Mecánica Computacional Vol. XXII
M. B. Rosales, V. H. Cortínez y D. V. Bambill (Editores)
Bahía Blanca, Argentina, Noviembre 2003.

PIEZELETRIC ACTUATORS PLACEMENT STUDY THROUGH SINGULAR ANALYSIS VALUE ON THE CANTILEVER PLATE

Aguinaldo Soares de Oliveira[*], José Juliano Lima Junior[†]

[*] Federal University of Itajubá – Minas Gerais - Brazil
Prof. José Rodrigues Seabra Campus, 1303, BPS Av. –Pinheirinho District, ZIP – 37500-903
aguinald@unifei.edu.br

[†] José Juliano Lima Junior
Federal University of Itajubá – Minas Gerais - Brazil
Prof. José Rodrigues Seabra Campus, 1303, BPS Av. – Pinheirinho District, ZIP – 37500-903
limajr@unifei.edu.br

Key words: actuators, sensors, placement, singular analysis value

Abstract. *The piezoelectric actuator and sensor, have received lot of attention by researcher. The reason for this, it is because these devices present the piezoelectricity effect. This effect is the conversion between mechanical energy in electric energy and vice versa. So this effect is very useful in active vibration control, AVC, and its results are more effective than passive vibration control. The intelligent structures are the units compound by: actuator, sensor, controller and structures (Lima Jr, 1999). Intelligent structure good design, the actuators and sensors placement are a fundamental part, because misplacement can cause lack of system controllability and observability. So this paper intends to propose actuators placement technique, through index obtained from singular value decomposition of control matrix [B]. The structure about the study is a cantilever plate and the results check with the simulation done in finite elements.*

1 INTRODUCTION

Vibrations control new aim of the flexible structures has been received more attention by many researchers. According to new aim, the control is more effective if we use active elements. So integrating elements such as: sensor, actuator and controller, the mechanical vibrations could be minimized better than the use of passive elements. Nowadays these systems joining sensors, actuators, controllers and structures, are called as intelligent structures (Lima Jr, 1999).

Several technologies were proposal and investigated by researchers. Among these technologies are the piezoelectric elements. These elements, present the piezoelectricity effect that permit the conversion between electric and mechanical energy and vice versa The piezoelectric direct effect was discovered by Curie brothers and piezoelectric inverse effect was deduced by Lippman (Rao & Sunar, 1994). Among these elements, there are the piezoelectric materials, especially the ceramics, PZT – piezoelectric lead zirconate titanate and polymer films, PVDF – piezoelectric vinylidene fluoride (Lima Jr. & Arruda, 1999). The ceramics have high stiffness, therefore they are used as actuators. While that the polymer films are more handler than ceramics and can be produced in complex geometric shapes, for this reason, they are used as sensors. (Lima Jr, 1999). Piezoelectric materials are small, lightweight and resilient against adverse working environments. Moreover piezoelectric materials have been used as both actuators and sensors, (Wang, 2001).

One of the pioneers in using piezoelectric actuators as elements of intelligent structure was (Crawley & De Luis, 1987). He worked with an aluminum beam with piezoelectric actuator attached and he worked also, with graffiti/ epoxy beam and glass/epoxy beam. It was used, velocity proportional feedback controller in his work

The intelligent structure design is divide in three areas, such as: Modeling in finite element method (FEM); Actuators and sensors placement; System controller. In a good intelligent structure design, actuators and sensors placement study is a fundamental part to avoid undesirable effects in structure under active control, such as: Lack of observability and controllability system. This paper purpose is to suggest, optimum piezoelectric actuators placement, in a flexible structure, using modal and spatial controllability measurements. To quantify the controllability index, we intend to use the singular value analysis of the [B] control matrix.

2 KIRCHHOFF PLATE

According to Kirchhoff hypothesis, showed in the figure 1, the field displacement *u*, *v*, and *w* can be express such as (Lima Jr, 1999):

$$\begin{cases} u = -z\dfrac{\partial w}{\partial x} \\ v = -z\dfrac{\partial w}{\partial y} \\ w = w(x.y) \end{cases} \tag{1}$$

Where: x and y are cartesian system coordinator placed in the plate medium surface and z is direction along of plate thickness.

