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WAVE PROPAGATION ANALYSIS AS A GLOBAL STIFFNESS MEASUREMENT IN THE INVESTIGATION OF FIBER ORIENTATION EFFECT OF ORTHOTROPIC BEAMS

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Abstract. As products made of composite materials become more applicable in many engineering sectors throughout the years, arises the need for advanced numerical methods which can enhance project development regarding innovation in these areas. Specifically for applications of constant cross section parts, i.e pultruded structures, understanding which and how geometric design aspects and material properties change the stiffness is a key point for the design of the springs. Such parameter can be influenced by several design variables such as fiber orientations and its distribution along the cross-section since the orthotropic behavior of composite materials are mainly guided by it. Methodologies to numerically evaluate the stiffness of structures often use the finite element method, in which the results are highly dependent on symmetry and shapes of boundary conditions. To overcome this, a global analysis can be carried out by solving an eigenvalue problem using the wave propagation approach, which relates the wave speed propagation inside the structure with its stiffness. In that sense, this work presents a numerical evaluation of the fiber orientation effect in wave propagation phenomenon to globally analyze the stiffness of a straight, orthotropic and perfectly coupled beam. SAFE and Block-Floquet methodologies are carried out together with the finite element method to obtain the elastodynamic behavior and the results are validated with modal analysis, which is used to study and compare the fundamental modes: bar, shaft and beam modes. Once validated, different sections and fiber orientations are explored while maintaining the same beam cross-section properties: area and moment of inertia. Results demonstrate how bending and longitudinal stiffness are decreased to increase torsion stiffness when fiber orientation or cross-sectional shape changes. Finally, this paper successfully delivers the study of how the combination of geometrical aspects and fiber orientation globally affects stiffness, employing a relatively simple methodology. http://www.amcaonline.org.ar.

1 INTRODUCTION

As the world walks into sustainable development in many sectors, composites arise as promising materials which can contribute to these changes. Their high strength-to-ratio, durability, and corrosion resistance, among other properties, make their use valuable in several applications. Among the manufacturing methods of these materials, pultrusion is particularly efficient when a constant cross-section is required with a wide range of lengths.

This technique is capable of producing both closed and open-section profiles and with a wide range of fiber orientation distribution across different regions of the cross-section. Pultrusion happens by pulling reinforcement fibers through guides, bathing them in resin material and then placing them in a heated chamber, allowing the resin to cure. This is a continuous process - simultaneously, a saw cuts the profiles into parts of the desired lengths. Pultruded profiles present interesting performance in many sectors while still being cost-efficient and satisfying mechanical, physical and environmentally resistant properties.

These profiles are widely adopted in different applications such as bridge construction, bridge decks, cooling tower components, building elements, marine construction, transportation and energy systems. Often among these applications, structural components can be simplified into beam-sort of components in terms of design, as shown in Figure 1. In each application these components are subjected to different loading conditions - usually in a combination of torsion, flexion and traction, which have different contributions to the total load (Vedernikov et al., 2020).

