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AMORTIGUADOR DE MASA SINTONIZADA ROBUSTO PARA LA REDUCCION DE VIBRACIONES INDUCIDAS POR VIENTO

ROBUST MULTIPLE TUNED MASS DAMPER FOR WIND-INDUCED VIBRATION REDUCTION

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Palabras clave: Vibraciones, amortiguador de masa sintonizada, disipación de energía.

Resumen. Una nueva configuración de múltiples amortiguadores de masa sintonizada (MTMD) se propone para la reducción de vibraciones de estructuras elásticas. Se analiza el comportamiento dinámico de MTMD acoplados con disipadores viscosos (VCTMD) conectados a una estructura principal sometida a excitaciones de banda ancha. Se analiza la reducción de la desviación estándar de la deformación de la estructura principal sujeta a excitación de ruido blanco y se compara con las reducciones obtenidas con un amortiguador de masa convencional (TMD) y con MTMD desacoplados en paralelo. El desempeño de modelos estructurales con parámetros nominales y la sensibilidad del desempeño ante cambios en los parámetros dinámicos (masa, rigidez) de la estructura principal se evalúan para los casos de aplicación de TMD, MTMD en paralelo y VCTMD. Usando modelos numéricos simples se demuestra que una estructura elástica con el dispositivo de disipación de energía propuesto muestra mejor desempeño y menor sensibilidad a cambios en los parámetros mecánicos de la estructura principal en relación a la estructura con TMD o MTMD.

Keywords: Vibrations, tuned mass damper, energy dissipation

Abstract. A novel configuration of multiple tuned-mass damper (MTMD) is proposed for vibration reduction of elastic structures. Viscously coupled multiple tuned mass dampers (VCTMD) connected to a main structure subjected to broad-band external force are analyzed. The reduction of root mean square deformation of the main structural system subjected to white noise is analyzed and compared with those of a conventional optimal TMD and of a parallel MTMD configuration without viscous coupling between TMDs. The VCTMD outperforms both the conventional TMD and parallel uncoupled MTMDs in terms of deformation reduction. The performance of the structural model with nominal values and sensitivity to varied dynamic parameters (mass and stiffness) of the main structure are assessed for the case of supplemental TMD and for the case of supplemental VCTMD. Using simple numerical models it is demonstrated that the structure with the proposed energy dissipation device shows better performance and more robust performance to changes in main structure parameters.

1. TMD AND MTMD APPLICATIONS IN WIND VIBRATION REDUCTION

The use of tuned mass dampers (TMD) to increase damping ratios of specific modes of vibration of an undamped structure or a structure with low modal damping ratios subjected to broad band excitation has been proposed and applied during the last decades around the world in different structures. Some applications of TMD in wind vibration reduction for comfort assurance in high-rise flexible buildings are: the John Hancock Tower, 241 m in height, in Boston, the Citicorp Center, 279 m in height, in New York, and the Taipei 101 Tower (508 m) (Taipei-101.com.tw) in Taiwan. TMD were also installed in the Millennium Bridge in London for pedestrian induced vibration reduction (Fitzpatrick et al., 2001). A recent application of a combination of viscous dampers in combination with TMD to reduce peak floor accelerations in the project of a high-rise building (185 m) subjected to wind load to improve occupants' comfort is described by Inaudi et al. (2017). The dampers are designed to provide an increase of equivalent damping ratios of about 1% to 1.5% in the first three modes of vibration of the structure at low intensity one-year return-period wind condition. To augment an additional 1.5% to 2% the modal damping ratios in low intensity vibration levels, a bidirectional TMD is designed to improve the building performance.

During the design process of a TMD the uncertainty of main-structure dynamic parameters (deviation from nominal values defined in the structural model of the building) plays an important role. Changes of these parameters during construction or operation stage with respect to design values are frequent due to occupancy changes, variation of internal wall configuration, or changes in tangent stiffness parameters after medium or high intensity seismic event (due to damage in secondary structural elements and/or main structural elements). For this reason, the capability of fine tuning of mechanical parameters is an aspect to be considered in a sound design of a TMD. Such fine tuning could be avoided if the TMD design provided low sensitivities to main structure modal mass and modal stiffness changes. This robustness feature is precisely the aim of the proposed MTMD configuration proposed in this paper.