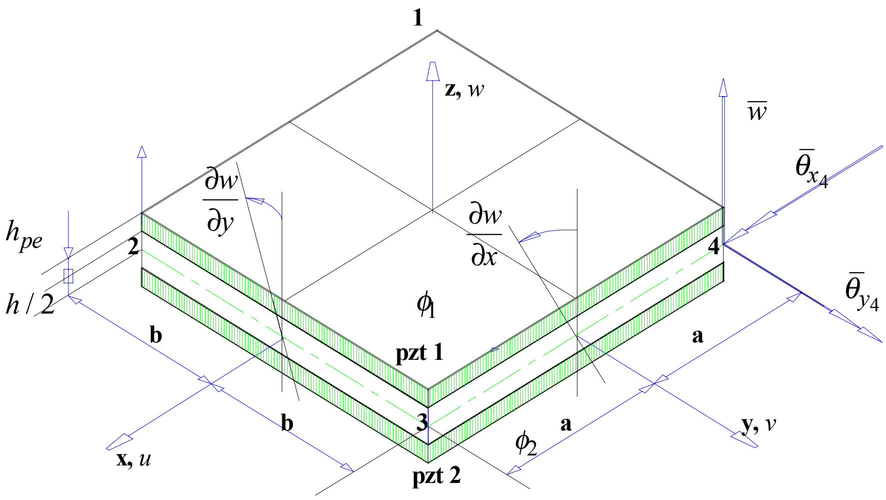

Figure 1: Plate Elements (Lima Jr, 1999).

Due the fact of shear effect isn't taken in to consideration, the deformation field can be write in function of displacement such as:

$$\begin{cases} \varepsilon_x = \dfrac{\partial u}{\partial x} = -z\dfrac{\partial^2 w}{\partial x^2} \\ \varepsilon_y = \dfrac{\partial v}{\partial y} = -z\dfrac{\partial^2 w}{\partial y^2} \\ \gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} = -2z\dfrac{\partial^2 w}{\partial x \partial y} \end{cases} \tag{2}$$

3 FINITE ELEMENTS APROXIMATION

We considerate four nodes in the rectangular plate element, according to plate classic theory (Bathe, 1996), in each node it has three degrees of freedom such as: $\overline{w}$ displacement in direction z, $\overline{\theta}_x$ rotation in relation to axe x and $\overline{\theta}_y$ rotation in relation to axe y. So the displacement function, w, is:

$$\begin{aligned} w(x_i, y_i) &= d_1 + d_2 x_i + d_3 y_i + d_4 x_i^2 + d_5 x_i y_i + d_6 y_i^2 + d_7 x_i^3 + d_8 x_i^2 y_i \\ &+ d_9 x_i y_i^2 + d_{10} y_i^3 + d_{11} x_i^3 y_i + d_{12} x_i y_i^3 \end{aligned} \tag{3}$$

Where:

$$\begin{cases} i = 1,2,\ldots,4 \\ x_1 = -a;\ y_1 = -b;\ x_2 = a;\ y_2 = -b \\ x_3 = a;\ y_3 = b;\ x_4 = -a;\ y_4 = b \end{cases} \tag{4}$$

In the matrix form, the equation (4) is:

$$w = \{P\}^T \{d\} \tag{5}$$

The vector $\{q_i\}$ is defined such node displacement field, in the rectangular element, such as:

$$\{q_i\} = \left\{ \overline{w}_1 \quad \overline{\theta}_{x_1} \quad \overline{\theta}_{y_1} \quad \ldots \quad \overline{w}_4 \quad \overline{\theta}_{x_4} \quad \overline{\theta}_{y_4} \right\}^T \tag{6}$$

4 PIEZOELETRIC VARIATIONAL EQUATION

The behavior of piezoelectric material, there are mechanics and electric effects which can be written in the matrix form, (Lima Jr e Arruda, 1997) and (Lima Jr., 1999) such as:

$$\begin{aligned} &\iiint_V \rho \{\delta u\}^T \{\ddot{u}\} dV + \iiint_V \{\delta\varepsilon\}^T \{\sigma\} dV - \iiint_V \{\delta E\}^T \{D\} dV = \\ &\iiint_V \{\delta u\}^T \{\bar{f}_V\} dV + \iint_{S_F} \{\delta u\}^T \{\bar{f}_S\} ds - \iint_{S_q} \delta\phi \sigma_q dS \end{aligned} \tag{7}$$