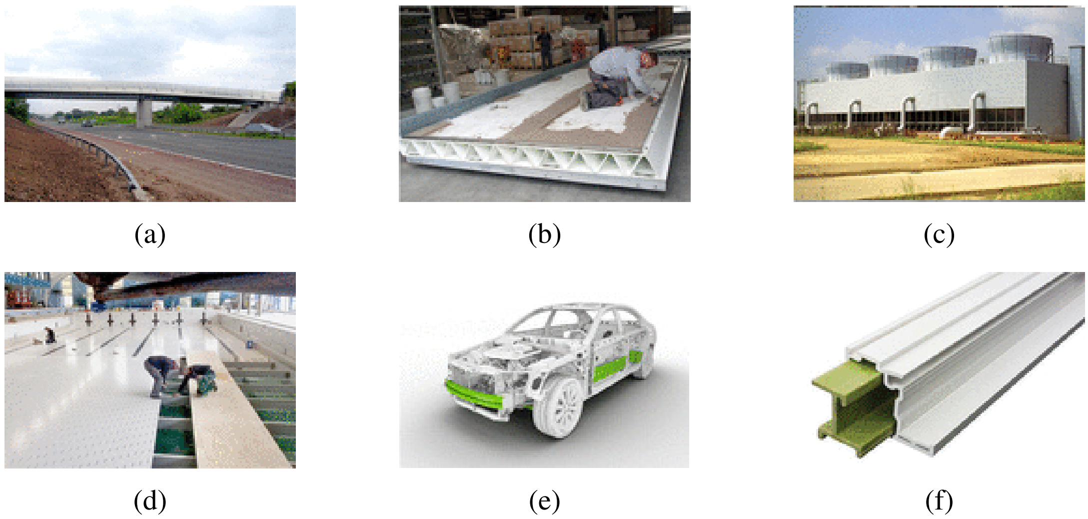

Figure 1: Examples of constant cross-section composites structures: a) Bridge construction; b) Bridge deck; c) Cooling towers; d) Water-related construction; e) Automotive parts; f) Window elements (Vedernikov et al., 2020).

In this scenario, new projects of composite parts can become a hard and time-consuming task due to the high number of project variables involved - in traditional approaches such as static analysis using FEM, how boundary conditions are applied have a significant impact on the stiffness of the structure and are generally more computationally expensive than modal analysis. In this matter, the current work takes advantage of the constant cross-section of extruded parts to evaluate their free vibrational modes and frequencies to extract information about the stiffness distribution, such characteristic is directly proportional to the frequency value, which in turn is directly proportional to fiber orientation distribution when mass remains constant. Moreover, this can be evaluated for all different vibrational modes, but in this study, only modes of flexion, traction and torsion are investigated, which are known as fundamental modes for waveguides.

One of the approaches that can be used to achieve this goal is through guided wave propagation simulating methods like Bloch-Floquet and Semi-Analytical Finite Element (SAFE). Such a theme is well established in cases of infinitely wide plates, rods and pipes (Rose, 2014) and has a wide range of applications in non-destructive testing, as can be seen in Ahmad and Gabbert (2012); Mazzotti et al. (2012) among others. Numerical methods to model guided wave modes in bars with rectangular cross-section exist, being first described by Mindlin and Fox (1960). One other numerical tool for stiffness evaluation is through modal analysis, using the standard finite element method (FEM), which is easily applicable.

The dispersion curve of a structure is simple the pairs frequency f and wavenumber k (or wavelength λ) in which it can responses to some excitation. This dispersion curve is very useful to evaluate properties (Nečiūnas et al., 2018) and predict dynamic behaviour in real applications (Lissenden et al., 2015).

Using commercial software packages and custom routines developed by the authors, the behavior of guided waves inside a rectangular-shaped beam made of composite material was evaluated to explore the effect of different fiber orientations on structure stiffness and compare them with a reference case of isotropic material acting as a “benchmark” for the stiffness evaluation. The anisotropic behavior is verified to be correctly modeled inside the commercial software used by submitting the models through a test in which a constant strain is applied to a cube, more details are given in chapter 3.3. The numerical methods used are compared qualitatively in the case of modal analysis and quantitatively between SAFE and Bloch-Floquet for the main cases. Results presented in the form of dispersion curves and vibration modes demonstrate the stiffness of each composite structure when subjected to bending, traction, and torsion loads.

2 METHODOLOGY

2.1 Applied materials

Material properties considered for this study are maintained constant along all routines and were obtained by a micromechanical approach using *Mech G-Comp* (Angrizani et al., 2017) online tool with 78% in weight of *Fiberglass S2* and 22% of epoxy resin considered, with a homogeneous density of ρ = 1992.95 kg/m³ and the following constitutive matrix in case of unidirectional fibers oriented longitudinally shown in Equation 1.

$$\boldsymbol{C} = \begin{bmatrix} 60.20 & 8.07 & 8.07 & 0 & 0 & 0 \\ 8.07 & 21.56 & 8.31 & 0 & 0 & 0 \\ 8.07 & 8.31 & 21.56 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.89 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6.20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6.20 \end{bmatrix} \text{GPa} \tag{1}$$

2.2 Modeled geometries

The geometry studied was a prismatic beam of width w = 10 mm of side and length L = 1000 mm (for the modal analysis, since SAFE and Bloch-Floquet require only the cross-section to be modeled). The models evaluated are formed by a nucleous and until two layers of different fiber orientations, respectively, are counted from the center to the boundary of the structure.