The most frequent technology proposed in the literature to provide performance robustness to changes in main structure parameters is the MTMD with several uncoupled TMDs connected in parallel with different frequencies tuned in an interval close to the natural frequency of the targeted mode of the structure as Figure 1b shows.

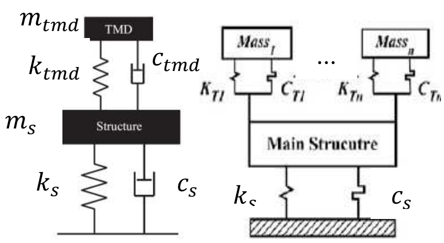

Figure 1. a) Conventional single TMD b) Parallel uncoupled MTMDs

Several authors have investigated optimum parameters of MTMD for different loading conditions. Studies by Abé and Fujino (1994), Igusa and Xu (1994) and Bakre and Jangid (2007) have shown that MTMD can be more effective and robust than a single TMD with the same total mass. This implies that the response of the main system is not much influenced by small changes or estimation errors in the assumed values of system parameters used in the TMD design process in relation to the real built structure. Different configurations have been proposed and studied. Avila and Gonçalves (2009) studied different configurations of double mass dampers with optimized parameters using a minimax procedure that considers mass, tuning frequency and damping ratio of each damper as free variables.

To increase robustness using the same total mass of a conventional TMD, the consideration of a 3-TMD configuration with central frequency close to optimal frequency of the classical TMD is proposed and analyzed in this paper. To improve performance and lower sensibility to changes in main structure parameters a novel configuration of MTMD is considered. The idea is to use a set of closely spaced natural frequency TMDs connected elastically in parallel to the main structure and viscously coupled among them (not between the TMDs and the structure) to improve induced modal damping ratios of the TMDs-main structural system and reduce deformation demands and sensitivity to main structure parameters with respect to classical TMD and MTMD applications. Figure 2 illustrates the proposed viscously coupled TMDs.

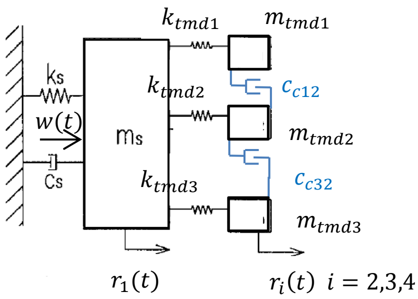

Figure 2. Proposed VCMTMD.

In section 2.1 the conventional TMD is analyzed and the optimal parameters that reduce rms deformation of the main structure subjected to white noise are revisited. In section 2.2 a MTMD configuration with three TMDs operating in parallel and closely spaced natural frequencies is explored and compared to conventional TMD. In section 2.3 the proposed viscously coupled multiple TMD (VCMTMD) is modelled and analyzed for a configuration of three TMDs. In section 2.4 the robustness of the three configurations is compared. Finally, some conclusions and future lines of research are presented in section 3.

2. CONVENTIONAL TMD, MTMD AND VCMTMD

2.1 Conventional single TMD

Let us consider a simple model of TMD attached to a main structure subjected modelled as a two-degree of freedom model subjected to external load applied on the main structure (Figure 1.a). The equations of motion of the model are

$$M\,\ddot{r}(t) + C\,\dot{r}(t) + K\,r(t) = L_w w(t)$$

$$M = \begin{bmatrix} m_s & 0 \\ 0 & m_{tmd} \end{bmatrix},\; C = \begin{bmatrix} c_s + c_{tmd} & -c_{tmd} \\ -c_{tmd} & c_{tmd} \end{bmatrix},\; K = \begin{bmatrix} k_s + k_{tmd} & -k_{tmd} \\ -k_{tmd} & k_{tmd} \end{bmatrix}, L_w = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (1)$$

Where m_s, c_s, k_s, m_{tmd}, c_{tmd} and k_{tmd} are the mass, viscous constant and stiffness of the main structure and the TMD, respectively; $w(t)$ is the applied load, L_w the load influence vector for load applied on the main structure, and $r(t)$ the displacement vector.