The linear piezoelectricity constructive equation is:

$$\begin{aligned} \{\sigma\} &= \left[c^E\right]\{\varepsilon\} - [e]\{E\} \\ \{D\} &= [e]^T \{\varepsilon\} - \left[\xi^\varepsilon\right]\{E\} \end{aligned} \tag{8}$$

Where:

$$\begin{aligned} [e] &= \left[c^E\right][d] \\ \left[\xi^\varepsilon\right] &= \left[\xi^\sigma\right] - [d]^T \left[c^E\right][d] \end{aligned} \tag{9}$$

Where: $\{\sigma\}$- stress tensor; $\{\varepsilon\}$- deformation tensor; $\{E\}$- electric field vector; $\{D\}$- electric displacement vector; $[C^E]$- elasticity matrix for constant electric field; $[e]$- piezoelectric

constants matrix; [ξ^{ε}]- dielectric constants tensor for constant deformation [ξ^{σ}]- dielectric constant matrix for constant stress; [d]- constant matrix of piezoelectric deformations.
The variational principle equation for piezoelectric material (Lima Jr, 1999), is obtained put equation (8) in (7), so it is given by:

$$\begin{aligned}&\iiint_V \rho\{\delta u\}^T\{\ddot{u}\}dV+\iiint_V \{\delta\varepsilon\}^T\left[c^E\right]\{\varepsilon\}dV-\iiint_V \{\delta\varepsilon\}^T[e]^T\{E\}dV-\iiint_V \{\delta E\}^T[e]\{\varepsilon\}dV\\&-\iiint_V \{\delta E\}^T[\xi]\{E\}dV=\iiint_V \{\delta u\}^T\{\bar{f}_V\}dV+\iint_{S_f}\{\delta u\}^T\{\bar{f}_S\}dS-\iint_{Sq}\delta\phi\sigma_q dS\end{aligned} \tag{10}$$

From of Hamilton principle and eletromechanic variational principle to piezoelectric materials and applying it in the rectangular plate, we obtain the mass matrix of structure without or with the piezoelectric element attached, given by:

$$\left[m_{st}\right]=\rho_{st}h\iint_{A_{st}}\left[N_w\right]^T\left[h_{st}\right]\left[N_w\right]dA_{st} \tag{11}$$

$$\left[m_{pe}\right]=\rho_{st}h_c\iint_{A_{pe}}\left[N_w\right]^T\left[h_{pe}\right]\left[N_w\right]dA_{pe} \tag{12}$$

Where: [m_{st}] is the structure mass matrix and [m_{pe}] is the piezoelectric mass matrix.
The structural and piezoelectric stiffness matrixes are:

$$\left[k_{qq}\right]=\iint_{A_{pe}}\left[B_K\right]^T\left[c_k^{pe}\right]\left[B_k\right]dA_{pe} \tag{13}$$

$$\left[k_{q\phi}\right]=h_b^2\iint_{A_{pe}}\left[B_k\right]^T[e]\left[B_\phi\right]dA_{pe} \tag{14}$$

$$\left[k_{\phi q}\right]=h_b^2\iint_{A_{pe}}\left[B_\phi\right]^T[e]^T\left[B_k\right]dA_{pe} \tag{15}$$

$$\left[k_{\phi\phi}\right]=-h_c\iint\left[B_\phi\right]^T\left[\xi^{\varepsilon}\right]\left[B_\phi\right]dA_{pe} \tag{16}$$

Finally the force and electric loads outside vectors are:

$$\{f_s\}=\iint_{A_{pe}}\left[N_w\right]^T\{\bar{f}_s\}dA \tag{17}$$

Each one of these matrixes the elements are assembled in order to obtain a global matrixes system, that is given by:

$$\begin{cases} [M_{qq}]\{\ddot{q}_i\} + [k_{qq}]\{q_i\} + [k_{q\phi}]\{\phi_i\} = \{F_S\} \\ [k_{\phi q}]\{q_i\} + [k_{\phi\phi}]\{\phi_i\} = \{Q_s\} \end{cases} \tag{18}$$

