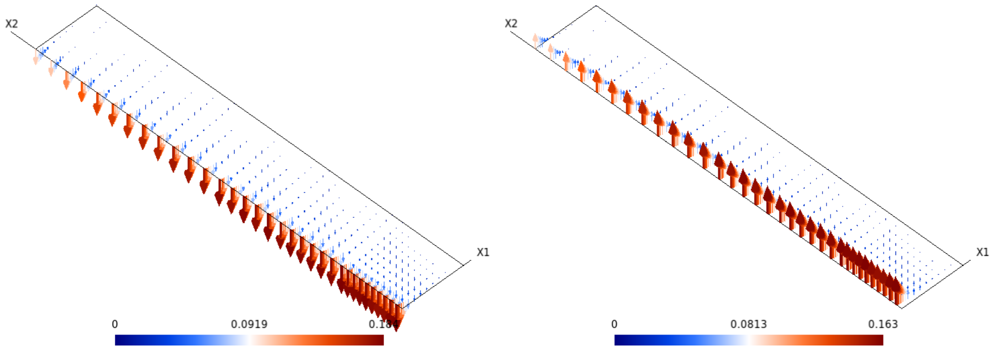

Figure 2: Wing dimensions (a), example of VS distribution (b) and internal structure details (c).

Two static load cases are considered, aiming to represent points of a flight envelope (V-n diagram), with negative and positive loading on the wings at cruise speed or maximum design speed, for example . The speeds of 100 and 110 m/s are considered, and the resulting loads are shown in Fig. 3 for both load cases. The aircraft certification usually requires that flutter or divergence must not occur inside the flight envelope, and a safety margin is established. To evaluate the ability of the proposed framework to satisfy that requirement, limiting speeds of 120 and 180 m/s are used here as constraints ($V_{lim}$), to represent increasing requirements to be satisfied.

As usual in heuristic methods, the optimization study considered multiple runs of the PSO method, with different parameters $n_p$ and $n_{iter}$. Table 2 shows a summary of results obtained for UD and VS laminates. For UD, only one orientation is sought for each ply, and for VS two design variables are defined for each ply in the optimization set. The parameters $h$ and $p$ from Eq.(3) are both set equal to 2, what showed to guarantee that unfeasible designs be avoided. The interations history of the objective function (Fig. 4a) shows that many unfeasible designs are found in the begging of the process (higher values not shown). Some of the design variables converge slowly, mostly after 200 steps (Fig. 4b).

The failure factor below the limit of 0.5 is easily met, which presented only feasible designs, avoiding constraints violation. Failure constraints are not active for most of the iterations, as seen in Fig. 5.

The aeroelastic stability, on the other hand, is more difficult to guarantee for the upper limiting case of 180 m/s. It was observed that for the present lamination orders and thickness, the




(a) Case 1: 100 m/s, $\alpha = -10^o$

(b) Case 2: 110 m/s, $\alpha = +8^o$

Figure 3: Force vectors for each load case.

$V_g$ speed is not found below 100 m/s, also seen in Fig. 5, even at initial stages of PSO, where random orientations are considered for evaluation. However, when the limit is set at 180 m/s, the speed constraints are active for most part of the search, and the best value is defined at the verge of the envelope. Figure 6 shows the $VGF$ curves for the best designs when considering 120 and 180 m/s as limits. It is observed that the instability occurs exactly at the limiting speeds for 180 m/s, but for 120 m/s a safety margin is still available at the best design point found (UD and VS in Table 2).

The best design for a UD and VS laminates are shown in Fig. 7, for both top and bottom skins. The UD solution, in Fig. 7a, tend to combine directions mostly aligned with the span, helping to reduce bending, with orientations that increase torsional stiffness ($12.95^o$ and $14.84^o$), what acts in the way of preventing divergence and flutter at lower flight speeds. It is observed that the VS laminate offer more options for minimizing the compliance, and thus displacements, by aligning the fibers with $x_2$ direction near the wing root. The orientations at the tip are more effective in reducing torsion, in directions predicted by Shirk et al. (1985) to prevent aeroelastic instability.