A cylindrical tube of radius $R = 114.15$ mm with the same length L was also tested, keeping the moment of inertia of cross section constant in all cases Equation 2. The fiber orientations follow the usual micromechanical local coordinate system convention: the 1-axis being the direction of the fibers and the 2-axis and 3-axis are perpendicular to them, while the orientation definition is the two decomposed angles θ and ϕ of the new 1'-axis and the reference 1-axis (see Figure 2).

$$I_{rectangular} = \frac{wh^3}{12} \qquad I_{tube} = \frac{\pi(2R)^4}{64} \qquad R = \frac{1}{2}\sqrt[4]{\frac{16wh^3}{3\pi}}. \tag{2}$$

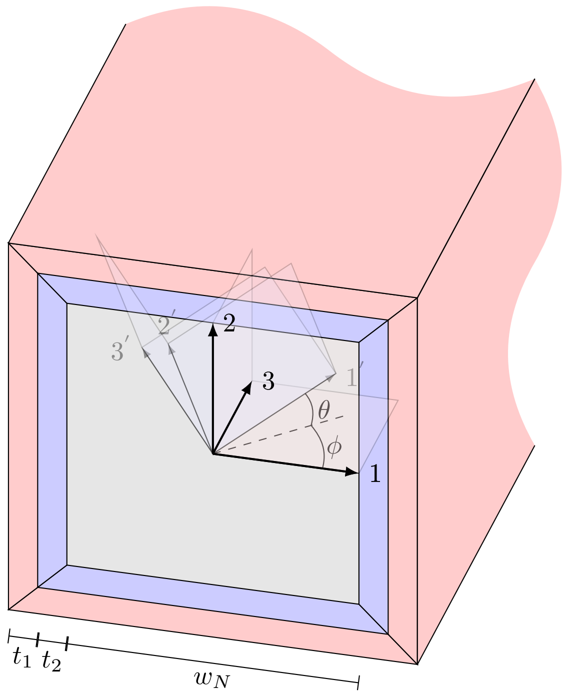

Figure 2: 3D beam representation and axis orientation with rotations.

The Table 1 shows all orientations of the nucleous and the layers if it is present in that case. More ilustrations about the beams are shown in results section Figure 4.

2.2.1 Optimization analysis

Table 1 presents the rotational angles studied. Not all the possibles angle orientations were covered and performing a full matrix analysis was not possible due to computational time.

To overcome this restriction, an amoeba optimizer (Lagarias et al., 1998) was applied only in a four-layer pipe (1 in Figure 4). The main idea is find high stiffness. To do so, dispersion curve of the pipes were evaluated. The dispersion curves were calculated using the methodology presented in subsection 2.5. From the dispersion curve the stiffness of the structure was estimated - the higher the velocity, higher is the stiffness. As for the objective function in the optimization analysis, minimizing slowness (i.e. the inverse of velocity) was chosen and its equation is given by Equation 3:

$$O(\theta_l) = \sum_{k=k_i}^{k_f} \sum_{\text{mode}=1}^{4} \frac{1}{c_p^{\text{mode}}(\theta_i, k)} \tag{3}$$

Model	t_1		t_2		w_N	
	ϕ [°]	θ [°]	ϕ [°]	θ [°]	ϕ [°]	θ [°]
1	-		-		90	45
2	90	45	90	45	90	0
3	90	45	90	30	90	0
4	90	45	90	0	empty	
5	90	45	-		90	0
6	90	45	0	0	90	0
7	-		-		90	30
8	-		-		90	45
9	0	0	90	45	90	0
10	90	30	-		90	0
11	-		-		90	0
12	0	0	-		90	0
13	-		-		90	60
14	-		-		0	0

Table 1: Fiber orientations of models evaluated.

where c_p^{mode} is the phase velocity of the wave mode, k is the wave number that was summed in the discrete range $[0, 10]$ [1/m], the mode stands for the torsional (1), flexural a (2), flexural b (3), and longitudinal (4) modes, and θ_l is the angle of the layer l. In this case, there are four angles (one for each layer) supporting a range of $[0, 90]$ degrees.