In the case of a real building with multiple modes of vibration and a TMD tuned to a particular mode, minor interference of the dynamics of the TMD with other modes of vibration is observed if the main structure has well separated natural frequencies. This is the reason why the simple 2DOF model in Eq. (1) can be applied for preliminary design of TMD in multi-degree of freedom models of structures. To relate the modal coordinate of the tuned mode in the simplified model, the corresponding structural mode shape has to be scaled such that the TMD support displacement is one. The parameter m_s is the corresponding modal mass computed using the mode shape with unit TMD support displacement $\tilde{\phi}$ as

$$m_s = \tilde{\phi}^T M \tilde{\phi} \tag{2}$$

And the stiffness parameter k_s

$$k_s = \tilde{\phi}^T K \tilde{\phi} = m_s\, \omega_s^2 \tag{3}$$

where ω_s is the natural frequency of the target mode shape of the main structure.

To analyze the effect of the TMD on a main structure subjected to external loads we analyze a simplified model of broad-band excitation, modelling $w(t)$ as a white-noise Gaussian stationary process of intensity W with autocorrelation function

$$R_{ZZ}(t) = E[w(t)w(t+\tau)] = W\delta(\tau) \tag{4}$$

Optimal parameters of the TMD can be computed as those that minimize rms structure deformation for a given mass ratio $\mu = m_{tmd}/m_{st}$. We use a standard state-space formulation

$$\dot{x}(t) = A\, x(t) + B\, w(t) \tag{5}$$

where

$$x(t) = \begin{bmatrix} r(t) \\ \dot{r}(t) \end{bmatrix},\quad A = \begin{bmatrix} O & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix},\quad B = \begin{bmatrix} 0 \\ M^{-1}L_w \end{bmatrix} \tag{6}$$

Defining the tuning parameter $\beta = \omega_{tmd}/\omega_s$ where $\omega_s = \sqrt{k_s/m_s}$ and TMD damping ratio $\xi_{tmd} = c_{tmd}/(2\, m_{tmd}\, \omega_{tmd})$, we can compute the stationary covariance matrix $P_{xx} = E[x(t)x(t)^T]$ as a function of the dimensionless parameters using the stationary Lyapunov equation:

$$A(\mu,\beta,\, \xi_{tmd})P + PA(\mu,\beta,\, \xi_{tmd})^T + B\, WB^T = 0 \tag{7}$$

For a given mass ratio μ we can find the optimal TMD tuning parameters β and ξ_{tmd} that minimize rms displacement of the main structure, $\sigma_{r_1} = P(1{,}1)^{1/2}$, subjected to white noise excitation. The optimal parameters are independent of the white noise intensity W for a linear model. Table 1 shows analytical expressions of these optimal parameters by Den Hartog and Warburton.

Table 1. Analytical expressions of tuning parameters for external load applied to main structure.

Criterion	Frequency tuning parameter $\beta = \frac{\omega_{tmd}}{\omega_{st}}$	TMD damping ratio ξ_{tmd}
Den Hartog for undamped main structural model subjected to harmonic loading	$\frac{1}{1+\mu}$	$\sqrt{\frac{3\mu}{8(1+\mu)}}$
Warburton for undamped main structure subjected to white noise force	$\frac{\sqrt{1-\mu/2}}{1+\mu}$	$\sqrt{\frac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}}$

Figure 3 shows the optimal values of β and ξ_{tmd} as functions of the mass ratio μ for undamped main structures with $c_s/(2\,m_{st}\,\omega_{st}) = 0$ computed numerically to minimize rms structure deformation and the corresponding values estimated with analytical expressions proposed by Den Hartog and Warburton for an undamped main structure with external load applied to the main structure defined in Table 1. The optimal parameters for broad-band force on the main structure show very similar parameters for low main structure damping $\xi_s <$ 0.015, and vary for $\xi_s > 0.05$ (not typical in vibration reduction with TMD applications in low intensity wind conditions). Therefore, the analytical expression of Warburton provides quite accurate optimal parameters for TMD design of lightly damped structures subjected to broad-band external force. Den Hartog parameters (shown in dashed line in Figure 3) are not precise estimates of optimum tuning TMD parameters for white-noise force input. The reason for this is that the performance index minimized Den Hartog's parameters is the maximum absolute value of the frequency response from external load to displacement of the main structure, while the performance index considered by Warburton is the rms of the structure deformation, square root of the variance of this deformation (integral in frequency of the square of the absolute value of the frequency response function times $W/2\pi$ in the case of white noise).