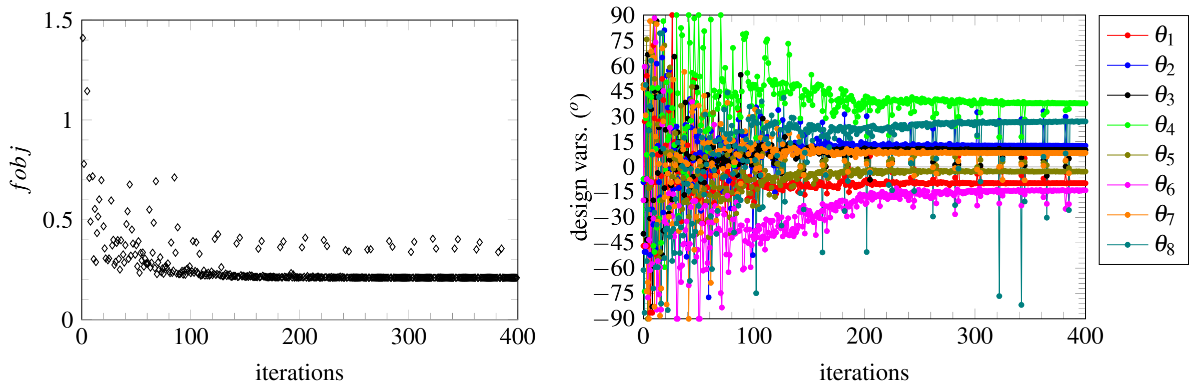
| laminate | $n_{vars}$ | $n_p$ | $n_{iter}$ | $V_{lim}$ | $F_{max}$ | $f_{obj}$ | $V_d$ | $F$ |
|---|---|---|---|---|---|---|---|---|
| UD | 4 | 20 | 200 | 120 | 0.5 | 0.2244 | 128.28 | 0.1561 |
| UD | 4 | 20 | 200 | 180 | 0.5 | 0.2478 | 180.51 | 0.1276 |
| UD | 4 | 40 | 400 | 180 | 0.5 | 0.2116 | 180.40 | 0.1261 |
| VS | 8 | 20 | 200 | 120 | 0.5 | 0.2668 | 121.66 | 0.1886 |
| VS | 8 | 20 | 200 | 180 | 0.5 | 0.3181 | 180.23 | 0.1276 |
| VS | 8 | 20 | 400 | 180 | 0.5 | 0.2102 | 180.05 | 0.1189 |
| VS | 8 | 40 | 400 | 120 | 0.5 | 0.2781 | 127.08 | 0.1787 |
| VS | 8 | 40 | 400 | 180 | 0.5 | 0.2511 | 181.61 | 0.1813 |

Table 2: Best values found for VS laminates.

## 5 CONCLUSIONS

A strategy for aeroelastic optimization of wings made of laminate composite materials was presented and discussed. The process is applied to laminates made of single orientation plies (unidirectional stiffness) and to plies with continuously variable orientation (variable stiffness).

As it can be seen in results section, the strength constraint is not active at the optimized




(a) (b) $V_{lim}$=180 m/s and $n_{max}$ = 400

Figure 4: Iterations history of objective function and design variables for VS laminate with $V_{lim}$=180 m/s and $n_{max}$ = 400.

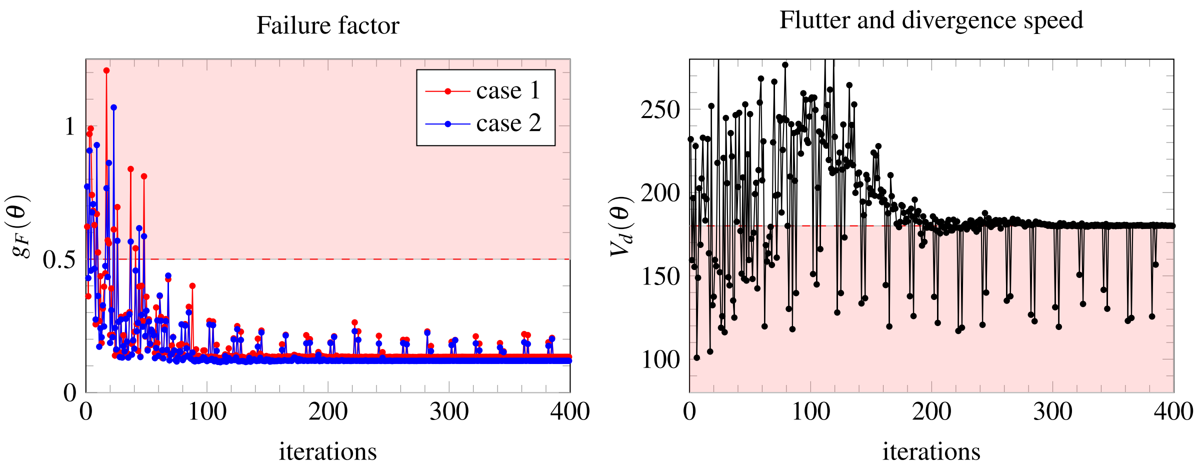



Figure 5: Iterations history of failure factors and instability speed for VS laminate, $V_{lim}$=180 m/s and $n_{max}$ = 400. The unfeasible regions are highlighted.

designs, however, during the search it showed to be important as a penalization factor on the objective function. The aeroelastic stability onset (flutter speed), however, is activated in the found solutions, favoring the optimized design definition. This is a very interesting result, since it raises the question on the necessity in considering this failure function on the optimization problem. The compliance minimization problem also helps to lower the wing mechanical failure, by indirectly lowering the tip displacement, stabilizing it in a safe region.

This research is a first approach on the study of instability analyses along with some mechanical strength criteria for the wing's composite material. Although this issue needs more investigation, this research gives important insight on this subject.