Figure 3 depict the behavior of the optimizer vs the sum of the four layers angle. It was observed, by inspection, that low angle values produced a high stiffness in the wave guide with low dispersion in the data. Higher angles also produces low dispersion in the objective function. Those observations leads to the choosing of angles around 0/90 degree angle for the others geometry.

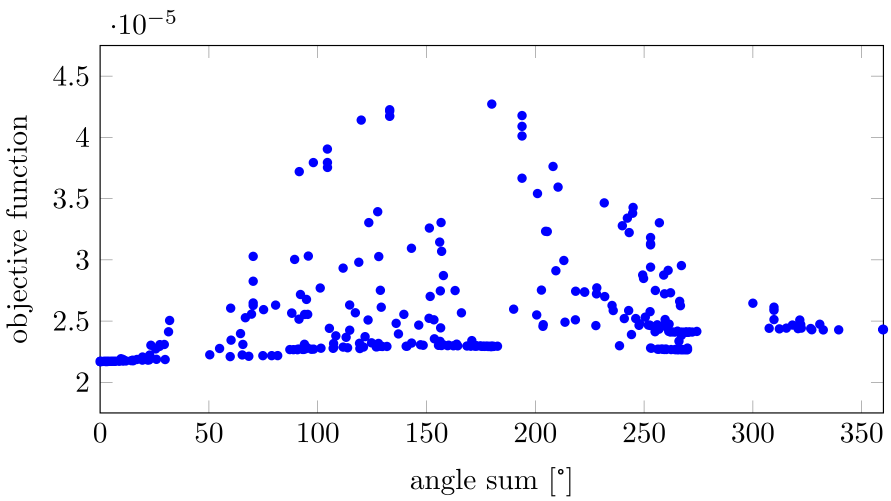

Figure 3: Evaluation of Equation 3 by the sum of θ_i.

2.3 Method #1 - Modal analysis

The finite element method (FEM) is probably the most famous numerical method nowadays to solve complex engineering problems, there are many references explaining the fundamentals in the literature, some of them are Bathe (1996); Reddy (2006); Zienkiewicz and Taylor (2005). The calculation performed using this method is through modal analysis, which outputs natural frequencies and modes of a structure under free vibration. It can be broken down into a resolution of an eigenvalue and eigenvector problem of the following system of equations, where $\boldsymbol{M}$ refers to the mass matrix, $\boldsymbol{Q}$ the degrees of freedom vector (eigenvectors), $\boldsymbol{K}$ the stiffness matrix as can be shown in Equation 4. Damping in all cases evaluated in this work is disregarded. The resulting natural frequencies are related to the stiffness and mass of the structure, so if the mass is considered as uniformly distributed and the only variable becomes the fiber orientation distribution, stiffness can be controlled according to how fibers are aligned inside the composite structure.

$$\left[\boldsymbol{K} - \omega^2 \boldsymbol{M}\right] \boldsymbol{Q} = 0 \tag{4}$$

Because of the anisotropy of the fibers, the dispersion couldn't be found through pure modal analysis because the wavenumber k (nor wavelength λ) are not explicitly in Equation 4. To overcome this fact the study is made qualitatively for this case. The solver for the analysis is *Ansys Mechanical APDL Lanczos Block*. In isotropic cases, the wavenumber parameter can be found using post-processing over the deformed shape of the beam through fast-Fourier-transform (FFT), for more details see (Groth et al., 2020; Sorohan et al., 2011).

2.4 Method 2 - SAFE

The semi-analytical finite element method (SAFE) works like standard plane FEM from a mathematical point of view. Only the cross-section is discretized and a harmonic function is applied to the displacement formulation to emulate a plane wave propagation in the axial direction. In this harmonic function the wavenumber k parameter appears explicitly in the derivatives, due to this, in the final eigenvalue problem the stiffness $\boldsymbol{K}$ depends on k. The formulation is shown in Equation 5. In this work, the SAFE is all implemented in Matlab and uses its linear eigenvalue solver.