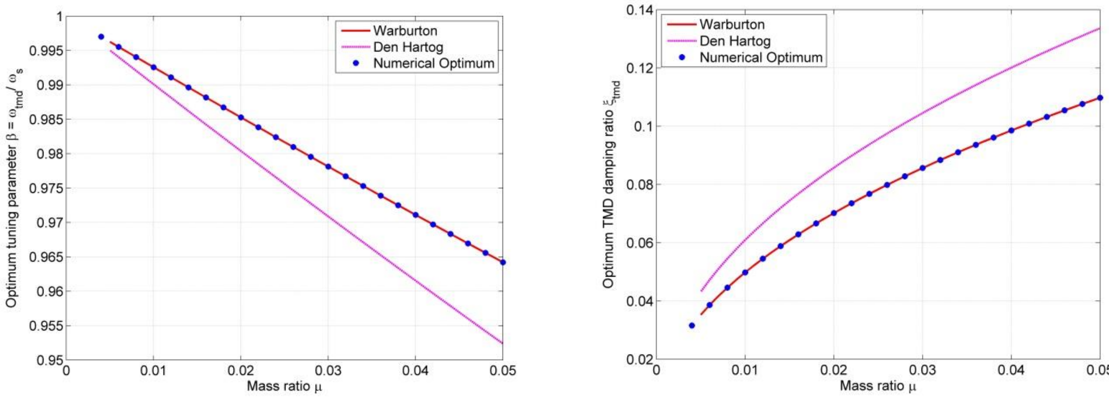

Figure 3. Optimal TMD-tuning parameters and TMD damping ratio for undamped main structure (dashed line Den Hartog; thick line Warburton; stars and circles: numerical optimization)

It is worth mentioning that in wind applications, the loading process on a specific modal coordinate is a broad-band process but shows decreasing power spectral density with increasing frequency while white noise shows a constant PSD. Furthermore, optimization criterion used is not rms deformation, but rms acceleration, mean peak acceleration, or one-third octave rms filtered acceleration (performance variables usually used to assess occupant comfort). In these cases, optimal tuning parameters can be searched by a computational optimization process for the corresponding power spectral density of the external force and performance index, using Warburton's optimal parameters as starting values in the optimization method (Inaudi et al., 2017).

2.2 Multiple uncoupled TMDs

MTMDs have been proposed to increase the frequency bandwidth of the imaginary part of the dynamic stiffness of the MTMD around a main structural frequency to improve robustness under uncertainty or changes of the dynamic properties of the main structure around nominal values. In this case, tuning of the TMDs is achieved with closely spaced natural frequencies of each TMD. To analyze the performance and robustness improvement with respect to a conventional single TMD, let's analyze a case of triple TMD (Figure 1.b) with identical masses $m_{tmd1} = m_{tmd2} = m_{tmd3} = m_{tmd}/3$ equal to the third of the mass m_{tmd} of a conventional TMD. Each TMD, labeled as 1, 2 and 3 is defined with different natural frequency: ω_{tmd2} = central frequency equal to the optimal frequency of the conventional TMD ω_{tmd} for a given mass ratio, $\omega_{tmd1} = (1+\Delta_\omega)\omega_{tmd2}$, $\omega_{tmd3} = (1-\Delta_\omega)\omega_{tmd2}$ where $\Delta_\omega$is a dimensionless parameter that controls the bandwidth of the MTMD. For simplicity, all TMDs are assigned with the same damping ratio, equal to the optimal value of the conventional TMD for a given mass ratio μ defined in the previous section: $\xi_{tmd1} = \xi_{tmd2} = \xi_{tmd3} = \xi_{tmd}$. Although the mass, damping and stiffness properties of each mass damper of the MTMD could be defined as independent variables for the optimization, the selected parameter configuration allows a simpler optimization process to focus on the effect of the bandwidth parameter Δ_ω on the performance of the system.

The equations of motion of the model subjected to broad-band excitation applied on the main structural system can be obtained as:

$$M_{mtmd}\ddot{r}(t) + C_{mtmd}\dot{r}(t) + K_{mtmd}r(t) = L_w\, w(t) \tag{8}$$

where

$$M_{mtmd} = \begin{bmatrix} m_{st} & 0 & 0 & 0 \\ 0 & m_{tmd}/3 & 0 & 0 \\ 0 & 0 & m_{tmd}/3 & 0 \\ 0 & 0 & 0 & m_{tmd}/3 \end{bmatrix}$$