In the piezoelectric sensor there isn't voltage apply ($Q_s = 0$). So the electric potential yield by sensor is:

$$\{\phi_s\} = -[K_{\phi\phi}]^{-1}[K_{\phi q}]\{q_i\} \tag{19}$$

Replace the Eq. (19) in the Eq. (18), we get the equation global system for a beam with actuator attached, that is:

$$[M_{qq}]\{\ddot{q}_i\} + [K^*_{qq}]\{q_i\} = \{F_S\} + \{F_{el}\} \tag{20}$$

Where:

$$[K^*_{qq}] = [K_{qq}] - [K_{q\phi}][K_{\phi\phi}]^{-1}[K_{\phi q}] \tag{21}$$

$$\{F_{el}\} = -[K_{q\phi}]\{\phi_a\} \tag{22}$$

5 CONTROLLABILITY INDEX

The system controllability comes origin from the modern control theory. It is used to determine if a system can be controlled there being a controller. The decomposition of singular matrix [*S*] yields a measure quantity of system controllability. This index shows the energy that is need in the actuator to control a given input. The Eq. (18) can write in the state space:

$$\begin{aligned} \{\dot{x}\} &= [A]\{x\} + [B]\{u\} \\ \{y\} &= [C]\{x\} \end{aligned} \tag{23}$$

Where:

$$[B] = \begin{bmatrix} [0] \\ [M_{qq}]^{-1}\{F_{el}\} \end{bmatrix} \quad \{y\} = \{\phi_S\} \quad [C] = \left[\left([0]_i - [K_{\phi\phi}]_S^{-1}[K_{\phi q}]_S\right) \quad [0]\right] \tag{24}$$

The rank of state space matrixes, depend of modes numbers that are considered and the actuators number in the structure. From Eq. (23), the control force applied can be written, such as:

$$\{f_c\} = [B]\{u\} \tag{25}$$

Where {u} is the electric potential vector, we have that:

$$\{f_c\}^T\{f_c\}=\{u\}^T[B]^T[B]\{u\} \tag{26}$$

Writing:

$$[B]=[M][S][N]^T \tag{27}$$

Using singular analysis value, where:

$$[M],[N]\in R^n \; e \; [M]^T[M]=[I] \; e \, [N]^T\,[N]=[I] \tag{28}$$

Where:

$$[S]=\begin{bmatrix}\sigma_1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \sigma_k\end{bmatrix} \tag{29}$$

The biggest value σ_i, show the optimum place for actuator.

6 NUMERIC SIMULATION

We simulated a cantilever plate according to following table 1:

Table 1: Properties of simulated cantilever plate

Parameters	Value	Unit
Length	1.5	*m*
Width	1.0	*m*
Thickness	0.075	*m*
Elasticity Module (*E*)	210	*GPa*
Specific density (ρ)	7800	kg/m^3

At first place, we simulated, figure 2, the 1º and 2º mode shape of cantilever plate and we showed also its singular values between 1º and 2º mode, figure 3. So we can see the optimum place to actuators are the ends of the plate, because there the singular values are bigger than others places. According to 1º and 2º mode shape, the maximum displacement is the ends of the plate, therefore in agreement with the singular value.

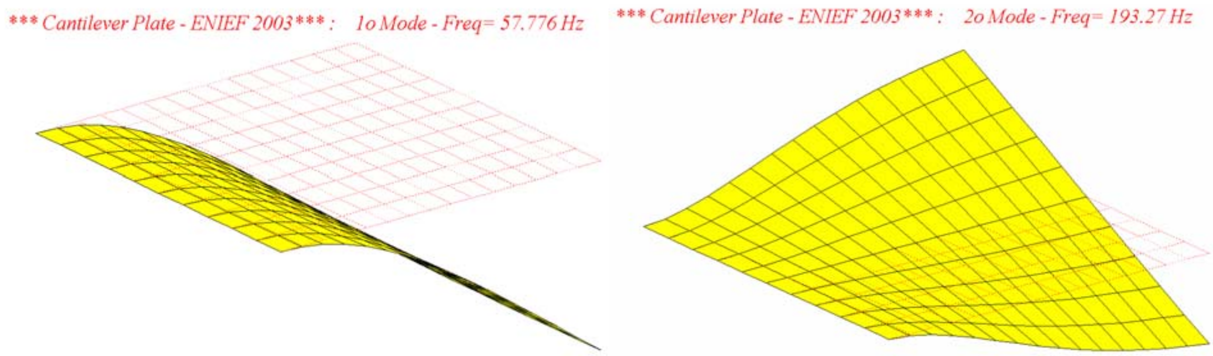

Figure 2: Cantilever in 1^o and 2^o mode shape.