One of the fundamentals of SAFE is that in axial directions the properties are constant, which limits its use in isotropic and transversely isotropic structures. For more details, see the refs Viola et al. (2007); Hayashi et al. (2003); Ahmad et al. (2013).

$$\left[\boldsymbol{K}(k) - \omega^2 \boldsymbol{M}\right] \boldsymbol{Q} = 0 \tag{5}$$

2.5 Method 3 - Bloch-Floquet

The Bloch-Floquet periodicity is an extension of SAFE capabilities, its formulation is the same as standard FEM which the degrees of freedom matrix $\boldsymbol{Q}$ is transformed by coupling boundary nodes through with a function that emulates a wave crossing this structure. The effect of this coupling is the appearance of k parameter in both mass $\boldsymbol{M}$ and stiffness $\boldsymbol{K}$ matrices. Because of this coupling, in the axial direction, the discretization needs a minimal axial length related directly to the maximum wavenumber k which can be formed. The final eigenvalue problem is shown at Equation 6.

$$\left[\boldsymbol{K}(k) - \omega^2 \boldsymbol{M}(k)\right] \boldsymbol{Q} = 0 \tag{6}$$

This method is extensively applied to periodic structures as can be seen at Gomez Garcia and Fernández-Álvarez (2015); Collet et al. (2011); Hakoda et al. (2018) and does not have any limitations on the isotropy of structure.

3 RESULTS

3.1 Modal analysis

The modal analysis is performed for all 14 cases shown in Table 1 and the results are shown at Figure 4. It can be noticed that for some cases as 11 and 12 the bending and traction stiffnesses are high while being low for torsion, and this difference diminishes as more external layers with $\theta = 90$ and $\phi = 45$ orientations are inserted in the cross-section. $\pm$ 45° orientation is known to be the stiffest fiber orientation under torsion load, so if the objective is maximizing stiffness in this condition, choosing this fiber angle near the borders of the cross-section is a good option. Plus, because of the J inertia parameter in torsion, the most efficient geometry for this case is the pipe, model number 1. On the other hand, it significantly sacrifices flexion and traction mode frequency value.

Now, if the main goal is considered to find the best geometry for support bending and traction (longitudinal movement) the most satisfactory results are found when fibers are oriented purely axially in models with the greater nucleus oriented at $\theta = 90$ and $\phi = 0$, in models 10, 11 and 12. In case when the objective is to have an average stiffness for the three load cases, then the models that would best fit are 4 or 5.

3.2 SAFE and Bloch-Floquet

The SAFE is applied only for isotropic and transversely isotropic cases, which are the numbers 11 and 14. The Bloch-Floquet periodicity was used in the other two cases of interest, the pipe model number 1 and the model number 5. The results of these dispersion curves for modes longitudinal, bending, and torsion are shown in Figure 5, being the velocity graphs computed as shown in Equation 7.

$$c_p = \frac{\omega}{2\pi k} = \frac{f}{k} \tag{7}$$

As can be seen in Figure 5 the bending mode varies with k parameter (or λ parameter) and in this case the stiffness varies. It is also valid to say that only the lower stiffness bending mode is calculated, the same was computed in modal analysis. Because the modal analysis search algorithm points to the lower stiffness modes, it corresponds to lower k values qualitatively in dispersion curves, which in the graph show that models number 1 and 11 are the stiffer ones. For upper k values, the graph shows the stiffer models as the numbers 11 and 14 because of the bending mode quadratic variation ($f(k^2)$). In this case, the best configuration needs to be selected based on predicting the wavenumbers k and wavelengths λ which could be formed.