$$C_{mtmd} = \begin{bmatrix} c_{st} + c_{tmd1} + c_{tmd2} + c_{tmd3} & -c_{tmd1} & -c_{tmd2} & -c_{tmd3} \\ -c_{tmd1} & c_{tmd1} & 0 & 0 \\ -c_{tmd2} & 0 & c_{tmd2} & 0 \\ -c_{tmd3} & 0 & 0 & c_{tmd3} \end{bmatrix}$$

$$K_{mtmd} = \begin{bmatrix} k_{st} + k_{tmd1} + k_{tmd2} + k_{tmd3} & -k_{tmd1} & -k_{tmd2} & -k_{tmd3} \\ -k_{tmd1} & k_{tmd1} & 0 & 0 \\ -k_{tmd2} & 0 & k_{tmd2} & 0 \\ -k_{tmd3} & 0 & 0 & k_{tmd3} \end{bmatrix}, \; L_w = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

According to the defined dimensionless parameters in this case

$$c_{tmd1} = \frac{2\xi_{tmd}\omega_{tmd}\,(1+\Delta_\omega)\,m_{tmd}}{3}, \quad c_{tmd2} = \frac{2\xi_{tmd}\,\omega_{tmd}\,m_{tmd}}{3}, \quad c_{tmd3} = \frac{2\xi_{tmd}\,\omega_{tmd}\,(1-\Delta_\omega)\,m_{tmd}}{3} \tag{10}$$

where $\omega_{tmd} = \beta(\mu)\omega_{st}$ and $\xi_{tmd}(\mu)$ are the optimal parameters selected for the conventional TMD, computed with Warburton expressions (defined in Table 1).

For a white-noise input, the optimal bandwidth parameter Δ_ω of the MTMD and its corresponding minimum rms displacement of the main structure can be computed in an optimization procedure with a performance index solved by the stationary Lyapunov equation as we proceeded in the case of the structure-TMD model analyzed in the previous section: defining the appropriate state vector, assembling the corresponding system matrices A and B using the mass, damping and stiffness matrices defined in Eqs. (9).

Figure 4a shows the optimal bandwidth parameter Δ_ω for the model considered with no main-structure damping (ξ_s=0). The optimal bandwidth parameter increases with mass ratio as observed by other authors. As shown in Figure 4b, a performance improvement of 2% to 3% is achieved with the triple MTMD compared with a single optimal TMD for the range of mass ratio μ from 0.4% to 5%.

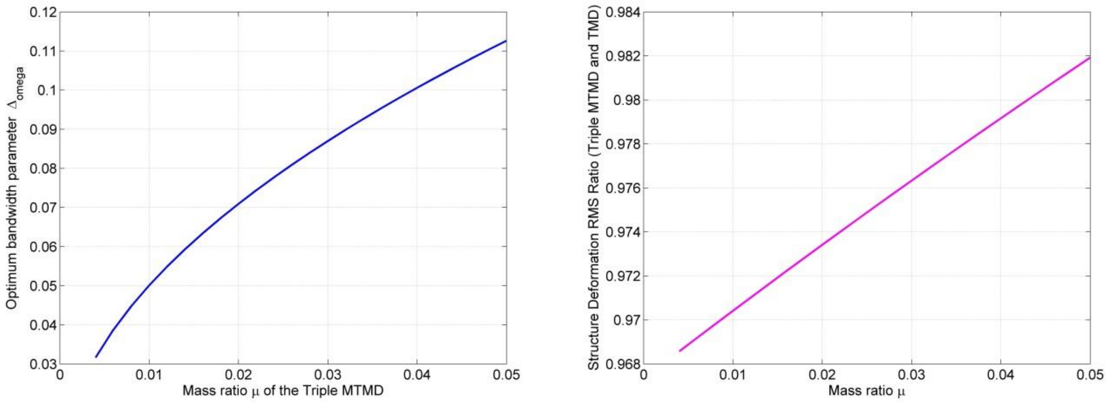

Figure 4. (a) Optimal bandwidth parameter for MTMD considered. (b) Performance comparison of MTMD and TMD (structure displacement rms ratio).

The constraint imposed of equal damping ratios of individual TMDs of the MTMD could be released defining these parameters as free variables of the optimization process to achieve a better performance with the MTMD. I do not include such analysis to focus more on the proposed novel configuration of MTMD. Before comparing robustness (or sensitivity) of TMD and MTMD, let´s introduce the proposed VCMTD configuration.