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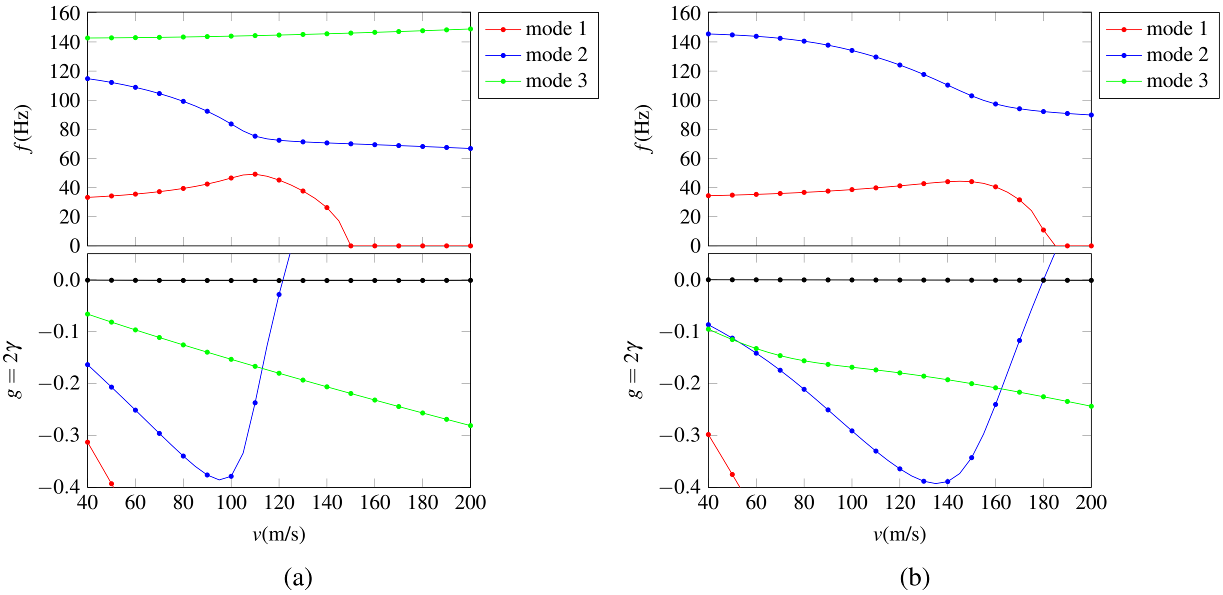


Figure 6: Aeroelastic evolution curves at the best points for: (a) $V_{lim}$=120 m/s and (b) $V_{lim}$=180 m/s.

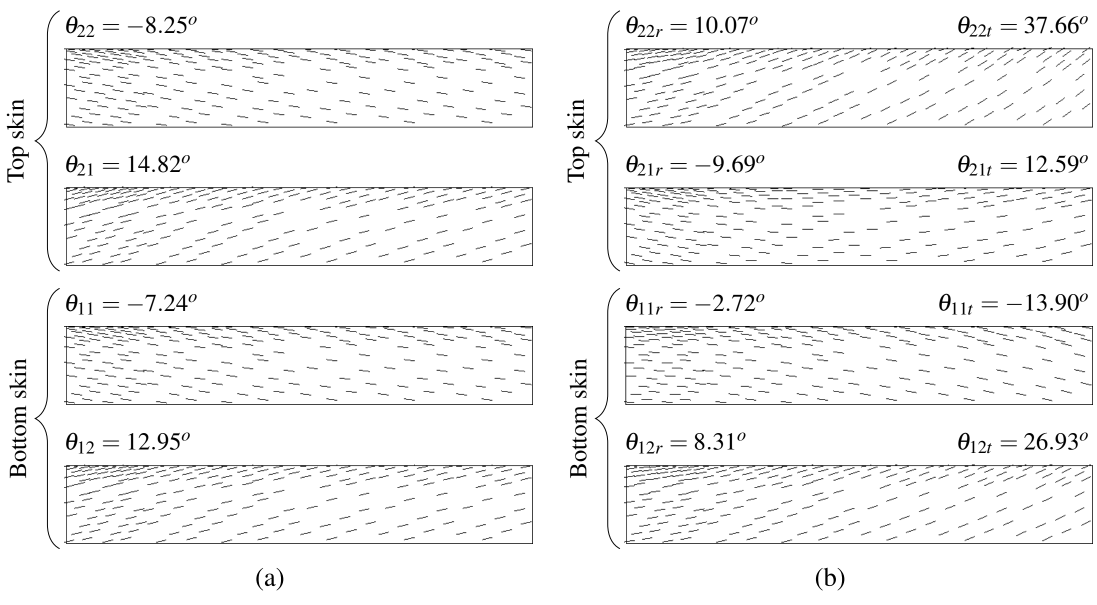


Figure 7: The best designs for $V_{lim} = 180$ m/s: (a) UD laminates and (b) VS laminates (results sampled every 5 elements).

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