The torsion and longitudinal modes have a linear relationship until certain ranges of k values ($f(k)$) with constant velocity c_p, then it is possible to find an equivalent shear modulus G^* and equivalent axial modulus E^*, both for torsion mode and longitudinal mode, respectively. These values are computed based on Equation 8 derived from standard bar and shaft theories (for more details, see (Graff, 2012)). These results are shown in Table 2.

$$\begin{aligned} E^* &= c_p^2 \rho \quad \text{longitudinal mode} \\ G^* &= c_p^2 \rho \quad \text{torsion mode} \end{aligned} \tag{8}$$

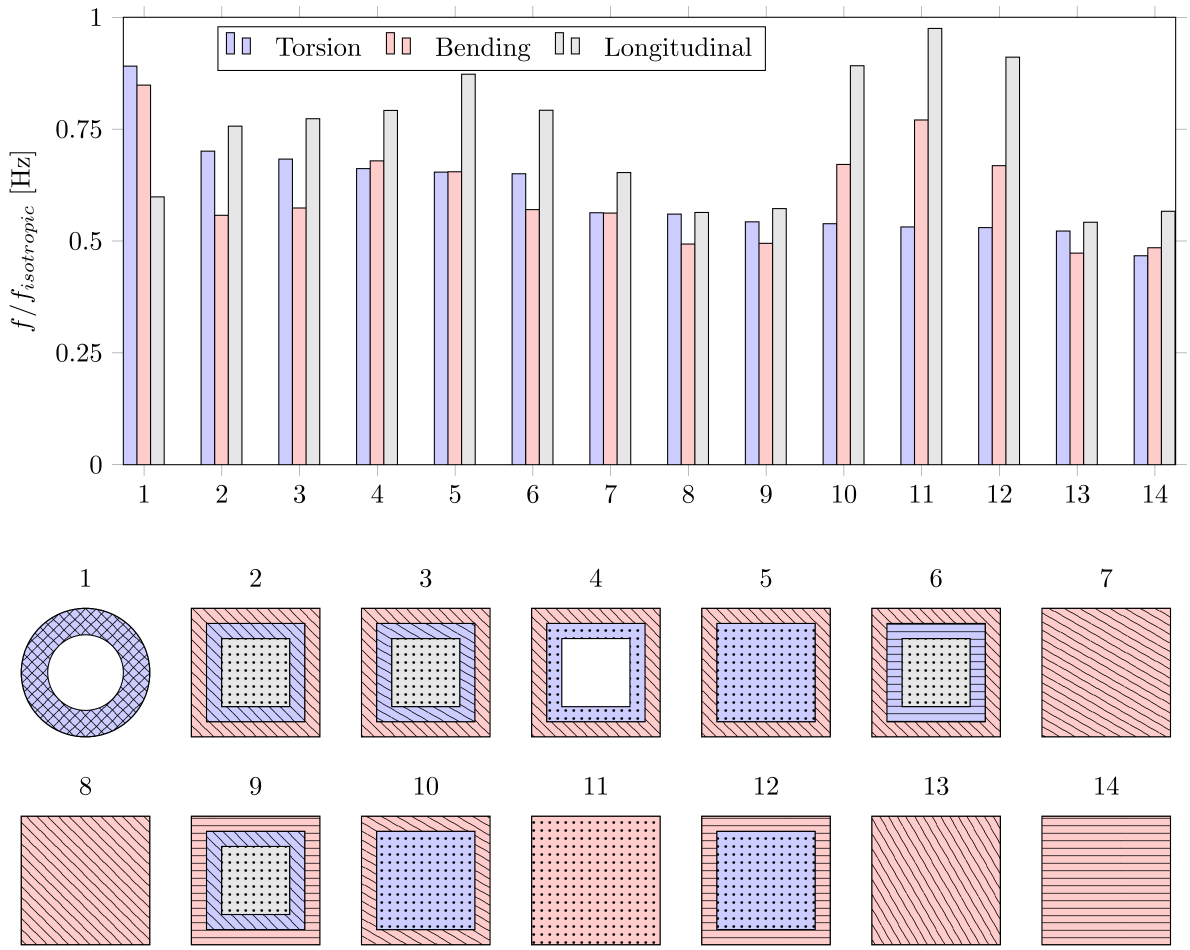

Figure 4: Results of modal analysis.

From Table 2 is possible to see that because of J inertia moment of a circle being bigger, the pipe model is the stiffest among them, however, it sacrifices too much for traction resistances as could be seen for model number 11 with fibers oriented axially. In the dispersion curves always the torsion mode is less stiff than the longitudinal mode, independent of configurations selected for the fibers.