2.3 Multiple TMDs coupled with viscous dampers (VCMTMD)

The proposed innovation in the MTMD consists of a variation of a MTMD in parallel with viscous coupling among the TMDs and no viscous dampers connecting the TMD with the main structure. As illustrated in Figure 2 no elastic elements are used connecting the 3 TMDs to avoid the appearance of spurious natural frequencies. The central frequency (TMD number 2) is designed without a viscous damper connecting the TMD to the structure ($c_{tmd2} = 0$) and although direct dampers of TMD1 and TMD3 could be considered, in the seminal conceptual scheme I considered $c_{tmd1} = c_{tmd3} = 0$. Some brief comments on the effect of these parameters on performance are done at the end of this section.

The strategy looks for pole damping-ratio augmentation of the coupled MTMD-main structure system in the vecinity of the central frequency and improved robustness under uncertainties or changes of main structure dynamic parameters by an energy dissipation mechanism based on to interaction between TMDs (relative deformation between TMDs), not due to relative deformation of the TMDs with respect to the support point of the structure. The concept is basically a change in the energy dissipation mechanism, based more on relative displacements between parallel TMDs coupled with viscous dampers instead of dissipation with direct dampers of each individual TMD connected to the main structure. Incorporating coupling dampers and lowering or eliminating direct viscous dampers of each TMD determines that the imaginary part of the dynamic stiffness of the MTMD increases significantly at a frequency band centered at the central frequency of the second TMD, tuned to the main structure natural frequency.

Let's evaluate the performance of this energy dissipation mechanism. The equations of motion of the VCMTMD-main structure system are

$$M_{cvmt}\ddot{r}(t) + C_{cvmt}\dot{r}(t) + K_{cvmt}r(t) = L_w\, w(t) \tag{11}$$

where

$$M_{mtmd} = \begin{bmatrix} m_{st} & 0 & 0 & 0 \\ 0 & m_{tmd}/3 & 0 & 0 \\ 0 & 0 & m_{tmd}/3 & 0 \\ 0 & 0 & 0 & m_{tmd}/3 \end{bmatrix}$$

$$C_{mtmd} = \begin{bmatrix} c_{st} + c_{tmd1} + c_{tmd3} & -c_{tmd1} & 0 & -c_{tmd3} \\ -c_{tmd1} & c_{tmd1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -c_{tmd3} & 0 & 0 & c_{tmd3} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & c_{c12} & -c_{c12} & 0 \\ 0 & -c_{c12} & c_{c12} + c_{c23} & -c_{c23} \\ 0 & 0 & -c_{c23} & c_{c23} \end{bmatrix}$$

$$K_{mtmd} = \begin{bmatrix} k_{st} + k_{tmd1} + k_{tmd2} + k_{tmd3} & -k_{tmd1} & -k_{tmd2} & -k_{tmd3} \\ -k_{tmd1} & k_{tmd1} & 0 & 0 \\ -k_{tmd2} & 0 & k_{tmd2} & 0 \\ -k_{tmd3} & 0 & 0 & k_{tmd3} \end{bmatrix}, \; L_w = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

Although more parameters of the VCMTMD could be optimized (for example: individual variation of each damping ratio of the TMDs 1 and 3, a shift in the central frequency of TMD 2, and different damping ratios for TMDs 1 and 3, for the sake of simplicity the coupling damping terms between TMDs are defined equal ($c_{c23} = c_{c12} = c_c$), direct TMD damping ratios are defined as $\xi_{tmd1} = \xi_{tmd3} = \Delta_\xi \, \xi_{tmd}$, where Δ_ξ is defined as a direct TMD damping parameter (assumed as zero in the first parametric analysis) and ξ_{tmd} is the optimal conventional TMD damping ratio for the total mass considered. The central frequency is defined using the optimal tuning factor $\beta = \omega_{tmd2}/\omega_s$ computed for conventional TMD for a given mass ratio μ. For fixed values of mass ratio and Δ_ξ, the optimization is done searching for optimal frequency band parameter Δ_ω (defined before as in the case of MTMDs in section 2.2) and an additional dimensionless parameter Δ_c that defines the viscous coupling term $c_c = c_{c12} = c_{c23}$ between TMDs 1 and 2 and between TMDs 2 and 3 as a function of optimal damping ratio ξ_{tmd} of the classical TMD model:

$$\Delta_c = \frac{c_c}{2 \, m_{tmd} \, \xi_{tmd} \, \beta \, \omega} \quad (13)$$

Figure 5 shows the computed optimal parameter Δ_c as a function of the mass ratio for the design for $\Delta_\xi = 0$ (no direct TMD dampers connecting TMDs and main structure) and the optimal values of Δ_ω of the VCMTMD. Both Δ_ω and Δ_c parameters increase monotonically with μ. Comparing Figure 5 and Figure 6 we can observe that $\Delta_\omega(\mu)$ for the VCMTMD is significantly larger than that of MTMD for the mass-ratio range considered.