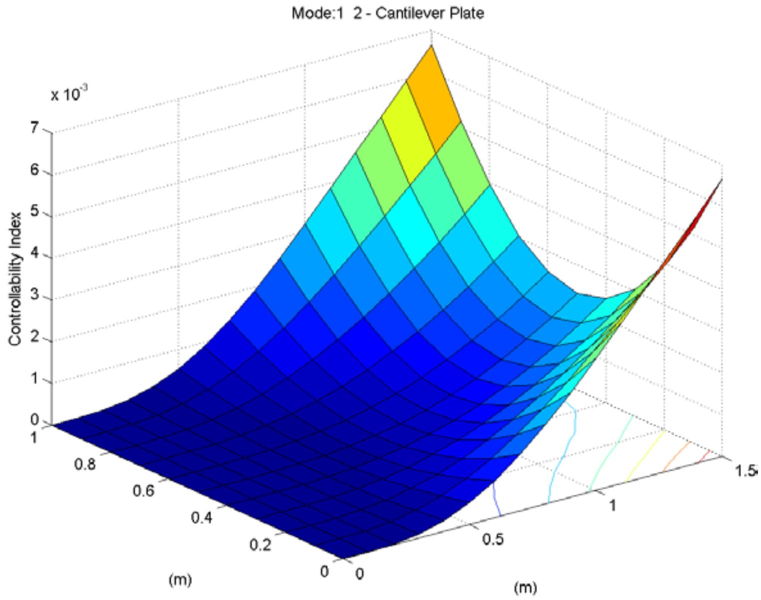

Figure 3: Controllability Index - singular values between 1^o and 2^o mode.

Then simulated, figure 4, the 2^o and 3^o shape mode of cantilever plate and showed singular values between 2^o and 3^o mode, figure 5. The optimum place to actuators are the ends of plate yet and the valley is the nodal line according to 2^o and 3^o shape mode. This area isn't suitable to place actuators.

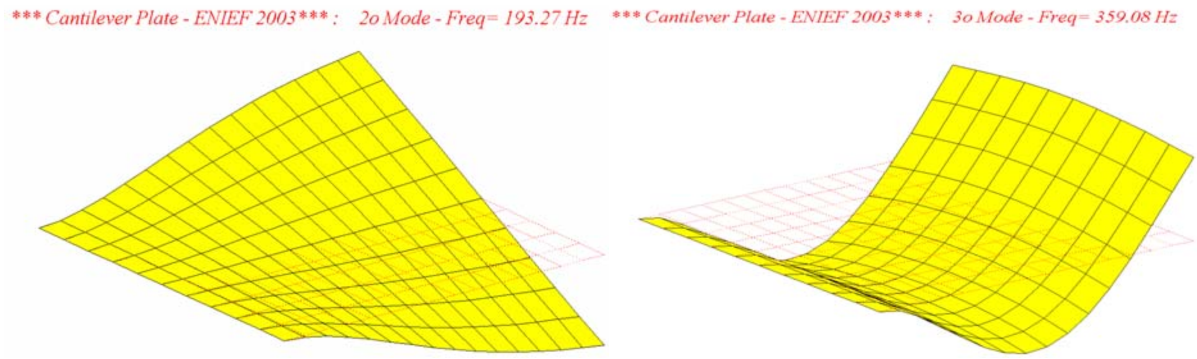

Figure 4: Cantilever in 2^{o} and 3^{o} mode shape.