4 CONCLUSIONS

This work presented numerical approaches to globally evaluate a constant-cross-section composite part stiffness through its natural frequencies without boundary condition application while still considering its fiber orientation distribution. Modal analysis showed that the structure's stiffness can become higher under torsion when adding fiber orientations at $\phi = 90$ and $\theta = 45$ and lower under bending and traction. Plus, maintaining the same moment of inertia, adding void to the center of the cross-section the torsion stiffness slightly rises due to its geometrical shape but sacrifices a lot on traction. In regards to the results obtained via the SAFE and Bloch-Floquet methodologies, the work showed that not only they are in agreement with the results obtained via the modal analysis, but also that this framework can complement the latter, as it allows for the analysis of higher k behavior, not necessarily captured by modal analysis classic algorithms as could be seen for the case of bending behavior which changes

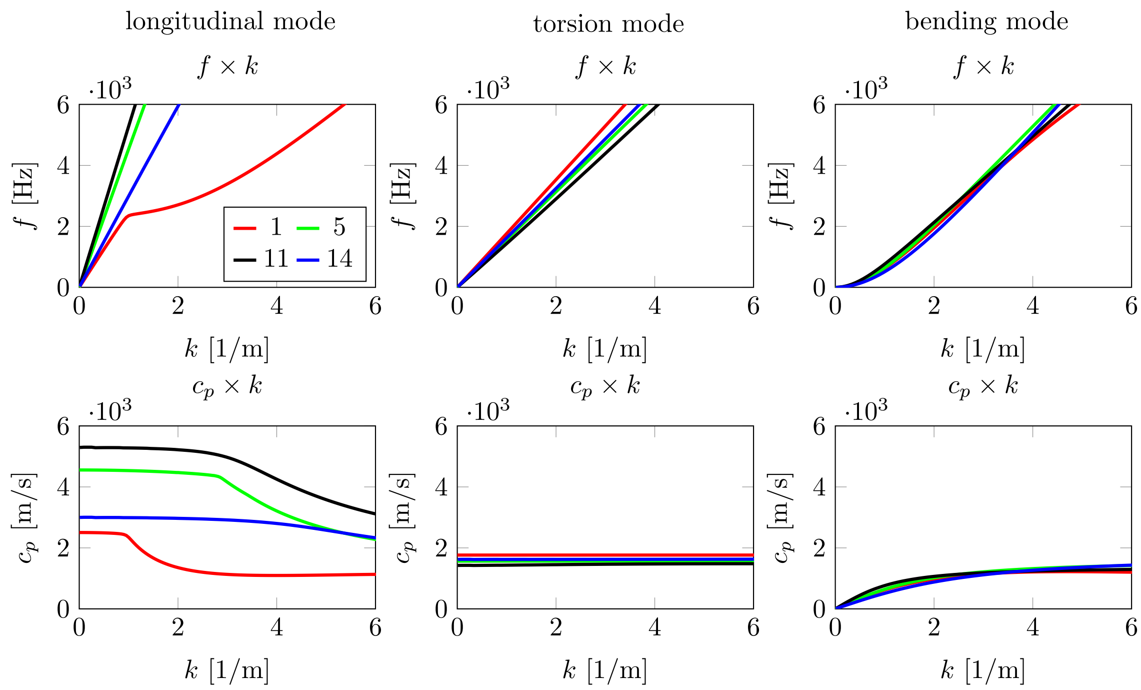

Figure 5: Dispersion curves found with SAFE and Bloch-Floquet for fundamental modes of models 1, 5, 11 and 14.

Model	E^* [GPa]	G^* [GPa]
1	12.49	6.20
5	41.35	4.89
11	56.02	4.06
14	17.98	5.26

Table 2: Equivalent stiffness for torsion and traction of models 1, 5, 11 and 14.

with k. The best configuration for all models is found for $\phi = 90$ and $\theta = 0$ which is confirmed by the optimization analysis performed.

In general, the results shows that wave propagation analysis can perform as a tool of evaluating the trade-off between stiffness in different modes when varying fiber orientation distribution along the cross section.

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