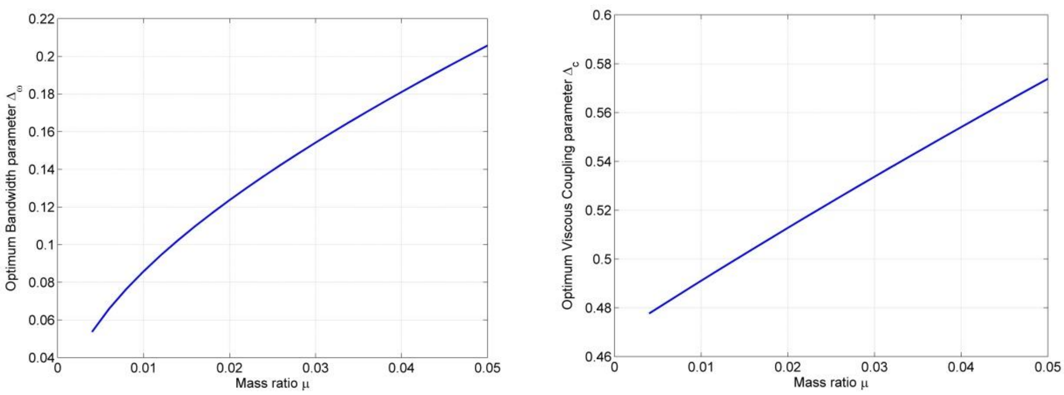

Figure 5. Optimal parameters for VCMTMD for $\Delta_\xi = 0$: a) Optimum bandwidth parameter b) optimum viscous coupling parameters of the tuned mass dampers.

Figure 6.a compares the achievable optimal performance of the structure with MTMD and VCMTMD normalized with respect to the optimal performance of the classical single TMD for the same total mass in the devices (same μ). The VCMTD configuration outperforms the

MTMD for the range of mass ratios considered. VCMTD achieves between 5% and 7% reduction of main-structure deformation rms with respect to the conventional single TMD.

Figure 6.b shows the rms deformations of the TMDs normalized with respect to the rms of the main structure deformation of the optimum TMD for the Triple MTMD and for the VCMTMD with $\Delta_\xi = 0$. Normalized TMDs' deformations of the MTMD are larger than those of an optimal VCTMD, and significantly larger than those of the single TMD with the same total mass. Because each mass of the MTMD or VCMTMD is 1/3 of the single TMD, larger rms deformations are expected in TMDs.

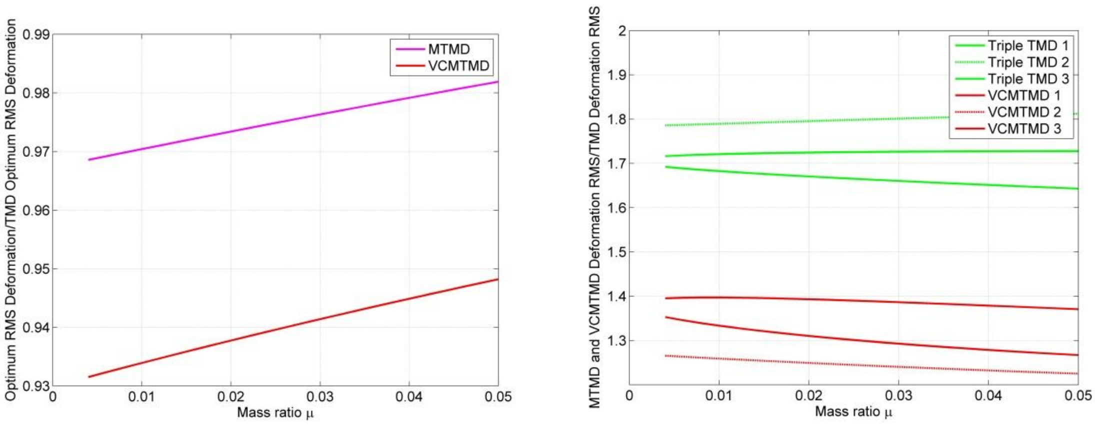

Figure 6. (a) Normalized optimal rms deformation ratio of the main structure for MTMD and VCMTMD with $\Delta_\xi = 0$. (b) Comparison of normalized TMDs' deformation rms for MTMD and VDMTMD.