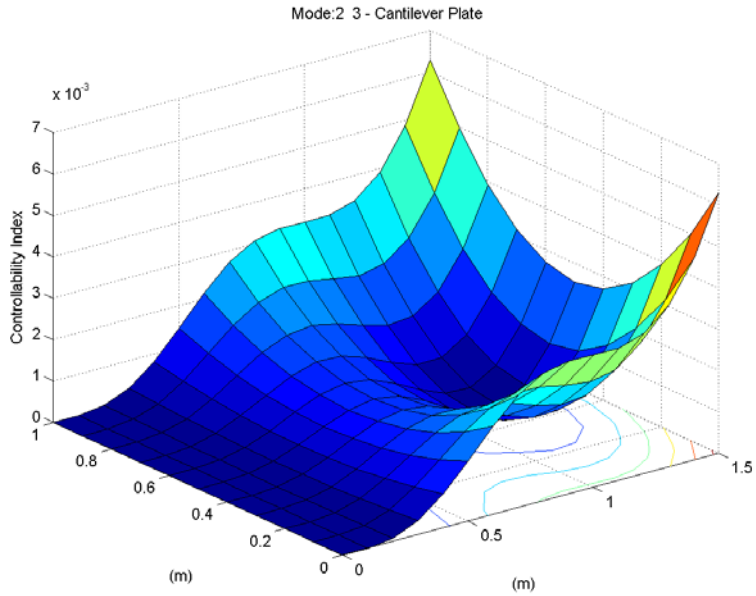

Figure 5: Controllability Index - singular values between 2^{o} and 3^{o} mode.

After, we simulated, figure 6, the 3^{o} and 4^{o} shape mode of cantilever plate and showed the singular values between 3^{o} and 4^{o} mode, figure 7. The optimum places to actuators are the ends of plate and valley is nodal line according to 3^{0} and 4^{0} mode shape.

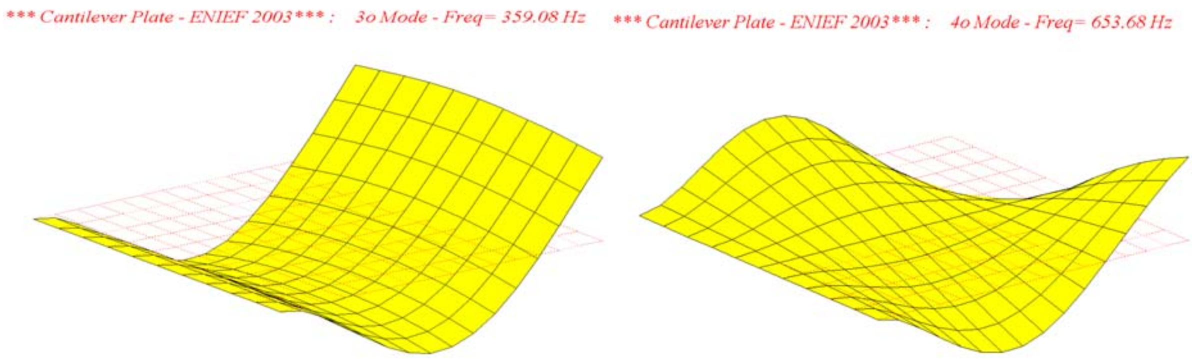

Figure 6: Cantilever in 3°and 4° mode shape.

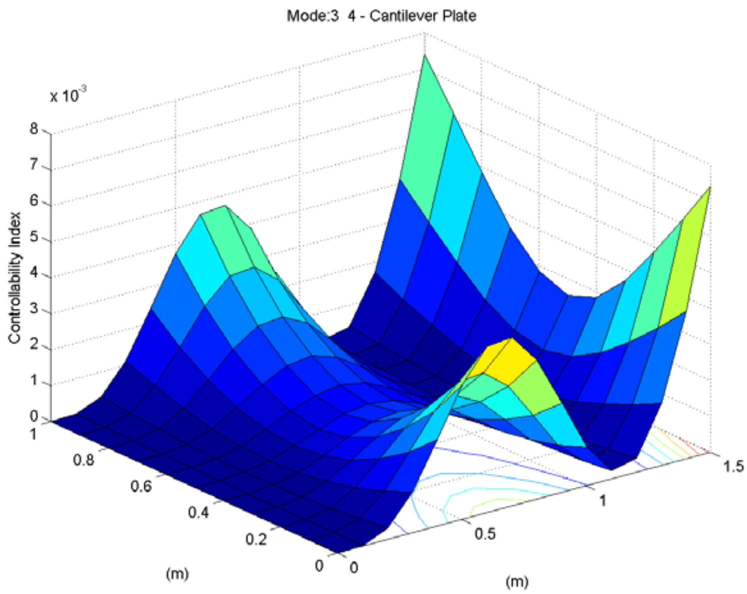

Figure 7: Controllability Index - singular values between 3° and 4° mode.