To assess the effect of direct viscous dampers connecting TMDs 1 and 3 with the main structural model in the VCMTMD configuration, the optimization is run for values of $\Delta_\xi = 0, 0.5$ and 1. The main structure rms deformations of the optimized VCMTMD do not vary significantly when Δ_ξ is modified with respect to the case $\Delta_\xi = 0$. The results show that for a larger Δ_ξ parameter, the optimum Δ_c parameter of the VCMTMD reduces.

2.4 Performance sensitivity of TMD designs

To analyze performance sensitivity to structural parameter changes, the rms deformation of the structure-TMD model is computed for small changes in structural stiffness and mass around nominal values $({}_{nom}k_s, {}_{nom}m_s)$ with an optimal TMD for the nominal structural model. Sensitivity under changes in main structure parameter for conventional TMD, MTMD and VCTMD is analyzed. Natural frequency of the undamped structure without TMDs is assumed as $\omega_s = 10\, rad/s$ and total TMD mass ratio $\mu = 0.02$. Figure 7 shows main structure rms deformation normalized by the optimum value for the structure with TMD as a function of stiffness and mass variations of the main structural system defined with dimensionless parameters α_{k_s} and α_{m_s}, such that

$$k_s = \alpha_{k_s\ nom}k_s \qquad m_s = \alpha_{m_s\ nom}m_s \tag{14}$$

Figure 7 shows that the optimal MTMD design shows better performance than the conventional TMD design for nominal parameters and robustness for positive increments in structural mass or reductions of structural stiffness with respect to nominal parameters. VCMTMD configuration outperforms both TMD and MTMD and exhibits a robust performance to positive and negative changes in both stiffness and mass of the structure.

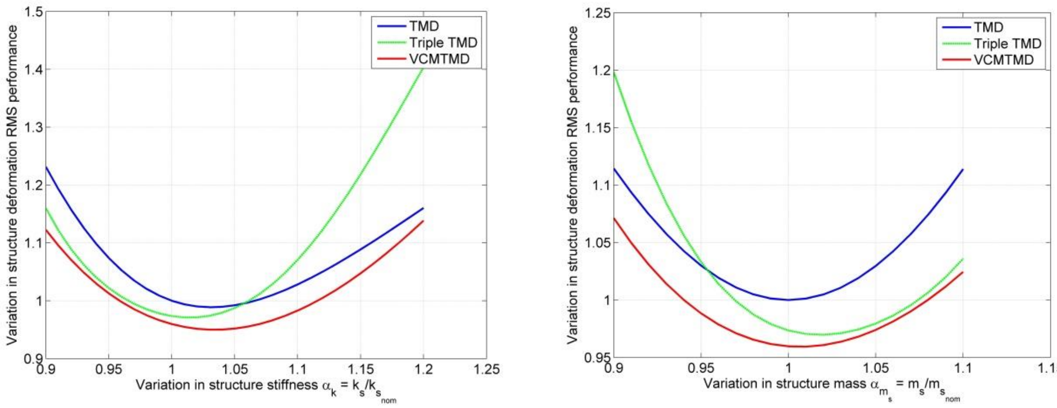

Figure 7. (a) Performance sensitivity to main structure stiffness changes and (b) Performance sensitivity to main structure mass changes.

3 CONCLUSIONS AND FUTURE RESEARCH

The proposed VCMTMD shows excellent performance and robustness compared to conventional TMD and uncoupled parallel MTMD strategies. Viscous coupling between undamped parallel TMDs shows a promising strategy for vibration reduction of lightly damped structures, outperforming TMD and conventional parallel MTMD in main structure rms deformation reduction and robustness. Future lines of research will be pursued on: i) a more detailed optimization of MTMD and VCMTD with more free design parameters, ii) the development and experimental verification of VCMTMD prototype implementations, iii) applications of VTMTMD in wind-induced vibration and low-intensity ground motion input to flexible structures, and iv) the combination of VCMTMD with inerters to reduce the total mass of the device and required space.

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