At last, we simulated, figure 8, the 4^0 and 5^o shape mode of cantilever plate and showed the singular values between the 4^o and 5^o, figure 9. The optimum place to actuators, have been continuing the ends of the plate. The two valley areas, in the singular values graphic, are the nodal lines according to 4^o and 5^0 mode shape. These areas aren't suitable to place actuators.

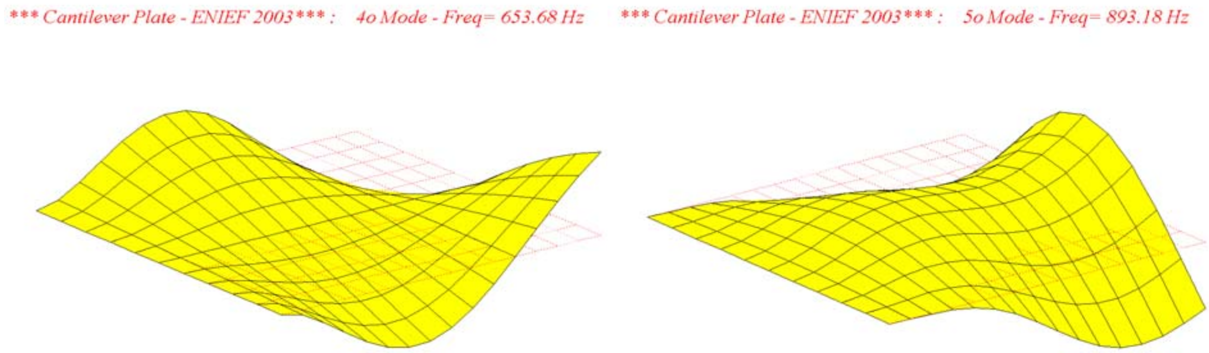

Figure 8: Cantilever in 4°and 5° mode shape.

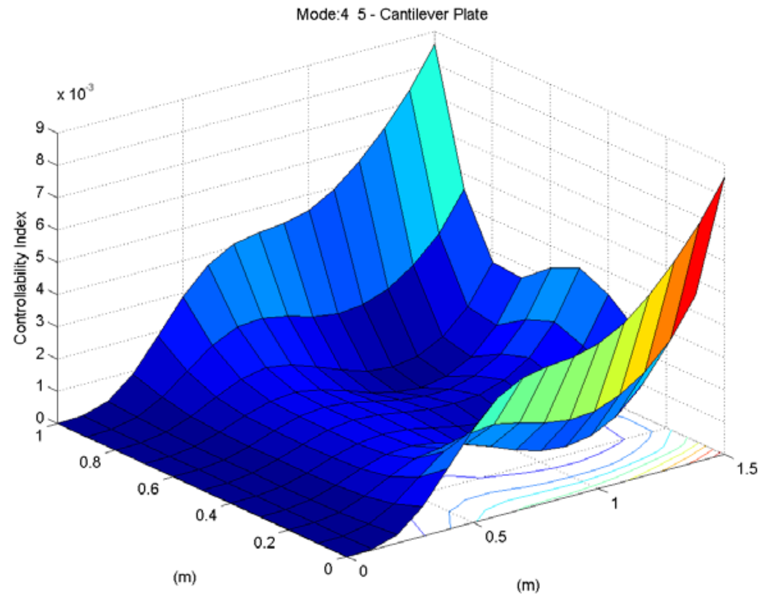

Figure 9: Controllability Index - singular values between 4° and 5° mode.

7 CONCLUSIONS

We showed an index to quantify controllability system of the cantilever plate with piezoelectric attached, in this paper. With this index, it is possible to determine the optimum place to actuators, this way we minimizing the controller effort. We showed that the singular value decomposition, of the control matrix, could be used like measurement to quantify the energy supplied to actuators. The performance of this index was good and the contribution of this paper is extending of work of Wang 2001 from one dimension to two dimensions structure, that is, a cantilever plate.

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